# 2-6 Ratios and Proportions

**Main Ideas**

- Determine whether two ratios form a proportion.
- Solve proportions.

**New Vocabulary**

ratio
proportion
extremes
means
rate

## GET READY for the Lesson

The ingredients in the recipe will make 4 servings of honey frozen yogurt. Keri can use ratios and equations to find the amount of each ingredient needed to make enough yogurt for her club meeting.

| Honey Frozen Yogurt | |
|---|---|
| 2 cups 2% milk | 2 eggs, beaten |
| $\frac{3}{4}$ cup honey | 2 cups plain low-fat |
| 1 dash salt | yogurt |
| | 1 tablespoon vanilla |

**Ratios and Proportions** A **ratio** is a comparison of two numbers by division. The ratio of $x$ to $y$ can be expressed in the following ways.

$$x \text{ to } y \qquad x{:}y \qquad \frac{x}{y}$$

The recipe above states that for 4 servings you need 2 cups of milk. The ratio of servings to milk may be written as 4 to 2, 4:2, or $\frac{4}{2}$. In simplest form, the ratio is written as 2 to 1, 2:1, or $\frac{2}{1}$.

Suppose you wanted to double the recipe to have 8 servings. The amount of milk required would be 4 cups. The ratio of servings to milk is $\frac{8}{4}$. When this ratio is simplified, the ratio is $\frac{2}{1}$. Notice that this ratio is equal to the original ratio. An equation stating that two ratios are equal is called a **proportion**. So, we can state that $\frac{4}{2} = \frac{8}{4}$ is a proportion.

$$\frac{4}{2} \overset{\div 2}{=} \frac{2}{1} \qquad \frac{8}{4} \overset{\div 4}{=} \frac{2}{1}$$

**Study Tip**

**Whole Numbers**
A ratio that is equivalent to a whole number is written with a denominator of 1.

### EXAMPLE Determine Whether Ratios Form a Proportion

**1** **Determine whether the ratios $\frac{4}{5}$ and $\frac{24}{30}$ form a proportion.**

$$\frac{4}{5} \overset{\div 1}{=} \frac{4}{5} \qquad \frac{24}{30} \overset{\div 6}{=} \frac{4}{5}$$

The ratios are equal. Therefore, they form a proportion.

**CHECK Your Progress**

**Determine whether each pair of ratios forms a proportion. Write *yes* or *no*.**

**1A.** $\frac{6}{10}, \frac{2}{5}$ no

**1B.** $\frac{1}{6}, \frac{5}{30}$ yes

There are special names for the terms in a proportion.

0.8 and 0.7 are called the **means**. They are the middle terms of the proportion.

$$0.4 : 0.8 = 0.7 : 1.4$$

0.4 and 1.4 are called the **extremes**. They are the first and last terms of the proportion.

**Vocabulary Link**

**Extremes**

**Everyday Use** something at one end or the other of a range, as in extremes of heat and cold

**Math Use** the first and last terms of a proportion

## KEY CONCEPT *Means–Extremes Property of Proportion*

**Words** In a proportion, the product of the extremes is equal to the product of the means.

**Symbols** If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

**Examples** Since $\frac{2}{4} = \frac{1}{2}$, $2(2) = 4(1)$ or $4 = 4$.

Another way to determine whether two ratios form a proportion is to use cross products. If the cross products are equal, then the ratios form a proportion.

## EXAMPLE Use Cross Products

**Study Tip**

**Cross Products** When you find cross products, you are said to be *cross multiplying*.

**2** **Use cross products to determine whether each pair of ratios forms a proportion.**

**a.** $\frac{0.4}{0.8}, \frac{0.7}{1.4}$

$\frac{0.4}{0.8} \stackrel{?}{=} \frac{0.7}{1.4}$ Write the equation.

$0.4(1.4) \stackrel{?}{=} 0.8(0.7)$ Find the cross products.

$0.56 = 0.56$ Simplify.

The cross products are equal, so the ratios form a proportion.

**b.** $\frac{6}{8}, \frac{24}{28}$

$\frac{6}{8} \stackrel{?}{=} \frac{24}{28}$ Write the equation.

$6(28) \stackrel{?}{=} 8(24)$ Find the cross products.

$168 \neq 192$ Simplify.

The cross products are not equal, so the ratios do not form a proportion.

**CHECK Your Progress**

**2A.** $\frac{0.2}{1.8}, \frac{1}{0.9}$ no

**2B.** $\frac{15}{36}, \frac{35}{42}$ no

**Solve Proportions** To solve proportions that involve a variable, use cross products and the techniques used to solve other equations.

## EXAMPLE Solve a Proportion

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**3** **Solve the proportion $\frac{n}{15} = \frac{24}{16}$.**

| | |
|---|---|
| $\frac{n}{15} = \frac{24}{16}$ | Original equation |
| $16(n) = 15(24)$ | Find the cross products. |
| $16n = 360$ | Simplify. |
| $\frac{16n}{16} = \frac{360}{16}$ | Divide each side by 16. |
| $n = 22.5$ | Simplify. |

**CHECK Your Progress**

**Solve each proportion. If necessary, round to the nearest hundredth.**

**3A.** $\frac{r}{8} = \frac{25}{40}$ 5

**3B.** $\frac{3.2}{4} = \frac{2.6}{n}$ 3.25

The ratio of two measurements having different units of measure is called a **rate.** For example, a price of \$1.99 per dozen eggs, a speed of 55 miles per hour, and a salary of \$30,000 per year are all rates. Proportions are often used to solve problems involving rates.

## Real-World EXAMPLE

**Study Tip**

To ensure that the proportion is set up correctly, you should label the units. For example, $\frac{15 \text{ mi}}{2 \text{ hrs}} = \frac{25 \text{ mi}}{x \text{ hrs}}$.

**4** **BICYCLING Trent goes on a 30-mile bike ride every Saturday. He rides the distance in 4 hours. At this rate, how far can he ride in 6 hours?**

**Explore** Let $m$ represent the number of miles Trent can ride in 6 hours.

**Plan** Write a proportion for the problem using rates.

miles → / hours → $\frac{30}{4} = \frac{m}{6}$ ← miles / ← hours

**Solve** Estimate: If he rides 30 miles in 4 hours, then he would ride 60 miles in 8 hours. So, in 6 hours, he would ride between 30 and 60 miles.

| | |
|---|---|
| $\frac{30}{4} = \frac{m}{6}$ | Original proportion |
| $30(6) = 4(m)$ | Find the cross products. |
| $180 = 4m$ | Simplify. |
| $\frac{180}{4} = \frac{4m}{4}$ | Divide each side by 4. |
| $45 = m$ | Simplify. |

**Check** Check the reasonableness of the solution. If Trent rides 30 miles in 4 hours, he rides 7.5 miles in 1 hour. So, in 6 hours, Trent can ride 6 × 7.5 or 45 miles. The answer is correct.

**CHECK Your Progress**

4. **EXERCISE** It takes 7 minutes for Isabella to walk around the track twice. At this rate, how many times can she walk around the track in a half hour? **about 8.6 times**

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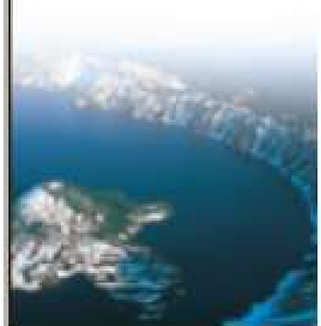
A ratio or rate called a **scale** compares the size of a model to the actual size of the object using a proportion. Maps and blueprints are two common scale drawings.



**Real-World Link**

Crater Lake is a volcanic crater in Oregon that was formed by an explosion 42 times the blast of Mount St. Helens.

**Source:** travel.excite.com

## Real-World EXAMPLE

**5** **CRATER LAKE** **The scale of a map for Crater Lake National Park is 2 inches = 9 miles. The distance between Discovery Point and Phantom Ship Overlook on the map is about $1\frac{3}{4}$ inches. What is the distance $d$ between these two places?**

scale → $\frac{2}{9} = \frac{1\frac{3}{4}}{d}$ ← scale
actual → ← actual

$2(d) = 9\left(1\frac{3}{4}\right)$ Find the cross products.

$2d = \frac{63}{4}$ Simplify.

$2d \div 2 = \frac{63}{4} \div 2$ Divide each side by 2.

$d = \frac{63}{8}$ or $7\frac{7}{8}$ Simplify.

The actual distance is about $7\frac{7}{8}$ miles.

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**CHECK Your Progress**

5. **AIRPLANES** On a model airplane, the scale is 5 centimeters = 2 meters. If the wingspan of the model is 28.5 centimeters, what is the wingspan of the actual airplane? **11.4 m**