## Example: Irrational Roots

Solve $x^{2}-10 x+25=7$ by taking the square root of each side.
Round to the nearest tenth if necessary.

$$
\begin{aligned}
x^{2}-10 x+25 & =7 & & \text { Original equation } \\
(x-5)^{2} & =7 & & x^{2}-10 x+25 \text { is a perfect square trinomial. } \\
\sqrt{(x-5)^{2}} & =\sqrt{7} & & \text { Take the square root of each side. } \\
|x-5| & =\sqrt{7} & & \text { Simplify. } \\
x-5 & = \pm \sqrt{7} & & \text { Definition of absolute value } \\
x-5+5 & = \pm \sqrt{7}+5 & & \text { Add } 5 \text { to each side. } \\
x & =5 \pm \sqrt{7} & & \text { Simplify. }
\end{aligned}
$$

Use a calculator to evaluate each value of $x$.

$$
\begin{array}{rlrlrl}
x & =5+\sqrt{7} & \text { or } & x & =5-\sqrt{7} & \\
\text { Write each solution. } \\
& \approx 7.6 & & & \approx 2.4 & \\
\text { Simplify. }
\end{array}
$$

The solution set is $\{2.4,7.6\}$.

## Check your progress:

1) $m^{2}+18 m+81=90$

Since we are told we can take the square root of each side, we know that the left side of the equation is a perfect square trinomial and will factor to $\left(x+\frac{b}{2}\right)^{2}$.
$(m+9)^{2}=90$
$\sqrt{(m+9)^{2}}=\sqrt{90}$
$m+9= \pm \sqrt{90}$
$-9 \quad-9$
$m=-9 \pm \sqrt{90}$
$m \approx-18.5$ or $m \approx 0.5$

## Example: Solve an equation by Completing the Square

Solve $a^{2}-14 a+3=-10$ by completing the square.
Isolate the $a^{2}$ and $a$ terms. Then complete the square and solve.

$$
\begin{array}{rlrlrl}
a^{2}-14 a+3 & =-10 & & \text { Original equation } \\
a^{2}-14 a+3-3 & =-10-3 & & \text { Subtract 3 from each side. } \\
a^{2}-14 a & =-13 & & \text { Simplify. } \\
a^{2}-14 a+49 & =-13+49 & & \text { Since }\left(\frac{-14}{2}\right)^{2}=49, \text { add } 49 \text { to each side. } \\
(a-7)^{2} & =36 & & \text { Factor } a^{2}-14 a+49 . \\
a-7 & = \pm 6 & & \text { Take the square root of each side. } \\
& a & =7 \pm 6 & & \text { Add 7 to each side. } \\
a=7+6 \text { or } a & =7-6 & & \text { Separate the solutions. } \\
=13 & & & =1 & & \text { Simplify. }
\end{array}
$$

The solution set is $\{1,13\}$.

Step 1: Make sure the coefficient on the quadratic term is 1.
Step 2: Leave a blank spot after the linear term and any constant $\left(x^{2}+b x \quad\right)+d=f$
Step 3: Determine what number to add to the blank. It should always be half the linear term squared $c=\left(\frac{b}{2}\right)^{2}$. Remember to make sure that your net change on the equation is zero.

$$
x^{2}+b x+c+d-c=f
$$

Step 4: Factor the perfect square trinomial and simplify. $\left(x+\frac{b}{2}\right)^{2}+d-c=f$
Step 5: Solve the equation

Check your progress:

1) $x^{2}-8 x=4$

Step 1: Since the coefficient on the quadratic term is 1 , this step is complete.
Step 2:
$\left(x^{2}-8 x\right)=4$
Step 3:
$\left(\frac{-8}{2}\right)^{2}=(-4)^{2}=16$
$\left(x^{2}-8 x+16\right)-16=4$
Step 4:
$(x-4)^{2}-16=4$
Step 5:

$$
\begin{aligned}
& (x-4)^{2}-16=4 \\
& +16+16 \\
& (x-4)^{2}=20 \\
& \sqrt{(x-4)^{2}}=\sqrt{20} \\
& x-4= \pm \sqrt{20} \\
& +4+4 \\
& x=4 \pm \sqrt{20} \\
& x \approx-0.5 \text { or } x \approx 8.5
\end{aligned}
$$

2) $3 n^{2}-18 n=30$

Step 1: Since the coefficient on the quadratic term is not 1 , we must factor out that coefficient.
$3\left(n^{2}-6 n\right)=30$
Step 2:
$3\left(n^{2}-6 n\right) \quad=30$
Step 3:
$\left(\frac{-6}{2}\right)^{2}=(-3)^{2}=9$
$3\left(n^{2}-6 n+9\right)-27=30 \quad * *$ I am subtracting 27 because I actually added 27 if you multiply the 3 back through the set of parentheses.

Step 4:
$3(n-3)^{2}-27=30$
Step 5:
$3(n-3)^{2}-27=30$

$$
+27+27
$$

$3(n-3)^{2}=57$
$\frac{3(n-3)^{2}}{3}=\frac{57}{3}$
$(n-3)^{2}=19$
$\sqrt{(n-3)^{2}}=\sqrt{19}$
$n-3= \pm \sqrt{19}$

$$
+3 \quad+3
$$

$x=3 \pm \sqrt{19}$
$x \approx-1.4$ or $x \approx 7.4$

## Practice

Solve each equation by taking the square root of each side. Round to the nearest tenth if necessary.

1) $b^{2}-6 b+9=25$

Since we are told we can take the square root of each side, we know that the left side of the equation is a perfect square trinomial and will factor to $\left(x+\frac{b}{2}\right)^{2}$.
$(b-3)^{2}=25$
$\sqrt{(b-3)^{2}}=\sqrt{25}$
$b-3= \pm 5$
$+3+3$
$b=3 \pm 5$
$b=8$ or $b=-2$
2) $m^{2}+14 m+49=20$

Since we are told we can take the square root of each side, we know that the left side of the equation is a perfect square trinomial and will factor to $\left(x+\frac{b}{2}\right)^{2}$.
$(m+7)^{2}=20$
$\sqrt{(m+7)^{2}}=\sqrt{20}$
$m+7= \pm \sqrt{20}$
$-7 \quad-7$
$m=-7 \pm \sqrt{20}$
$m \approx-11.5$ or $m \approx-2.5$

Find the value of $c$ that makes each trinomial a perfect square.
3) $a^{2}-12 a+c$

This problem is asking us to find the value we would use in step 3 of our completing the square process.
$\left(\frac{-12}{2}\right)^{2}=(-6)^{2}=36$
$c=36$
4) $t^{2}+5 t+c$

This problem is asking us to find the value we would use in step 3 of our completing the square process.
$\left(\frac{5}{2}\right)^{2}=\frac{25}{4}=6 \frac{1}{4}$
$c=6 \frac{1}{4}$

Solve each equation by completing the square. Round to the nearest tenth if necessary.
5) $c^{2}-6 c=7$

Step 1: Since the coefficient on the quadratic term is 1 , this step is complete.
Step 2:
$\left(c^{2}-6 c\right)=7$
Step 3:
$\left(\frac{-6}{2}\right)^{2}=(-3)^{2}=9$
$\left(c^{2}-6 c+9\right)-9=7$
Step 4:
$(c-3)^{2}-9=7$
Step 5:
$(c-3)^{2}-9=7$

$$
+9+9
$$

$(c-3)^{2}=16$
$\sqrt{(c-3)^{2}}=\sqrt{16}$
$c-3= \pm 4$

$$
+3+3
$$

$c=3 \pm 4$
$c=7$ or $c=-1$
6) $x^{2}+7 x=-12$

Step 1: Since the coefficient on the quadratic term is 1 , this step is complete.
Step 2:
$\left(x^{2}+7 x\right)=-12$
Step 3:
$\left(\frac{7}{2}\right)^{2}=\frac{49}{4}=12 \frac{1}{4}$
$\left(x^{2}+7 x+12 \frac{1}{4}\right)-12 \frac{1}{4}=-12$

Step 4:
$\left(x+\frac{7}{2}\right)^{2}-12 \frac{1}{4}=-12$
Step 5:
$\left(x+\frac{7}{2}\right)^{2}-12 \frac{1}{4}=-12$

$$
+12 \frac{1}{4}+12 \frac{1}{4}
$$

$\left(x+\frac{7}{2}\right)^{2}=\frac{1}{4}$
$\sqrt{\left(x+\frac{7}{2}\right)^{2}}=\sqrt{\frac{1}{4}}$
$x+\frac{7}{2}= \pm \frac{1}{2}$
$-\frac{7}{2} \quad-\frac{7}{2}$
$x=-\frac{7}{2} \pm \frac{1}{2}$
$x=-4$ or $x=-3$
7) $v^{2}+14 v-9=6$

Step 1: Since the coefficient on the quadratic term is 1 , this step is complete.
Step 2:
$\left(v^{2}+14 v\right)-9=6$
Step 3:
$\left(\frac{14}{2}\right)^{2}=(7)^{2}=49$
$\left(v^{2}+14 v+49\right)-9-49=6$
Step 4:
$(v+7)^{2}-58=6$
Step 5:
$(v+7)^{2}-58=6$

$$
+58+58
$$

$(v+7)^{2}=64$
$\sqrt{(v+7)^{2}}=\sqrt{64}$
$v+7= \pm 8$
$-7 \quad-7$
$v=-7 \pm 8$
$v=1$ or $v=-15$
8) $r^{2}-4 r=2$

Step 1: Since the coefficient on the quadratic term is 1 , this step is complete.
Step 2:
$\left(r^{2}-4 r\right)=2$
Step 3:
$\left(\frac{-4}{2}\right)^{2}=(-2)^{2}=4$
$\left(r^{2}-4 r+4\right)-4=2$
Step 4:
$(r-2)^{2}-4=2$
Step 5:

$$
\begin{aligned}
& (r-2)^{2}-4=2 \\
& +4+4 \\
& (r-2)^{2}=6 \\
& \sqrt{(r-2)^{2}}=\sqrt{6} \\
& r-2= \pm \sqrt{6} \\
& +2+2 \\
& r=2 \pm \sqrt{6} \\
& r \approx-0.4 \text { or } r \approx 4.4
\end{aligned}
$$

9) $4 a^{2}+9 a-1=0$

Step 1: Since the coefficient on the quadratic term is not 1, we must factor out that coefficient. $4\left(a^{2}+\frac{9}{4} a\right)-1=0$

Step 2:
$4\left(a^{2}+\frac{9}{4} a\right)-1=0$
Step 3:
$\left(\frac{\frac{9}{4}}{2}\right)^{2}=\left(\frac{9}{8}\right)^{2}=\frac{81}{64}$
$4\left(a^{2}+\frac{9}{4} a+\frac{81}{64}\right)-1-\frac{81}{16}=0 \quad * *$ I am subtracting $\frac{81}{16}$ because I actually added $\frac{81}{16}$ if you multiply the 4 back through the set of parentheses.

Step 4:
$4\left(a+\frac{9}{8}\right)^{2}-\frac{97}{16}=0 \quad * *-1=-\frac{16}{16} \quad-\frac{16}{16}-\frac{81}{16}=-\frac{97}{16}$
Step 5:
$4\left(a+\frac{9}{8}\right)^{2}-\frac{97}{16}=0$

$$
+\frac{97}{16}+\frac{97}{16}
$$

$4\left(a+\frac{9}{8}\right)^{2}=\frac{97}{16}$
$\frac{4\left(a+\frac{9}{8}\right)^{2}}{4}=\frac{\frac{97}{16}}{4}$
$\left(a+\frac{9}{8}\right)^{2}=\frac{97}{64}$
$\sqrt{\left(a+\frac{9}{8}\right)^{2}}=\sqrt{\frac{97}{64}}$
$a+\frac{9}{8}= \pm \sqrt{\frac{97}{64}}$
$-\frac{9}{8} \quad-\frac{9}{8}$
$a=-\frac{9}{8} \pm \sqrt{\frac{97}{64}}$
$a \approx 0.1$ or $a \approx-2.4$

$$
\text { 10) } 7=2 p^{2}-5 p+8
$$

Let's switch the order of the equation so that it looks more familiar.
$2 p^{2}-5 p+8=7$
Step 1: Since the coefficient on the quadratic term is not 1 , we must factor out that coefficient.
$2\left(p^{2}-\frac{5}{2} p\right)+8=7$
Step 2:
$2\left(p^{2}-\frac{5}{2} p\right)+8=7$
Step 3:
$\left(\frac{-\frac{5}{2}}{2}\right)^{2}=\left(-\frac{5}{4}\right)^{2}=\frac{25}{16}$
$2\left(p^{2}-\frac{5}{2} p+\frac{25}{16}\right)+8-\frac{25}{8}=7 \quad * *$ I am subtracting $\frac{25}{8}$ because I actually added $\frac{25}{8}$ if you multiply the 2 back through the set of parentheses.

Step 4:
$2\left(p-\frac{5}{4}\right)^{2}+\frac{39}{8}=7$
$* * 8=\frac{64}{8}$
$\frac{64}{8}-\frac{25}{8}=\frac{39}{8}$
Step 5:
$2\left(p-\frac{5}{4}\right)^{2}+\frac{39}{8}=\frac{56}{8}$
$-\frac{39}{8}-\frac{39}{8}$
$2\left(p-\frac{5}{4}\right)^{2}=\frac{17}{8}$
$\frac{2\left(p-\frac{5}{4}\right)^{2}}{2}=\frac{\frac{17}{8}}{2}$
$\left(p-\frac{5}{4}\right)^{2}=\frac{17}{16}$

$$
\begin{aligned}
& \sqrt{\left(p-\frac{5}{4}\right)^{2}}=\sqrt{\frac{17}{16}} \\
& p-\frac{5}{4}= \pm \sqrt{\frac{17}{16}} \\
& p-\frac{5}{4}= \pm \sqrt{\frac{17}{16}} \\
& +\frac{5}{4} \quad+\frac{5}{4} \\
& p=\frac{5}{4} \pm \sqrt{\frac{17}{16}} \\
& p \approx 2.3 \text { or } p \approx 0.2
\end{aligned}
$$

