Example: Irrational Roots

Solve $x^2 - 10x + 25 = 7$ by taking the square root of each side. Round to the nearest tenth if necessary.

$x^2 - 10x + 25 = 7$	Original equation
$(x-5)^2 = 7$	$x^2 - 10x + 25$ is a perfect square trinomial.
$\sqrt{(x-5)^2} = \sqrt{7}$	Take the square root of each side.
$\lvert x-5\rvert = \sqrt{7}$	Simplify.
$x - 5 = \pm\sqrt{7}$	Definition of absolute value
$x - 5 + 5 = \pm\sqrt{7} + 5$	Add 5 to each side.
$x = 5 \pm \sqrt{7}$	Simplify.

Use a calculator to evaluate each value of x.

$x = 5 + \sqrt{7}$	or	$x = 5 - \sqrt{7}$	Write each solution.
≈ 7.6		≈ 2.4	Simplify.

The solution set is $\{2.4, 7.6\}$.

Check your progress:

1) $m^2 + 18m + 81 = 90$

Since we are told we can take the square root of each side, we know that the left side of the equation is a perfect square trinomial and will factor to $\left(x + \frac{b}{2}\right)^2$.

$(m+9)^2 = 90$

$\sqrt{(m+9)^2} = \sqrt{90}$

$m + 9 = \pm\sqrt{90}$

$-9 \quad -9$

$m = -9 \pm \sqrt{90}$

$m \approx -18.5$ or $m \approx 0.5$

Example: Solve an equation by Completing the Square

Solve $a^2 - 14a + 3 = -10$ by completing the square.

Isolate the a^2 and a terms. Then complete the square and solve.

$a^2 - 14a + 3 = -10$	Original equation
$a^2 - 14a + 3 - 3 = -10 - 3$	Subtract 3 from each side.
$a^2 - 14a = -13$	Simplify.
$a^2 - 14a + 49 = -13 + 49$	Since $\left(\frac{-14}{2}\right)^2 = 49$, add 49 to each side.
$(a-7)^2 = 36$	Factor $a^2 - 14a + 49$.
$a - 7 = \pm 6$	Take the square root of each side.
$a = 7 \pm 6$	Add 7 to each side.

$a = 7 + 6$	or	$a = 7 - 6$	Separate the solutions.
$= 13$		$= 1$	Simplify.

The solution set is $\{1, 13\}$.

Step 1: Make sure the coefficient on the quadratic term is 1.

Step 2: Leave a blank spot after the linear term and any constant $(x^2 + bx \qquad) + d \qquad = f$

Step 3: Determine what number to add to the blank. It should always be half the linear term squared $c = \left(\frac{b}{2}\right)^2$. Remember to make sure that your net change on the equation is zero.

$$x^2 + bx + c + d - c = f$$

Step 4: Factor the perfect square trinomial and simplify. $\left(x + \frac{b}{2}\right)^2 + d - c = f$

Step 5: Solve the equation

<u>Check your progress:</u>

1) $x^2 - 8x = 4$

Step 1: Since the coefficient on the quadratic term is 1, this step is complete.

Step 2:

$(x^2 - 8x \qquad) \qquad = 4$

Step 3:

$\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

$(x^2 - 8x + 16) - 16 = 4$

Step 4:

$(x - 4)^2 - 16 = 4$

Step 5:

$(x - 4)^2 - 16 = 4$

$\qquad +16 \quad +16$

$(x - 4)^2 = 20$

$\sqrt{(x - 4)^2} = \sqrt{20}$

$x - 4 = \pm\sqrt{20}$

$\quad +4 \quad +4$

$x = 4 \pm \sqrt{20}$

$x \approx -0.5$ or $x \approx 8.5$

2) $3n^2 - 18n = 30$

Step 1: Since the coefficient on the quadratic term is not 1, we must factor out that coefficient.

$3(n^2 - 6n) = 30$

Step 2:

$3(n^2 - 6n \qquad) \qquad = 30$

Step 3:

$\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

$3(n^2 - 6n + 9) - 27 = 30$

**I am subtracting 27 because I actually added 27 if you multiply the 3 back through the set of parentheses.

Step 4:

$3(n-3)^2 - 27 = 30$

Step 5:

$3(n-3)^2 - 27 = 30$

$\qquad +27 \quad +27$

$3(n-3)^2 = 57$

$\frac{3(n-3)^2}{3} = \frac{57}{3}$

$(n-3)^2 = 19$

$\sqrt{(n-3)^2} = \sqrt{19}$

$n - 3 = \pm\sqrt{19}$

$\quad +3 \quad +3$

$x = 3 \pm \sqrt{19}$

$x \approx -1.4$ or $x \approx 7.4$

<u>Practice</u>

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

1) $b^2 - 6b + 9 = 25$

Since we are told we can take the square root of each side, we know that the left side of the equation is a perfect square trinomial and will factor to $\left(x+\frac{b}{2}\right)^2$.

$(b-3)^2 = 25$

$\sqrt{(b-3)^2} = \sqrt{25}$

$b-3 = \pm 5$

$\quad +3 \quad +3$

$b = 3 \pm 5$

$b = 8$ or $b = -2$

2) $m^2 + 14m + 49 = 20$

Since we are told we can take the square root of each side, we know that the left side of the equation is a perfect square trinomial and will factor to $\left(x+\frac{b}{2}\right)^2$.

$(m+7)^2 = 20$

$\sqrt{(m+7)^2} = \sqrt{20}$

$m+7 = \pm\sqrt{20}$

$\quad -7 \quad -7$

$m = -7 \pm \sqrt{20}$

$m \approx -11.5$ or $m \approx -2.5$

Find the value of c that makes each trinomial a perfect square.

3) $a^2 - 12a + c$

This problem is asking us to find the value we would use in step 3 of our completing the square process.

$\left(\frac{-12}{2}\right)^2 = (-6)^2 = 36$

$c = 36$

4) $t^2 + 5t + c$

This problem is asking us to find the value we would use in step 3 of our completing the square process.

$\left(\frac{5}{2}\right)^2 = \frac{25}{4} = 6\frac{1}{4}$

$c = 6\frac{1}{4}$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

5) $c^2 - 6c = 7$

Step 1: Since the coefficient on the quadratic term is 1, this step is complete.

Step 2:

$(c^2 - 6c \qquad) \qquad = 7$

Step 3:

$\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

$(c^2 - 6c + 9) - 9 = 7$

Step 4:

$(c - 3)^2 - 9 = 7$

Step 5:

$(c - 3)^2 - 9 = 7$

$+9 \quad +9$

$(c - 3)^2 = 16$

$\sqrt{(c - 3)^2} = \sqrt{16}$

$c - 3 = \pm 4$

$+3 \quad +3$

$c = 3 \pm 4$

$c = 7$ or $c = -1$

6) $x^2 + 7x = -12$

Step 1: Since the coefficient on the quadratic term is 1, this step is complete.

Step 2:

$(x^2 + 7x \qquad) \qquad = -12$

Step 3:

$\left(\frac{7}{2}\right)^2 = \frac{49}{4} = 12\frac{1}{4}$

$\left(x^2 + 7x + 12\frac{1}{4}\right) - 12\frac{1}{4} = -12$

Step 4:

$\left(x + \frac{7}{2}\right)^2 - 12\frac{1}{4} = -12$

Step 5:

$\left(x + \frac{7}{2}\right)^2 - 12\frac{1}{4} = -12$

$+12\frac{1}{4} \quad + 12\frac{1}{4}$

$\left(x + \frac{7}{2}\right)^2 = \frac{1}{4}$

$\sqrt{\left(x + \frac{7}{2}\right)^2} = \sqrt{\frac{1}{4}}$

$x + \frac{7}{2} = \pm\frac{1}{2}$

$-\frac{7}{2} \quad -\frac{7}{2}$

$x = -\frac{7}{2} \pm \frac{1}{2}$

$x = -4$ or $x = -3$

7) $v^2 + 14v - 9 = 6$

Step 1: Since the coefficient on the quadratic term is 1, this step is complete.

Step 2:

$(v^2 + 14v \qquad) - 9 \qquad = 6$

Step 3:

$\left(\frac{14}{2}\right)^2 = (7)^2 = 49$

$(v^2 + 14v + 49) - 9 - 49 = 6$

Step 4:

$(v + 7)^2 - 58 = 6$

Step 5:

$(v + 7)^2 - 58 = 6$

$\qquad +58 \quad +58$

$(v + 7)^2 = 64$

$\sqrt{(v + 7)^2} = \sqrt{64}$

$v + 7 = \pm 8$

$\quad -7 \quad -7$

$v = -7 \pm 8$

$v = 1$ or $v = -15$

8) $r^2 - 4r = 2$

Step 1: Since the coefficient on the quadratic term is 1, this step is complete.

Step 2:

$(r^2 - 4r \qquad) \qquad = 2$

Step 3:

$\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$

$(r^2 - 4r + 4) - 4 = 2$

Step 4:

$(r - 2)^2 - 4 = 2$

Step 5:

$(r - 2)^2 - 4 = 2$

$\qquad +4 \quad +4$

$(r - 2)^2 = 6$

$\sqrt{(r - 2)^2} = \sqrt{6}$

$r - 2 = \pm\sqrt{6}$

$\quad +2 \quad +2$

$r = 2 \pm \sqrt{6}$

$r \approx -0.4$ or $r \approx 4.4$

9) $4a^2 + 9a - 1 = 0$

Step 1: Since the coefficient on the quadratic term is not 1, we must factor out that coefficient.

$$4\left(a^2 + \frac{9}{4}a\right) - 1 = 0$$

Step 2:

$$4\left(a^2 + \frac{9}{4}a \qquad\right) - 1 \qquad = 0$$

Step 3:

$$\left(\frac{\frac{9}{4}}{2}\right)^2 = \left(\frac{9}{8}\right)^2 = \frac{81}{64}$$

$$4\left(a^2 + \frac{9}{4}a + \frac{81}{64}\right) - 1 - \frac{81}{16} = 0$$

**I am subtracting $\frac{81}{16}$ because I actually added $\frac{81}{16}$ if you multiply the 4 back through the set of parentheses.

Step 4:

$$4\left(a + \frac{9}{8}\right)^2 - \frac{97}{16} = 0$$

**$-1 = -\frac{16}{16}$ $\quad -\frac{16}{16} - \frac{81}{16} = -\frac{97}{16}$

Step 5:

$$4\left(a + \frac{9}{8}\right)^2 - \frac{97}{16} = 0$$

$$+\frac{97}{16} \quad +\frac{97}{16}$$

$$4\left(a + \frac{9}{8}\right)^2 = \frac{97}{16}$$

$$\frac{4\left(a + \frac{9}{8}\right)^2}{4} = \frac{\frac{97}{16}}{4}$$

$$\left(a + \frac{9}{8}\right)^2 = \frac{97}{64}$$

$$\sqrt{\left(a + \frac{9}{8}\right)^2} = \sqrt{\frac{97}{64}}$$

$$a + \frac{9}{8} = \pm\sqrt{\frac{97}{64}}$$

$$-\frac{9}{8} \quad -\frac{9}{8}$$

$a = -\frac{9}{8} \pm \sqrt{\frac{97}{64}}$

$a \approx 0.1$or $a \approx -2.4$

10) $7 = 2p^2 - 5p + 8$

Let's switch the order of the equation so that it looks more familiar.

$2p^2 - 5p + 8 = 7$

Step 1: Since the coefficient on the quadratic term is not 1, we must factor out that coefficient.

$2\left(p^2 - \frac{5}{2}p\right) + 8 = 7$

Step 2:

$2\left(p^2 - \frac{5}{2}p \qquad\right) + 8 \qquad = 7$

Step 3:

$\left(\frac{-\frac{5}{2}}{2}\right)^2 = \left(-\frac{5}{4}\right)^2 = \frac{25}{16}$

$2\left(p^2 - \frac{5}{2}p + \frac{25}{16}\right) + 8 - \frac{25}{8} = 7$ **I am subtracting $\frac{25}{8}$ because I actually added $\frac{25}{8}$ if you multiply the 2 back through the set of parentheses.

Step 4:

$2\left(p - \frac{5}{4}\right)^2 + \frac{39}{8} = 7$ $**8 = \frac{64}{8}$ $\frac{64}{8} - \frac{25}{8} = \frac{39}{8}$

Step 5:

$2\left(p - \frac{5}{4}\right)^2 + \frac{39}{8} = \frac{56}{8}$

$-\frac{39}{8} \quad -\frac{39}{8}$

$2\left(p - \frac{5}{4}\right)^2 = \frac{17}{8}$

$\frac{2\left(p - \frac{5}{4}\right)^2}{2} = \frac{\frac{17}{8}}{2}$

$\left(p - \frac{5}{4}\right)^2 = \frac{17}{16}$

$$\sqrt{\left(p-\frac{5}{4}\right)^2}=\sqrt{\frac{17}{16}}$$

$$p-\frac{5}{4}=\pm\sqrt{\frac{17}{16}}$$

$$p-\frac{5}{4}=\pm\sqrt{\frac{17}{16}}$$

$$+\frac{5}{4}\quad +\frac{5}{4}$$

$$p=\frac{5}{4}\pm\sqrt{\frac{17}{16}}$$

$p\approx 2.3$ or $p\approx 0.2$