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[► For an interactive version of both investigations, see the **Dynamic Geometry Exploration Special Angles on Parallel Lines** at math.kendallhunt.com/DG ◀]

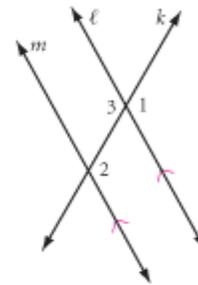


Developing Proof You used inductive reasoning to discover all three parts of the Parallel Lines Conjecture. However, if you accept any one of them as true, you can use deductive reasoning to show that the others are true.

Read the example below. Before you read the solution, write a deductive argument with your group. Remember the reasoning strategy of applying previous conjectures and definitions. Then compare your solution to the one presented. ■

EXAMPLE

Write a deductive argument explaining why the Alternate Interior Angles Conjecture is true. Assume that the Vertical Angles Conjecture and Corresponding Angles Conjecture are both true.



► **Solution**

Deductive Argument

In the diagram, lines ℓ and m are parallel and intersected by transversal k . If the Corresponding Angles Conjecture is true, the corresponding angles are congruent.

$$\angle 1 \cong \angle 2$$

If the Vertical Angles Conjecture is true, the vertical angles are congruent.

$$\angle 1 \cong \angle 3$$

Because both $\angle 2$ and $\angle 3$ are congruent to $\angle 1$, they're congruent to each other.

$$\angle 2 \cong \angle 3$$

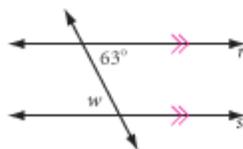
Alternate interior angles 2 and 3 are congruent. Therefore, if the corresponding angles are congruent, then the alternate interior angles are congruent. ■

It helps to visualize each statement and to mark all congruences you know on your paper.

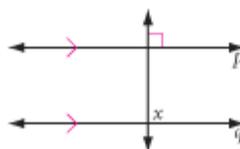


EXERCISES

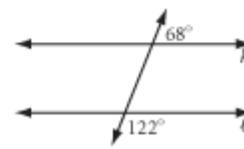
1. $w = ?$



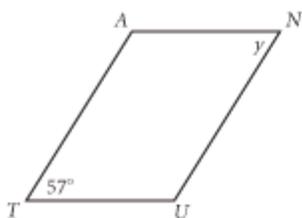
2. $x = ?$



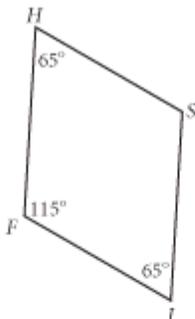
3. Is line k parallel to line ℓ ?



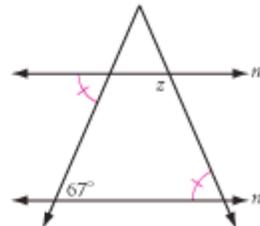
4. Quadrilateral *TUNA* is a parallelogram.
 $y = ?$ (h)



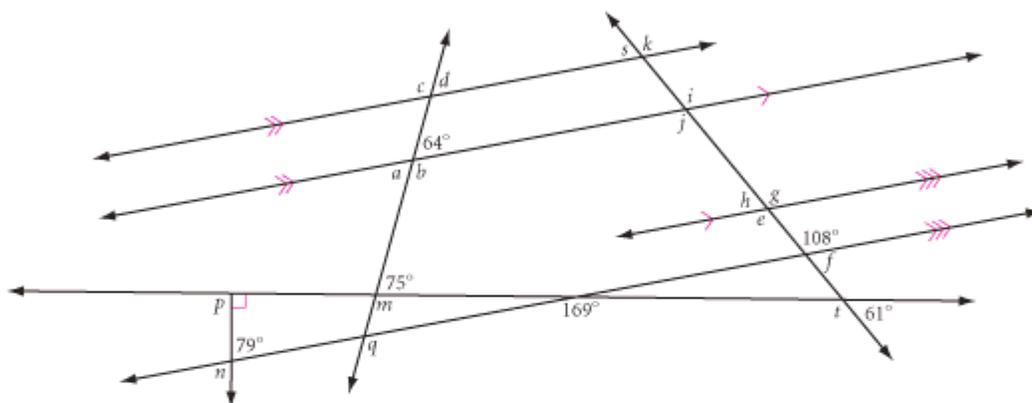
5. Is quadrilateral *FISH* a parallelogram?



6. $m \parallel n$
 $z = ?$ (h)



7. **Developing Proof** Trace the diagram below. Calculate each lettered angle measure. Explain how you determined measures n , p , and q . (h)

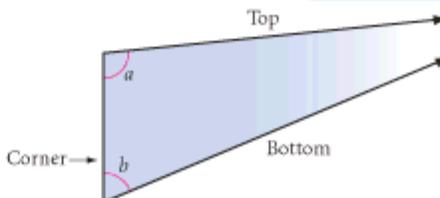


8. **Developing Proof** Write a deductive argument explaining why the Alternate Exterior Angles Conjecture is true. Assume that the Vertical Angles Conjecture and Corresponding Angles Conjecture are both true.

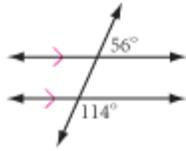
Cultural CONNECTION



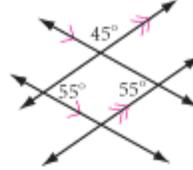
Sculptor Maya Lin designed the Vietnam Veterans Memorial Wall in Washington, D.C. Engraved in the granite wall are the names of United States armed forces service members who died in the Vietnam War or remain missing in action. Do the top and bottom of the wall meet in the distance, or are they parallel? How could you know from angle measures a and b in the diagram below? To learn more about the Memorial Wall and Lin's other projects, visit math.kendallhunt.com/DG



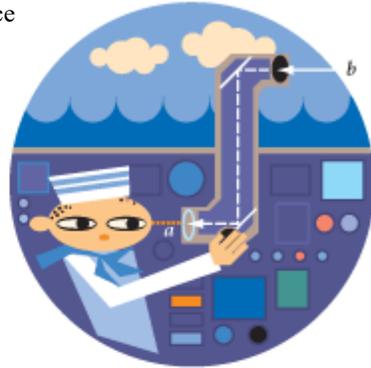
9. **Developing Proof** What's wrong with this picture?



10. **Developing Proof** What's wrong with this picture?



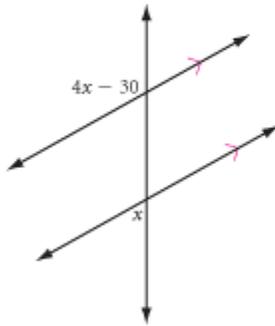
11. A periscope permits a sailor on a submarine to see above the surface of the ocean. This periscope is designed so that the line of sight a is parallel to the light ray b . The middle tube is perpendicular to the top and bottom tubes. What are the measures of the incoming and outgoing angles formed by the light rays and the mirrors in this periscope? Are the surfaces of the mirrors parallel? How do you know? **(h)**



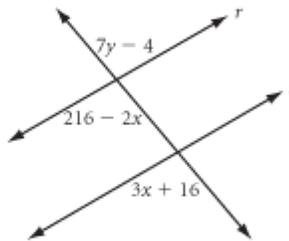
12. Draw a line on your paper and label it line AB . Place a point P about one to two inches from the line. Draw another line (a transversal) that passes through point P and line AB . Use your straightedge and protractor to draw line PQ that is parallel to line AB . Explain your method and why you know the lines are parallel.

13. Is the following statement true? "If yesterday was part of the weekend, then tomorrow is a school day." Write the converse of the statement. Is the converse true?

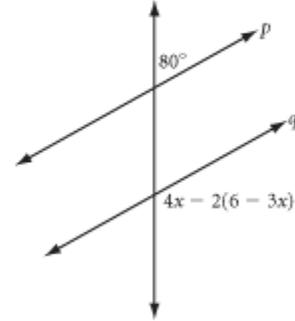
14. Find x .



15. If $r \parallel s$, find y .



16. If $x = 12^\circ$, is $p \parallel q$?



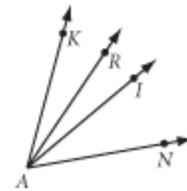
Review

17. What type (or types) of triangle has one or more lines of symmetry?

18. What type (or types) of quadrilateral has only rotational symmetry? **(h)**

19. If D is the midpoint of \overline{AC} and C is the midpoint of \overline{BD} , what is the length of \overline{AB} if $BD = 12$ cm?

20. If \overline{AI} is the angle bisector of $\angle KAN$ and \overline{AR} is the angle bisector of $\angle KAI$, what is $m\angle RAN$ if $m\angle RAK = 13^\circ$?



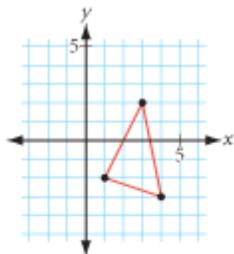
21. If everyone in the town of Skunk's Crossing (population 84) has a telephone, how many different lines are needed to connect all the phones to each other?



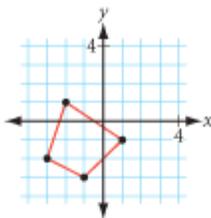
22. How many squares of all sizes are in a 4-by-4 grid of squares? (There are more than 16!) **(h)**

For Exercises 23–25, draw each polygon on graph paper. Relocate the vertices according to the rule. Connect the new points to form a new polygon. Describe what happened to the figure. Is the new polygon congruent to the original?

23. **Rule:** Subtract 1 from each x -coordinate. 



24. **Rule:** Reverse the sign of each x - and y -coordinate.



25. **Rule:** Switch the x - and y -coordinates. Pentagon *LEMON* with vertices:

- $L(-4, 2)$
- $E(-4, -3)$
- $M(0, -5)$
- $O(3, 1)$
- $N(-1, 4)$

26. Assume the pattern of blue and yellow shaded **T**'s continues. Copy and complete the table for blue and yellow squares and for the total number of squares. 

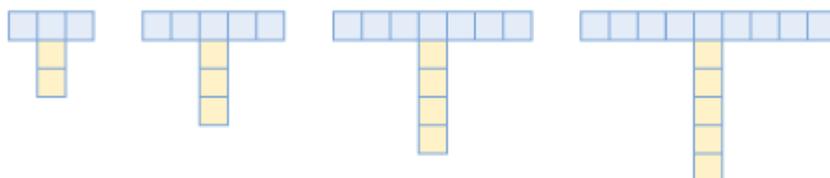


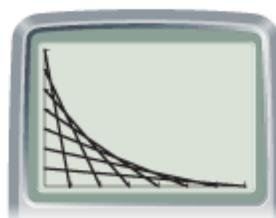
Figure number	1	2	3	4	5	6	...	n	...	35
Number of yellow squares	2						
Number of blue squares	3						
Total number of squares	5						

project

LINE DESIGNS

Can you use your graphing calculator to make the line design shown at right? You'll need to recall some algebra. Here are some hints.

- The design consists of the graphs of seven lines.
- The equation for one of the lines is $y = -\frac{1}{7}x + 1$.
- The x - and y -ranges are set to minimums of 0 and maximums of 7.
- There's a simple pattern in the slopes and y -intercepts of the lines.



Experiment with equations to create your own line design.

Your project should include

- ▶ A set of equations for the design shown here.
- ▶ Your own line design and a set of equations for it.