$$
\begin{aligned}
& \text { KEY CONCEPT } \\
& \begin{array}{ll}
\text { Symbols } & a^{2}-b^{2}=(a+b)(a-b) \text { or }(a-b)(a+b) \\
\text { Examples } & x^{2}-9=(x+3)(x-3) \text { or }(x-3)(x+3)
\end{array}
\end{aligned}
$$

Difference of Squares

In order to use this rule, we must know somethings about perfect squares.
$1^{2}=1,2^{2}=4,3^{2}=9,4^{2}=16,5^{2}=25,6^{2}=36,7^{2}=49,8^{2}=64,9^{2}=81,10^{2}=100$,
$11^{2}=121,12^{2}=144,13^{2}=169,14^{2}=196,15^{2}=225$, etc.
The numbers $1,4,9,16,25,36,49,81,100,121,144,169,196,225$, etc. are known as perfect squares.
**It is important when using the difference of squares rule that there is a difference (subtraction) between the two squares. If it is a sum (addition), it is not factorable and is prime.

## Example: Factor the Difference of Squares

## Factor each binomial.

a. $n^{2}-25$

$$
\begin{aligned}
n^{2}-25 & =n^{2}-5^{2} & & \text { Write in the form } a^{2}-b^{2} . \\
& =(n+5)(n-5) & & \text { Factor the difference of squares. }
\end{aligned}
$$

b. $36 x^{2}-49 y^{2}$

$$
\begin{aligned}
36 x^{2}-49 y^{2} & =(6 x)^{2}-(7 y)^{2} & & 36 x^{2}=6 x \cdot 6 x \text { and } 49 y^{2}=7 y \cdot 7 y \\
& =(6 x+7 y)(6 x-7 y) & & \text { Factor the difference of squares. }
\end{aligned}
$$

c. $48 a^{3}-12 a$

If the terms of a binomial have a common factor, the GCF should be factored out first before trying to apply any other factoring technique.

$$
\begin{aligned}
48 a^{3}-12 a & =12 a\left(4 a^{2}-1\right) & & \text { The GCF of } 48 a^{3} \text { and }-12 a \text { is } 12 a . \\
& =12 a\left[(2 a)-1^{2}\right] & & 4 a^{2}=2 a \cdot 2 a \text { and } 1=1 \cdot 1 \\
& =12 a(2 a+1)(2 a-1) & & \text { Factor the difference of squares. }
\end{aligned}
$$

## Check your progress:

1) $81-t^{2}$

We want to start by rewriting the binomial so that each term is a square.

$$
(9)^{2}-(t)^{2} \quad * * \text { We know that } 9^{2}=81 \text { and } 1^{2}=1
$$

To factor this, we then use 9 and $t$ and write one factor as the sum of those two things and one factor as the difference of those two things.
$(9+t)(9-t) \quad * *$ This is our solution.
2) $64 g^{2}-h^{2}$

We want to start by rewriting the binomial so that each term is a square.
$(8 g)^{2}-(h)^{2} \quad * *$ We know that $8^{2}=64$ and $1^{2}=1$.
To factor this, we then use 8 g and h and write one factor as the sum of those two things and one factor as the difference of those two things.
$(8 g+h)(8 g-h) \quad * *$ This is our solution.
3) $4 x^{2}+25$

This looks like an addition between the two terms. Even if I switch the order of the terms, it remains a sum of squares. Therefore, this is unfactorable and prime.

Prime $\quad$ **This is our solution.
4) $-4 y^{3}+9 y$

This looks like an addition between the two terms, but I can switch the order of the terms so that it becomes a subtraction.
$9 y-4 y^{3}$
If we look at this binomial, we can see that both terms are divisible by y . So, let's factor that out, first.
$y\left(\frac{9 y}{y}-\frac{4 y^{3}}{y}\right)$
$y\left(9-4 y^{2}\right)$
We sort of ignore the $y$ out front and we rewrite the binomial in parentheses so that each term is a square.
$y\left((3)^{2}-(2 y)^{2}\right) \quad * *$ We know that $2^{2}=4$ and $3^{2}=9$.
To factor this, we then use 3 and 2 y and write one factor as the sum of those two things and one factor as the difference of those two things.
$y(3+2 y)(3-2 y) \quad * *$ This is our solution.

## Example: Apply a Factoring Technique More Than Once

Factor $x^{4}-81$.

$$
\begin{aligned}
x^{4}-81 & =\left[\left(x^{2}\right)^{2}-9^{2}\right] & & x^{4}=x^{2} \cdot x^{2} \text { and } 81=9 \cdot 9 \\
& =\left(x^{2}+9\right)\left(x^{2}-9\right) & & \text { Factor the difference of squares. } \\
& =\left(x^{2}+9\right)\left(x^{2}-3^{2}\right) & & x^{2}=x \cdot x \text { and } 9=3 \cdot 3 \\
& =\left(x^{2}+9\right)(x+3)(x-3) & & \text { Factor the difference of squares. }
\end{aligned}
$$

## Check your progress:

1) $32 x^{4}-2$

If we look at this binomial, we can see that both terms are divisible by 2 . So, let's factor that out, first.
$2\left(\frac{32 x^{4}}{2}-\frac{2}{2}\right)$
$2\left(16 x^{4}-1\right)$
Rewrite the binomial in parentheses so that each term is a square.
$2\left(\left(4 x^{2}\right)^{2}-(1)^{2}\right) \quad * *$ We know that $\left(x^{2}\right)^{2}=x^{4}, 4^{2}=16$ and $1^{2}=1$.
Factor the binomials in parentheses:
$2\left(4 x^{2}+1\right)\left(4 x^{2}-1\right)$
At this point, we notice that one of our factors is again a difference of squares $\left(4 x^{2}-1\right) .4 x^{2}+$ 1 is a sum of squares and is not factorable.

Rewrite the term that is factorable so that each term of the binomials is a square. We leave everything else alone.
$2\left(4 x^{2}+1\right)\left((2 x)^{2}-(1)^{2}\right)$
Then we factor the last binomial.
$2\left(4 x^{2}+1\right)(2 x+1)(2 x-1) \quad * *$ This is our solution.
2) $4 a^{4}-4 b^{4}$

If we look at this binomial, we can see that both terms are divisible by 4. So, let's factor that out, first.
$4\left(\frac{4 a^{4}}{4}-\frac{4 b^{4}}{4}\right)$
$4\left(a^{4}-b^{4}\right)$
Rewrite the binomial in parentheses so that each term is a square.
$4\left(\left(a^{2}\right)^{2}-\left(b^{2}\right)^{2}\right) \quad * *$ We know that $\left(a^{2}\right)^{2}=a^{4}$ and $\left(b^{2}\right)^{2}=b^{4}$.
Factor the binomials in parentheses:
$4\left(a^{2}+b^{2}\right)\left(a^{2}-b^{2}\right)$
At this point, we notice that one of our factors is again a difference of squares $\left(a^{2}-b^{2}\right) . a^{2}+$ $b^{2}$ is a sum of squares and is not factorable.

Rewrite the term that is factorable so that each term of the binomials is a square. We leave everything else alone.
$4\left(a^{2}+b^{2}\right)\left((a)^{2}-(b)^{2}\right)$
Then we factor the last binomial.
$4\left(a^{2}+b^{2}\right)(a+b)(a-b) \quad * * T h i s$ is our solution.

## Example: Apply Several Different Factoring Techniques

$$
\begin{aligned}
& \text { Factor } 5 x^{3}+15 x^{2}-5 x-15 . \\
& \begin{aligned}
5 x^{3} & +15 x^{2}-5 x-15 & & \\
& =5\left(x^{3}+3 x^{2}-x-3\right) & & \text { Original polynomial } \\
& =5\left[\left(x^{3}-x\right)+\left(3 x^{2}-3\right)\right] & & \text { Groctor out the GCF. } \\
& =5\left[x\left(x^{2}-1\right)+3\left(x^{2}-1\right)\right] & & \text { Factor each grouping. } \\
& =5\left(x^{2}-1\right)(x+3) & & x^{2}-1 \text { is the common factor. } \\
& =5(x+1)(x-1)(x+3) & & \text { Factor the difference of squares, } x^{2}-1, \text { into }(x+1)(x-1) .
\end{aligned}
\end{aligned}
$$

## Check your progress:

1) $2 x^{3}+x^{2}-50 x-25$

Remember that to factor a polynomial with four terms, we split the polynomial in half and treat it like two binomials we are factoring separately.
$\left.\left.2 x^{3}+x^{2}\right\}-50 x-25\right)$
The first two terms are divisible by $x^{2}$. The last two terms are divisible by -25 .
$\left.x^{2}\left(\frac{2 x^{3}}{x^{2}}+\frac{x^{2}}{x^{2}}\right)\right\} 25\left(\frac{-50 x}{-25}+\frac{-25}{-2}\right)$
$x^{2}(2 x+1)-25(2 x+1)$

Both terms have a $(2 x+1)$, so we can factor that out.
$(2 x+1)\left(\frac{x^{2}(2 x+1)}{2 x+1}-\frac{25(2 x+1)}{2 x+1}\right)$
$(2 x+1)\left(x^{2}-25\right)$
At this point, we should notice that one of our factors is a difference of squares $\left(x^{2}-25\right)$.
Rewrite the term that is factorable so that each term of the binomials is a square. We leave everything else alone.
$(2 x+1)\left((x)^{2}-(5)^{2}\right)$
Then we factor the last binomial.
$(2 x+1)(x+5)(x-5) \quad * * T h i s$ is our solution.
2) $x^{3}-3 x^{2}-9 x+27$

Remember that to factor a polynomial with four terms, we split the polynomial in half and treat it like two binomials we are factoring separately.
$\left.\left.x^{3}-3 x^{2}\right\}-9 x+27\right)$
The first two terms are divisible by $x^{2}$. The last two terms are divisible by -9 .
$x^{2}\left(\frac{x^{3}}{x^{2}}-\frac{3 x^{2}}{x^{2}}\right)\left\{-9\left(\frac{-9 x}{-9}+\frac{27}{-9}\right)\right.$
$x^{2}(x-3)-9(x-3)$
Both terms have an $(x-3)$, so we can factor that out.
$(x-3)\left(\frac{x^{2}(x-3)}{x-3}-\frac{9(x-3)}{x-3}\right)$
$(x-3)\left(x^{2}-9\right)$
At this point, we should notice that one of our factors is a difference of squares $\left(x^{2}-9\right)$.
Rewrite the term that is factorable so that each term of the binomials is a square. We leave everything else alone.
$(x-3)\left((x)^{2}-(3)^{2}\right)$
Then we factor the last binomial.
$(x-3)(x+3)(x-3)$
Because two of our terms are exactly the same, we can write them as a square.
$(x-3)^{2}(x+3) \quad * *$ This is our solution.

## Example: Solve Equations by Factoring

In the equation $y=x^{2}-\frac{9}{16}$, which is a value of $x$ when $y=0$ ?

$$
\begin{array}{rlrlrl}
y & =x^{2}-\frac{9}{16} & & \text { Original equation } \\
0 & =x^{2}-\frac{9}{16} & & \text { Replace } y \text { with } 0 . \\
0 & =x^{2}-\left(\frac{3}{4}\right)^{2} & & x^{2}=x \cdot x \text { and } \frac{9}{16}=\frac{3}{4} \cdot \frac{3}{4} \\
0 & =\left(x+\frac{3}{4}\right)\left(x-\frac{3}{4}\right) & & \text { Factor the difference of squares. } \\
0 & =x+\frac{3}{4} & \text { or } & 0 & =x-\frac{3}{4} & \text { Zero Product Property } \\
-\frac{3}{4} & =x & \frac{3}{4} & =x & \text { Solve each equation. }
\end{array}
$$

## Check your progress:

1) $4 x^{2}=25$

We will start by getting one side of the equation to equal zero.
$4 x^{2}=25$
$-25 \quad-25$
$4 x^{2}-25=0 * *$ Notice that I am not combining these because they are not like terms.
Rewrite the binomial so that each term is a square.
$(2 x)^{2}-(5)^{2}=0$
Factor the difference of squares:
$(2 x+5)(2 x-5)=0$
Finally, we use the zero-product property to solve.
$(2 x+5)(2 x-5)=0$
We have two things $((2 x+5)$ and $(2 x-5))$ that are multiplied together and the product is zero. That means $2 x+5=0$ or $2 x-5=0$.

So, we write both of those equations down and solve.

| $2 x+5=0$ | or | $2 x-5=0$ |
| :---: | :---: | :---: |
| $-5-5$ |  | $+5+5$ |
| $2 x=-5$ | or | $2 x=5$ |

$$
\begin{array}{lll}
\frac{2 x}{2}=\frac{-5}{2} & \text { or } & \frac{2 x}{2}=\frac{5}{2} \\
x=-\frac{5}{2} & \text { or } & x=\frac{5}{2}
\end{array}
$$

The two solutions are $x=-\frac{5}{2}$ or $x=\frac{5}{2} . \quad * *$ These are our solutions, but it is always a good idea to double-check our solutions and see if we have the correct ones.

Check:
Replace x with $-5 / 2 \quad$ Replace x with $5 / 2$
$4 x^{2}=25 \quad 4 x^{2}=25$
$4\left(-\frac{5}{2}\right)^{2}=25 \quad 4\left(\frac{5}{2}\right)^{2}=25$

$$
\begin{array}{ll}
4\left(\frac{25}{4}\right)=25 & 4\left(\frac{25}{4}\right)=25 \\
\frac{100}{4}=25 & \frac{100}{4}=25 \\
25=25 & 25=25
\end{array}
$$

Because both equations are true, we know we have found the correct solutions.
2) $18 x^{3}=50 x$

We will start by getting one side of the equation to equal zero.
$18 x^{3}=50 x$
$-50 x-50 x$
$18 x^{3}-50 x=0 \quad * *$ Notice that I am not combining these because they are not like terms.

The binomial on the left has two terms are divisible by $2 x$. So, let's factor that out, first.
$2 x\left(\frac{18 x^{3}}{2 x}-\frac{50 x}{2 x}\right)=0$
$2 x\left(9 x^{2}-25\right)=0$
Rewrite the binomial in parentheses so that each term is a square.
$2 x\left((3 x)^{2}-(5)^{2}\right)=0$
Factor the difference of squares:
$2 x(3 x+5)(3 x-5)=0$
Finally, we use the zero-product property to solve.
$2 x(3 x+5)(3 x-5)=0$
We have three things $(2 x,(3 x+5)$ and $(3 x-5))$ that are multiplied together and the product is zero. That means $2 x=0,3 x+5=0$ or $3 x-5=0$.

So, we write all three of those equations down and solve.

$$
\begin{aligned}
& 2 x=0 \quad \text { or } \quad 3 x+5=0 \quad \text { or } \quad 3 x-5=0 \\
& \frac{2 x}{2}=\frac{0}{2} \\
& -5 \quad-5 \\
& x=0 \quad \text { or } \quad 3 x=-5 \quad \text { or } \quad 3 x=5 \\
& \frac{3 x}{3}=\frac{-5}{3} \quad \text { or } \quad \frac{3 x}{3}=\frac{5}{3} \\
& x=-\frac{5}{3} \quad \text { or } \quad x=\frac{5}{3}
\end{aligned}
$$

The three solutions are $x=0, x=-\frac{5}{3}$ or $x=\frac{5}{3} . \quad * *$ These are our solutions, but it is always a good idea to double-check our solutions and see if we have the correct ones.

Check:
Replace x with 0
Replace x with $-5 / 3$
Replace x with $5 / 3$
$18 x^{3}=50 x$

$$
18 x^{3}=50 x
$$

$$
18 x^{3}=50 x
$$

$$
18(0)^{3}=50(0)
$$

$$
18\left(-\frac{5}{3}\right)^{3}=50\left(-\frac{5}{3}\right)
$$

$$
18\left(\frac{5}{3}\right)^{3}=50\left(\frac{5}{3}\right)
$$

$$
18(0)=0
$$

$$
18\left(-\frac{125}{27}\right)=-\frac{250}{3}
$$

$$
18\left(\frac{125}{27}\right)=\frac{250}{3}
$$

$0=0$

$$
-\frac{2250}{27}=-\frac{250}{3} \quad \frac{2250}{27}=\frac{250}{3}
$$

$$
-\frac{250}{3}=-\frac{250}{3} \quad \frac{250}{3}=\frac{250}{3}
$$

Because all three equations are true, we know we have found the correct solutions.

