Vertical Motion Model:

A model for the vertical motion of a projected object is given by the equation $h = -16t^2 + vt + s$, where *h* is the height in feet, *t* is the time in seconds, *v* is the initial upward velocity in feet per second, and *s* is the initial height of the object in feet.

Example: Using the Vertical Motion Model:

PEP RALLY At a pep rally, small foam footballs are launched by cheerleaders using a sling-shot. How long is a football in the air if a student catches it on its way down 26 feet above the gym floor?

 $\begin{array}{ll} h = -16t^2 + vt + s & \text{Vertical motion model} \\ 26 = -16t^2 + 42t + 6 & h = 26, v = 42, s = 6 \\ 0 = -16t^2 + 42t - 20 & \text{Subtract 26 from} \\ each side. \\ 0 = -2(8t^2 - 21t + 10) & \text{Factor out } -2. \\ 0 = 8t^2 - 21t + 10 & \text{Divide each side by } -2. \\ 0 = (8t - 5)(t - 2) & \text{Factor } 8t^2 - 21t + 10. \\ 8t - 5 = 0 & \text{or} \quad t - 2 = 0 & \text{Zero Product Property} \\ 8t = 5 & t = 2 & \text{Solve each equation.} \\ t = \frac{5}{8} \end{array}$



The solutions are $\frac{5}{8}$ second and 2 seconds.

The first time represents how long it takes the football to reach a height of 26 feet on its way up. The later time represents how long it takes the ball to reach a height of 26 feet again on its way down. Thus, the football will be in the air for 2 seconds before the student catches it.

Check your progress:

The feet of a gymnast making a vault leave the horse at a height of 8 feet with an initial upward velocity of 8 feet per second. Use the model for vertical motion to find the time t in seconds it take for the gymnast's feet to reach the mat.



Step 1: Product = $-16 \cdot 8 = -128$

Sum = 8

**We need to find numbers that multiply to -128 and add to 8. Because the product is negative we must have one negative and one positive factor. Since the sum is positive, the larger factor must be positive.

Step 2:

Factors of -128:	Sum of factors:
-1, 128	-1 + 128 = 127
-2, 64	-2 + 64 = 62
-4, 32	-4 + 32 = 28
-8, 16	-8 + 16 = 8

-8 and 16 are the factors that will make a product of -128 and a sum of 8.

Step 3:

$$0 = -16t^{2} + 8t + 8$$

$$0 = -16t^{2} - 8t + 16t + 8$$

Step 4:

$$0 = -16t^2 - 8t + 16t + 8$$

The first two terms are divisible by -8t. The last two terms are divisible by 8.

$$0 = -8t\left(\frac{-16t^2}{-8t} + \frac{-8t}{-8t}\right) \left\{ + 8\left(\frac{16t}{8} + \frac{8}{8}\right) \\ 0 = -8t(2t+1) + 8(2t+1) \right\}$$

Both terms have a (2t + 1), so we can factor that out.

$$0 = (2t+1)\left(\frac{-8t(2t+1)}{2t+1} + \frac{8(2t+1)}{2t+1}\right)$$
$$0 = (2t+1)(-8t+8)$$

The last binomial has a common factor of 8, so let's factor that out.

$$0 = 8(2t+1)\left(\frac{-8t}{8} + \frac{8}{8}\right)$$
$$0 = 8(2t+1)(-t+1)$$

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Finally, we use the zero-product property to solve.

Since $0 \neq 8$, we can ignore that part.

Either 2t + 1 = 0 or -t + 1 = 0.

 $2t + 1 = 0 \quad \text{or} \quad -t + 1 = 0$ -1 -1 -1 -1 $2t = -1 \quad -t = -1$ $\frac{2t}{2} = \frac{-1}{2} \quad \frac{-t}{-1} = \frac{-1}{-1}$ $t = -\frac{1}{2} \quad \text{or} \quad t = 1$

The two solutions are $t = -\frac{1}{2}$ or t = 1. Since the negative solution does not makes sense in this situation, we will only consider the positive solution.

Check:

Replace x with 1

 $0 = -16t^{2} + 8t + 8$ $0 = -16(1)^{2} + 8(1) + 8$ 0 = -16(1) + 8 + 8 0 = -16 + 160 = 0

Because the equation is true, we know we have the correct solution.

The gymnast's feet reach the mat 1 second after she leaves the horse.

Example: Picture Frames

A picture is 60 cm long by 10 cm wide. The picture is framed using material that is x cm wide. The area of the frame and picture together is 1,056 square centimeters. What is the width of the framing material?



The length of the frame and picture together is 60 + 2x since we are adding x cm on each side of the picture.

The width of the frame and picture together is 10 + 2x since we are adding x cm on each side of the picture.

The area of the picture and the frame can be found by multiplying length times width.

(60 + 2x)(10 + 2x) = 1056

 $600 + 120x + 20x + 4x^2 = 1056$

 $600 + 140x + 4x^2 = 1056$

We need to get one side equal to zero.

 $600 + 140x + 4x^2 = 1056$

-1056 - 1056

 $-456 + 140x + 4x^2 = 0$

$$4x^2 + 140x - 456 = 0$$

All of the above terms are divisible by 4.

$$4(x^2 + 35x - 114) = 0$$

Divide by 4 on both sides:

$$\frac{4(x^2 + 35x - 114)}{4} = \frac{0}{4}$$

$$x^2 + 35x - 114 = 0$$
Step 1: Product = -114 · 1 = -114
Sum = 35

**We need to find numbers that multiply to -114 and add to 35. Because the product is negative we must have one negative and one positive factor. Since the sum is positive, the larger factor must be positive.

Step 2:

Factors of -114:	Sum of factors:
-1, 114	-1 + 114 = 113
-2, 57	-2 + 57 = 55
-3, 38	-3 + 38 = 35
-6, 19	-6 + 19 = 13

-3 and 38 are the factors that will make a product of -114 and a sum of 35.

Step 3:

$$x^{2} + 35x - 114 = 0$$
$$x^{2} - 3x + 38x - 114 = 0$$

Step 4:
$$x^2 - 3x + 38x - 114 = 0$$

The first two terms are divisible by x. The last two terms are divisible by 38.

$$x\left(\frac{x^2}{x} - \frac{3x}{x}\right) + 38\left(\frac{38x}{38} - \frac{114}{38}\right) = 0$$
$$x(x-3) + 38(x-3) = 0$$

Both terms have an (x - 3), so we can factor that out.

$$(x-3)\left(\frac{x(x-3)}{x-3} + \frac{38(x-3)}{x-3}\right) = 0$$

(x-3)(x+38) = 0

Finally, we use the zero-product property to solve.

Either x - 3 = 0 or x + 38 = 0.

x - 3 = 0	or	x + 38 = 0
+3 +3		-38 - 38
x = 3	or	x = -38

The two solutions are x = 3 or x = -38. Since the negative solution does not make sense in this situation, we will only consider the positive solution.

Check:

Replace x with 3

$$(60 + 2x)(10 + 2x) = 1056$$

 $(60 + 2 \cdot 3)(10 + 2 \cdot 3) = 1056$
 $(60 + 6)(10 + 6) = 1056$
 $(66)(16) = 1056$
 $1056 = 1056$

Because the equation is true, we know we have the correct solution.

The picture frame is 3 cm wide.

Guided Practice:

1) Suppose a diver leaps from the edge of a cliff 80 feet above the ocean with an initial upward velocity of 8 feet per second. How long will it take the diver to enter the water below? (Reminder: Vertical motion is modeled by $h = -16t^2 + vt + s$)

 $h = -16t^2 + vt + s$ $0 = -16t^2 + 8t + 80$

Step 1: $Product = -16 \cdot 80 = -1280$

Sum = 8

**We need to find numbers that multiply to -1280 and add to 8. Because the product is negative we must have one negative and one positive factor. Since the sum is positive, the larger factor must be positive.

Step 2:

Factors of -1280:	Sum of factors:
-1, 1280	-1 + 1280 = 1279
-2, 640	-2 + 640 = 638
-4, 320	-4 + 320 = 316
-8, 160	-8 + 160 = 152
-10, 128	-10 + 128 = 118
-16, 80	-16 + 80 = 64
-20, 64	-20 + 64 = 44
-32, 40	-32 + 40 = 8

-32 and 40 are the factors that will make a product of -1280 and a sum of 8.

Step 3:

$$0 = -16t^{2} + 8t + 80$$

$$0 = -16t^{2} - 32t + 40t + 80$$

Step 4:

$$0 = -16t^2 - 32t \left\{ +40t + 80 \right\}$$

The first two terms are divisible by -16t. The last two terms are divisible by 40.

$$0 = -16t \left(\frac{-16t^2}{-16t} + \frac{-32t}{-16t} \right) \left(+ 40 \left(\frac{40t}{40} + \frac{80}{40} \right) \right)$$

0 = -16t(t+2) + 40(t+2)

Both terms have a (t + 2), so we can factor that out.

$$0 = (t+2)\left(\frac{-16t(t+2)}{t+2} + \frac{40(t+2)}{t+2}\right)$$
$$0 = (t+2)(-16t+40)$$

The last binomial has a common factor of 8, so let's factor that out.

$$0 = 8(t+2)\left(\frac{-16t}{8} + \frac{40}{8}\right)$$
$$0 = 8(t+2)(-2t+5)$$

Finally, we use the zero-product property to solve.

Since $0 \neq 8$, we can ignore that part.

Either t + 2 = 0 or -2t + 5 = 0.

$$t + 2 = 0 \quad \text{or} \quad -2t + 5 = 0$$

-2 - 2 $-5 - 5$
$$t = -2 \quad -2t = -5$$

$$\frac{-2t}{-2} = \frac{-5}{-2}$$

$$t = -2 \quad \text{or} \quad t = \frac{5}{2} = 2\frac{1}{2}$$

The two solutions are t = -2 or $t = 2\frac{1}{2}$. Since the negative solution does not make sense in this situation, we will only consider the positive solution.

Check:

Replace x with 2 $\frac{1}{2}$

$$0 = -16t^{2} + 8t + 80$$

$$0 = -16(2.5)^{2} + 8(2.5) + 80$$

$$0 = -16(6.25) + 20 + 80$$

$$0 = -100 + 100$$

$$0 = 0$$

Because the equation is true, we know we have the correct solution.

The diver will reach the water 2 $\frac{1}{2}$ seconds after they leap.

2) A picture is 16 cm long by 20 cm wide. The picture is framed using material that is x cm wide. The area of the frame and picture together is 480 square centimeters. What is the width of the framing material?

The length of the frame and picture together is 16 + 2x since we are adding x cm on each side of the picture.

The width of the frame and picture together is 20 + 2x since we are adding x cm on each side of the picture.

The area of the picture and the frame can be found by multiplying length times width.

$$(16 + 2x)(20 + 2x) = 480$$
$$320 + 32x + 40x + 4x^{2} = 480$$

 $320 + 72x + 4x^2 = 480$

We need to get one side equal to zero.

 $320 + 72x + 4x^2 = 480$

 $-160 + 72x + 4x^2 = 0$

$$4x^2 + 72x - 160 = 0$$

All of the above terms are divisible by 4.

$$4(x^2 + 18x - 40) = 0$$

Divide by 4 on both sides:

$$\frac{4(x^2 + 18x - 40)}{4} = \frac{0}{4}$$

x² + 18x - 40 = 0
Step 1: Product = -40 \cdot 1 = -40
Sum = 18

**We need to find numbers that multiply to -40 and add to 18. Because the product is negative we must have one negative and one positive factor. Since the sum is positive, the larger factor must be positive.

Step 2:

Factors of -40:	Sum of factors:
-1, 40	-1 + 40 = 39
-2, 20	-2 + 20 = 18
-4, 10	-4 + 10 = 6
-5, 8	-5 + 8 = 3

-2 and 20 are the factors that will make a product of -40 and a sum of 18.

Step 3:

$$x^{2} + 18x - 40 = 0$$
$$x^{2} - 2x + 20x - 40 = 0$$

Step 4:

$$x^2 - 2x \left\{ + 20x - 40 = 0 \right\}$$

The first two terms are divisible by x. The last two terms are divisible by 20.

$$x\left(\frac{x^{2}}{x} - \frac{2x}{x}\right) \left\{ + 20\left(\frac{20x}{20} - \frac{40}{20}\right) = 0$$
$$x(x-2) + 20(x-2) = 0$$

Both terms have an (x - 2), so we can factor that out.

$$(x-2)\left(\frac{x(x-2)}{x-2} + \frac{20(x-2)}{x-2}\right) = 0$$
$$(x-2)(x+20) = 0$$

Finally, we use the zero-product property to solve.

Either x - 2 = 0 or x + 20 = 0.

x - 2 = 0	or	x + 20 = 0
+2 +2		-20 -20
x = 2	or	x = -20

The two solutions are x = 2 or x = -20. Since the negative solution does not makes sense in this situation, we will only consider the positive solution.

Check:

Replace x with 2

$$(16 + 2x)(20 + 2x) = 480$$
$$(16 + 2 \cdot 2)(20 + 2 \cdot 2) = 480$$
$$(16 + 4)(20 + 4) = 480$$
$$(20)(24) = 480$$
$$480 = 480$$

Because the equation is true, we know we have the correct solution.

The picture frame is 2 cm wide.