KEY CONCEPT *Difference of Squares*

Symbols $a^2 - b^2 = (a + b)(a - b)$ or $(a - b)(a + b)$

Examples $x^2 - 9 = (x + 3)(x - 3)$ or $(x - 3)(x + 3)$

In order to use this rule, we must know somethings about perfect squares.

$1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25, 6^2 = 36, 7^2 = 49, 8^2 = 64, 9^2 = 81, 10^2 = 100,$

$11^2 = 121, 12^2 = 144, 13^2 = 169, 14^2 = 196, 15^2 = 225$, etc.

The numbers 1, 4, 9, 16, 25, 36, 49, 81, 100, 121, 144, 169, 196, 225, etc. are known as perfect squares.

**It is important when using the difference of squares rule that there is a difference (subtraction) between the two squares. If it is a sum (addition), it is not factorable and is prime.

Example: Factor the Difference of Squares

Factor each binomial.

a. $n^2 - 25$

$n^2 - 25 = n^2 - 5^2$ Write in the form $a^2 - b^2$.

$= (n + 5)(n - 5)$ Factor the difference of squares.

b. $36x^2 - 49y^2$

$36x^2 - 49y^2 = (6x)^2 - (7y)^2$ $36x^2 = 6x \cdot 6x$ and $49y^2 = 7y \cdot 7y$

$= (6x + 7y)(6x - 7y)$ Factor the difference of squares.

c. $48a^3 - 12a$

If the terms of a binomial have a common factor, the GCF should be factored out first before trying to apply any other factoring technique.

$48a^3 - 12a = 12a(4a^2 - 1)$ The GCF of $48a^3$ and $-12a$ is $12a$.

$= 12a[(2a) \; - 1^2]$ $4a^2 = 2a \cdot 2a$ and $1 = 1 \cdot 1$

$= 12a(2a + 1)(2a - 1)$ Factor the difference of squares.

Check your progress:

1) $81 - t^2$

We want to start by rewriting the binomial so that each term is a square.

$(9)^2 - (t)^2$ **We know that $9^2 = 81$ and $1^2 = 1$.

To factor this, we then use 9 and t and write one factor as the sum of those two things and one factor as the difference of those two things.

$(9+t)(9-t)$ **This is our solution.

2) $64g^2 - h^2$

We want to start by rewriting the binomial so that each term is a square.

$(8g)^2 - (h)^2$ **We know that $8^2 = 64$ and $1^2 = 1$.

To factor this, we then use 8g and h and write one factor as the sum of those two things and one factor as the difference of those two things.

$(8g+h)(8g-h)$ **This is our solution.

3) $4x^2 + 25$

This looks like an addition between the two terms. Even if I switch the order of the terms, it remains a sum of squares. Therefore, this is unfactorable and prime.

Prime **This is our solution.

4) $-4y^3 + 9y$

This looks like an addition between the two terms, but I can switch the order of the terms so that it becomes a subtraction.

$9y - 4y^3$

If we look at this binomial, we can see that both terms are divisible by y. So, let's factor that out, first.

$$y\left(\frac{9y}{y} - \frac{4y^3}{y}\right)$$

$y(9-4y^2)$

We sort of ignore the y out front and we rewrite the binomial in parentheses so that each term is a square.

$y((3)^2 - (2y)^2)$ **We know that $2^2 = 4$ and $3^2 = 9$.

To factor this, we then use 3 and 2y and write one factor as the sum of those two things and one factor as the difference of those two things.

$y(3+2y)(3-2y)$ **This is our solution.

Example: Apply a Factoring Technique More Than Once

Factor $x^4 - 81$.

$x^4 - 81 = [(x^2)^2 - 9^2]$ — $x^4 = x^2 \cdot x^2$ and $81 = 9 \cdot 9$

$= (x^2 + 9)(x^2 - 9)$ — Factor the difference of squares.

$= (x^2 + 9)(x^2 - 3^2)$ — $x^2 = x \cdot x$ and $9 = 3 \cdot 3$

$= (x^2 + 9)(x + 3)(x - 3)$ — Factor the difference of squares.

Check your progress:

1) $32x^4 - 2$

If we look at this binomial, we can see that both terms are divisible by 2. So, let's factor that out, first.

$2\left(\frac{32x^4}{2} - \frac{2}{2}\right)$

$2(16x^4 - 1)$

Rewrite the binomial in parentheses so that each term is a square.

$2((4x^2)^2 - (1)^2)$ **We know that $(x^2)^2 = x^4, 4^2 = 16$ and $1^2 = 1$.

Factor the binomials in parentheses:

$2(4x^2 + 1)(4x^2 - 1)$

At this point, we notice that one of our factors is again a difference of squares $(4x^2 - 1)$. $4x^2 + 1$ is a sum of squares and is not factorable.

Rewrite the term that is factorable so that each term of the binomials is a square. We leave everything else alone.

$2(4x^2 + 1)((2x)^2 - (1)^2)$

Then we factor the last binomial.

$2(4x^2 + 1)(2x + 1)(2x - 1)$ **This is our solution.

2) $4a^4 - 4b^4$

If we look at this binomial, we can see that both terms are divisible by 4. So, let's factor that out, first.

$4\left(\frac{4a^4}{4} - \frac{4b^4}{4}\right)$

$4(a^4 - b^4)$

Rewrite the binomial in parentheses so that each term is a square.

$4((a^2)^2 - (b^2)^2)$ **We know that $(a^2)^2 = a^4$ and $(b^2)^2 = b^4$.

Factor the binomials in parentheses:

$4(a^2 + b^2)(a^2 - b^2)$

At this point, we notice that one of our factors is again a difference of squares $(a^2 - b^2)$. $a^2 + b^2$ is a sum of squares and is not factorable.

Rewrite the term that is factorable so that each term of the binomials is a square. We leave everything else alone.

$4(a^2 + b^2)((a)^2 - (b)^2)$

Then we factor the last binomial.

$4(a^2 + b^2)(a + b)(a - b)$ **This is our solution.

Example: Apply Several Different Factoring Techniques

Factor $5x^3 + 15x^2 - 5x - 15$.

$5x^3 + 15x^2 - 5x - 15$	Original polynomial
$= 5(x^3 + 3x^2 - x - 3)$	Factor out the GCF.
$= 5[(x^3 - x) + (3x^2 - 3)]$	Group terms with common factors.
$= 5[x(x^2 - 1) + 3(x^2 - 1)]$	Factor each grouping.
$= 5(x^2 - 1)(x + 3)$	$x^2 - 1$ is the common factor.
$= 5(x + 1)(x - 1)(x + 3)$	Factor the difference of squares, $x^2 - 1$, into $(x + 1)(x - 1)$.

Check your progress:

1) $2x^3 + x^2 - 50x - 25$

Remember that to factor a polynomial with four terms, we split the polynomial in half and treat it like two binomials we are factoring separately.

$2x^3 + x^2 - 50x - 25)$

The first two terms are divisible by x^2. The last two terms are divisible by -25.

$x^2\left(\frac{2x^3}{x^2} + \frac{x^2}{x^2}\right) - 25\left(\frac{-50x}{-25} + \frac{-25}{-25}\right)$

$x^2(2x + 1) - 25(2x + 1)$

Both terms have a $(2x+1)$, so we can factor that out.

$$(2x+1)\left(\frac{x^2(2x+1)}{2x+1}-\frac{25(2x+1)}{2x+1}\right)$$

$(2x+1)(x^2-25)$

At this point, we should notice that one of our factors is a difference of squares (x^2-25).

Rewrite the term that is factorable so that each term of the binomials is a square. We leave everything else alone.

$(2x+1)((x)^2-(5)^2)$

Then we factor the last binomial.

$(2x+1)(x+5)(x-5)$ **This is our solution.

2) $x^3-3x^2-9x+27$

Remember that to factor a polynomial with four terms, we split the polynomial in half and treat it like two binomials we are factoring separately.

$x^3-3x^2-9x+27)$

The first two terms are divisible by x^2. The last two terms are divisible by -9.

$x^2\left(\frac{x^3}{x^2}-\frac{3x^2}{x^2}\right)-9\left(\frac{-9x}{-9}+\frac{27}{-9}\right)$

$x^2(x-3)-9(x-3)$

Both terms have an $(x-3)$, so we can factor that out.

$$(x-3)\left(\frac{x^2(x-3)}{x-3}-\frac{9(x-3)}{x-3}\right)$$

$(x-3)(x^2-9)$

At this point, we should notice that one of our factors is a difference of squares (x^2-9).

Rewrite the term that is factorable so that each term of the binomials is a square. We leave everything else alone.

$(x-3)((x)^2-(3)^2)$

Then we factor the last binomial.

$(x-3)(x+3)(x-3)$

Because two of our terms are exactly the same, we can write them as a square.

$(x-3)^2(x+3)$ **This is our solution.

Example: Solve Equations by Factoring

In the equation $y = x^2 - \frac{9}{16}$, which is a value of x when $y = 0$?

$y = x^2 - \frac{9}{16}$	Original equation
$0 = x^2 - \frac{9}{16}$	Replace y with 0.
$0 = x^2 - \left(\frac{3}{4}\right)^2$	$x^2 = x \cdot x$ and $\frac{9}{16} = \frac{3}{4} \cdot \frac{3}{4}$
$0 = \left(x + \frac{3}{4}\right)\left(x - \frac{3}{4}\right)$	Factor the difference of squares.
$0 = x + \frac{3}{4}$ or $0 = x - \frac{3}{4}$	Zero Product Property
$-\frac{3}{4} = x$ $\quad\quad$ $\frac{3}{4} = x$	Solve each equation.

Check your progress:

1) $4x^2 = 25$

We will start by getting one side of the equation to equal zero.

$4x^2 = 25$

$-25 \quad -25$

$4x^2 - 25 = 0$ **Notice that I am not combining these because they are not like terms.

Rewrite the binomial so that each term is a square.

$(2x)^2 - (5)^2 = 0$

Factor the difference of squares:

$(2x + 5)(2x - 5) = 0$

Finally, we use the zero-product property to solve.

$(2x + 5)(2x - 5) = 0$

We have two things ($(2x + 5)$ and $(2x - 5)$) that are multiplied together and the product is zero. That means $2x + 5 = 0$ or $2x - 5 = 0$.

So, we write both of those equations down and solve.

$2x + 5 = 0$ or $2x - 5 = 0$

$-5 \quad -5$ $\quad\quad$ $+5 \quad +5$

$2x = -5$ or $2x = 5$

$\frac{2x}{2} = \frac{-5}{2}$ or $\frac{2x}{2} = \frac{5}{2}$

$x = -\frac{5}{2}$ or $x = \frac{5}{2}$

The two solutions are $x = -\frac{5}{2}$ or $x = \frac{5}{2}$. **These are our solutions, but it is always a good idea to double-check our solutions and see if we have the correct ones.

Check:

Replace x with -5/2	Replace x with 5/2
$4x^2 = 25$	$4x^2 = 25$
$4\left(-\frac{5}{2}\right)^2 = 25$	$4\left(\frac{5}{2}\right)^2 = 25$
$4\left(\frac{25}{4}\right) = 25$	$4\left(\frac{25}{4}\right) = 25$
$\frac{100}{4} = 25$	$\frac{100}{4} = 25$
$25 = 25$	$25 = 25$

Because both equations are true, we know we have found the correct solutions.

2) $18x^3 = 50x$

We will start by getting one side of the equation to equal zero.

$18x^3 = 50x$

$-50x \quad -50x$

$18x^3 - 50x = 0$ **Notice that I am not combining these because they are not like terms.

The binomial on the left has two terms are divisible by $2x$. So, let's factor that out, first.

$$2x\left(\frac{18x^3}{2x} - \frac{50x}{2x}\right) = 0$$

$2x(9x^2 - 25) = 0$

Rewrite the binomial in parentheses so that each term is a square.

$2x((3x)^2 - (5)^2) = 0$

Factor the difference of squares:

$2x(3x+5)(3x-5)=0$

Finally, we use the zero-product property to solve.

$2x(3x+5)(3x-5)=0$

We have three things ($2x$, $(3x+5)$ and $(3x-5)$) that are multiplied together and the product is zero. That means $2x=0$, $3x+5=0$ or $3x-5=0$.

So, we write all three of those equations down and solve.

$2x=0$	or	$3x+5=0$	or	$3x-5=0$
$\frac{2x}{2}=\frac{0}{2}$		$-5 \quad -5$		$+5 \quad +5$
$x=0$	or	$3x=-5$	or	$3x=5$
		$\frac{3x}{3}=\frac{-5}{3}$	or	$\frac{3x}{3}=\frac{5}{3}$
		$x=-\frac{5}{3}$	or	$x=\frac{5}{3}$

The three solutions are $x=0, x=-\frac{5}{3}$ or $x=\frac{5}{3}$.

**These are our solutions, but it is always a good idea to double-check our solutions and see if we have the correct ones.

Check:

Replace x with 0	Replace x with -5/3	Replace x with 5/3
$18x^3=50x$	$18x^3=50x$	$18x^3=50x$
$18(0)^3=50(0)$	$18\left(-\frac{5}{3}\right)^3=50\left(-\frac{5}{3}\right)$	$18\left(\frac{5}{3}\right)^3=50\left(\frac{5}{3}\right)$
$18(0)=0$	$18\left(-\frac{125}{27}\right)=-\frac{250}{3}$	$18\left(\frac{125}{27}\right)=\frac{250}{3}$
$0=0$	$-\frac{2250}{27}=-\frac{250}{3}$	$\frac{2250}{27}=\frac{250}{3}$
	$-\frac{250}{3}=-\frac{250}{3}$	$\frac{250}{3}=\frac{250}{3}$

Because all three equations are true, we know we have found the correct solutions.