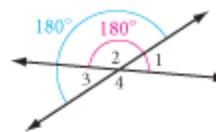▶ **Solution**

You can see from the diagram that the sum of the measures of angles 1 and 2 is equal to the sum of the measures of angles 2 and 3 because they are both linear pairs. Because angle 2 is the same in both sums, angle 1 must equal angle 3. To write a deductive argument, go through this logic one step at a time.

Deductive Argument

For any linear pair of angles, their measures add up to 180°.

$$m\angle 1 + m\angle 2 = 180°$$

$$m\angle 2 + m\angle 3 = 180°$$

Since both expressions on the left equal 180°, they equal each other.

$$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$$

Subtract $m\angle 2$ from both sides of the equation.

$$m\angle 1 = m\angle 3$$

Vertical angles 1 and 3 have equal measures, so they are congruent. ■

You discovered the Vertical Angles Conjecture: If two angles are vertical angles, then they are congruent. Does that also mean that all congruent angles are vertical angles? The **converse** of an "if-then" statement switches the "if" and "then" parts. The converse of the Vertical Angles Conjecture may be stated: If two angles are congruent, then they are vertical angles. Is this converse statement true? Remember that if you can find even one counterexample, like the diagram below, then the statement is false.
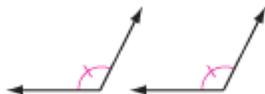
Therefore, the converse of the Vertical Angles Conjecture is false.

Practice Your Skills

EXERCISES

You will need

Geometry software
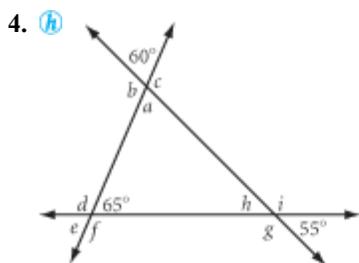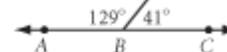for Exercise 12

▶ Without using a protractor, but with the aid of your two new conjectures, find the measure of each lettered angle in Exercises 1–5. Copy the diagrams so that you can write on them. List your answers in alphabetical order.
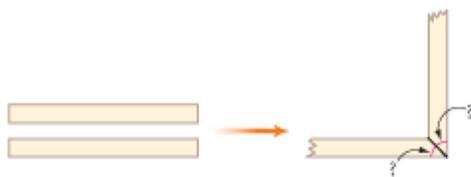
1.

2.

3.

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4. (*h*)

5.
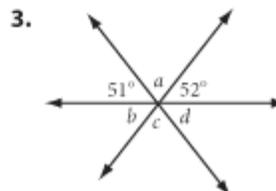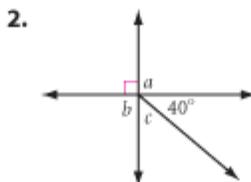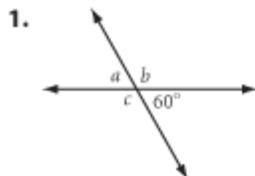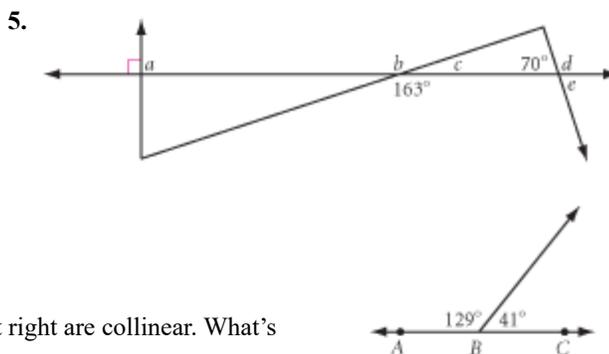
6. *Developing Proof* Points *A, B,* and *C* at right are collinear. What's wrong with this picture?

7. Yoshi is building a cold frame for his plants. He wants to cut two wood strips so that they'll fit together to make a right-angled corner. At what angle should he cut ends of the strips?

8. A tree on a 30° slope grows straight up. What are the measures of the greatest and smallest angles the tree makes with the hill? Explain.
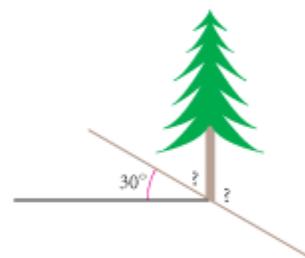
9. You discovered that if a pair of angles is a linear pair, then the angles are supplementary. Does that mean that all supplementary angles form a linear pair of angles? Is the converse true? If not, sketch a counterexample.

10. If two congruent angles are supplementary, what must be true of the two angles? Make a sketch, then complete the following conjecture: If two angles are both congruent and supplementary, then   ? .

11. *Developing Proof* Using algebra, write a paragraph proof that explains why the conjecture from Exercise 10 is true.

12. *Technology* Use geometry software to construct two intersecting lines. Measure a pair of vertical angles. Use the software to calculate the ratio of their measures. What is the ratio? Drag one of the lines. Does the ratio ever change? Does this demonstration convince you that the Vertical Angles Conjecture is true? Does it explain why it is true?

▶ Review

For Exercises 13–17, sketch, label, and mark the figure.

13. Scalene obtuse triangle *PAT* with *PA* = 3 cm, *AT* = 5 cm, and ∠*A* an obtuse angle

**14.** A quadrilateral that has rotational symmetry, but not reflectional symmetry

15. A circle with center at *O* and radii $\overline{OA}$ and $\overline{OT}$ creating a minor arc $\overset{\frown}{AT}$

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16. A pyramid with an octagonal base

17. A 3-by-4-by-6-inch rectangular solid rests on its smallest face. Draw lines on the three visible faces to show how you can divide it into 72 identical smaller cubes.

18. Miriam the Magnificent placed four cards face up (the first four cards shown below). Blindfolded, she asked someone from her audience to come up to the stage and turn one card 180°.
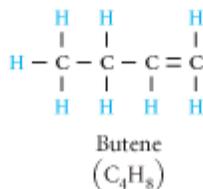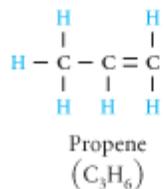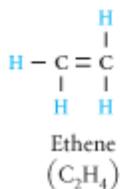
Before turn   After turn
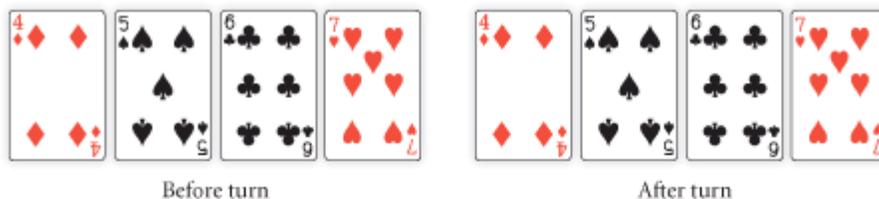
Miriam removed her blindfold and claimed she was able to determine which card was turned 180°. What is her trick? Can you figure out which card was turned? Explain.

19. If a pizza is cut into 16 congruent pieces, how many degrees are in each angle at the center of the pizza?

20. Paulus Gerdes, a mathematician from Mozambique, uses traditional *lusona* patterns from Angola to practice inductive thinking. Shown below are three *sona* designs. Sketch the fourth *sona* design, assuming the pattern continues.
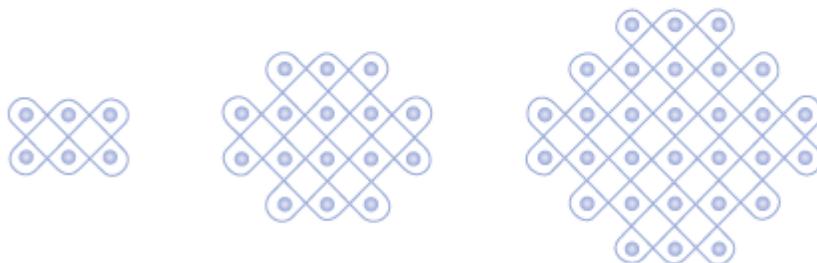
21. Hydrocarbon molecules in which all the bonds between the carbon atoms are single bonds except one double bond are called *alkenes*. The first three alkenes are modeled below.

$$H - \underset{\underset{H}{|}}{\overset{\overset{H}{|}}{C}} = \underset{\underset{H}{|}}{\overset{\overset{H}{|}}{C}}$$

Ethene (C_2H_4)

Propene (C_3H_6)

Butene (C_4H_8)

Sketch the alkene with eight carbons in the chain. What is the general rule for alkenes $(C_nH_?)$? In other words, if there are n carbon atoms (C), how many hydrogen atoms (H) are in the alkene?

Science CONNECTION

Organic chemistry is the study of carbon compounds and their reactions. Drugs, vitamins, synthetic fibers, and food all contain organic molecules. To learn about new advances in organic chemistry, go to **math.kendallhunt.com/DG**

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