## Vertical Motion Model:

A model for the vertical motion of a projected object is given by the equation $h=-16 t^{2}+v t+s$, where $h$ is the height in feet, $t$ is the time in seconds, $v$ is the initial upward velocity in feet per second, and $s$ is the initial height of the object in feet.

## Example: Using the Vertical Motion Model:

PEP RALLY At a pep rally, small foam footballs are launched by cheerleaders using a sling-shot. How long is a football in the air if a student catches it on its way down 26 feet above the gym floor?

$$
\begin{array}{rl}
\begin{array}{l}
h=-16 t^{2}+v t+s \\
26
\end{array}=-16 t^{2}+42 t+6 & \text { Vertical motion model } \\
0=-16 t^{2}+42 t-20 & \text { Subtract } 26 \text { from } \\
0=-2\left(8 t^{2}-21 t+10\right) & \text { each side. } \\
0=8 a c t o r ~ o u t-2 . \\
0=8 t^{2}-21 t+10 & \text { Divide each side by }-2 . \\
0=(8 t-5)(t-2) & \text { Factor } 8 t^{2}-21 t+10 . \\
8 t-5=0 \text { or } t-2=0 & \text { Zero Product Property } \\
8 t=5 & t=2
\end{array} \text { Solve each equation. }
$$



The solutions are $\frac{5}{8}$ second and 2 seconds.
The first time represents how long it takes the football to reach a height of 26 feet on its way up. The later time represents how long it takes the ball to reach a height of 26 feet again on its way down. Thus, the football will be in the air for 2 seconds before the student catches it.

## Check your progress:

The feet of a gymnast making a vault leave the horse at a height of 8 feet with an initial upward velocity of 8 feet per second. Use the model for vertical motion to find the time $t$ in seconds it take for the gymnast's feet to reach the mat.

$h=-16 t^{2}+v t+s$
$0=-16 t^{2}+8 t+8$

Step 1: $\quad$ Product $=-16 \cdot 8=-128$

$$
\text { Sum }=8
$$

**We need to find numbers that multiply to -128 and add to 8 . Because the product is negative we must have one negative and one positive factor. Since the sum is positive, the larger factor must be positive.

Step 2:
Factors of -128: Sum of factors:
-1, 128
$-1+128=127$
-2, 64
$-2+64=62$
$-4,32$
$-4+32=28$
$-8,16$
$-8+16=8$
-8 and 16 are the factors that will make a product of -128 and a sum of 8 .

Step 3:

$$
\begin{gathered}
0=-16 t^{2}+8 t+8 \\
0=-16 t^{2}\ulcorner-8 t+16 t+8
\end{gathered}
$$

Step 4:
$\left.0=-16 t^{2}-8 t\right\}+16 t+8$
The first two terms are divisible by $-8 t$. The last two terms are divisible by 8 .
$\left.0=-8 t\left(\frac{-16 t^{2}}{-8 t}+\frac{-8 t}{-8 t}\right)\right\}+8\left(\frac{16 t}{8}+\frac{8}{8}\right)$
$0=-8 t(2 t+1)+8(2 t+1)$
Both terms have a $(2 t+1)$, so we can factor that out.
$0=(2 t+1)\left(\frac{-8 t(2 t+1)}{2 t+1}+\frac{8(2 t+1)}{2 t+1}\right)$
$0=(2 t+1)(-8 t+8)$
The last binomial has a common factor of 8, so let's factor that out.
$0=8(2 t+1)\left(\frac{-8 t}{8}+\frac{8}{8}\right)$
$0=8(2 t+1)(-t+1)$

Finally, we use the zero-product property to solve.
Since $0 \neq 8$, we can ignore that part.
Either $2 t+1=0$ or $-t+1=0$.

$$
\begin{array}{rrr}
2 t+1=0 & \text { or } & -t+1=0 \\
-1-1 & -1-1 \\
2 t=-1 & -t=-1 \\
\frac{2 t}{2}=\frac{-1}{2} & \frac{-t}{-1}=\frac{-1}{-1} \\
t=-\frac{1}{2} & \text { or } & t=1
\end{array}
$$

The two solutions are $t=-\frac{1}{2}$ or $t=1$. Since the negative solution does not makes sense in this situation, we will only consider the positive solution.

Check:
Replace x with 1

$$
\begin{aligned}
& 0=-16 t^{2}+8 t+8 \\
& 0=-16(1)^{2}+8(1)+8 \\
& 0=-16(1)+8+8 \\
& 0=-16+16 \\
& 0=0
\end{aligned}
$$

Because the equation is true, we know we have the correct solution.
The gymnast's feet reach the mat 1 second after she leaves the horse.

## Example: Picture Frames

A picture is 60 cm long by 10 cm wide. The picture is framed using material that is $x \mathrm{~cm}$ wide. The area of the frame and picture together is 1,056 square centimeters. What is the width of the framing material?


The length of the frame and picture together is $60+2 x$ since we are adding $x \mathrm{~cm}$ on each side of the picture.

The width of the frame and picture together is $10+2 x$ since we are adding $x \mathrm{~cm}$ on each side of the picture.

The area of the picture and the frame can be found by multiplying length times width.
$(60+2 x)(10+2 x)=1056$
$600+120 x+20 x+4 x^{2}=1056$
$600+140 x+4 x^{2}=1056$
We need to get one side equal to zero.
$600+140 x+4 x^{2}=1056$
$-1056 \quad-1056$
$-456+140 x+4 x^{2}=0$
$4 x^{2}+140 x-456=0$
All of the above terms are divisible by 4 .
$4\left(x^{2}+35 x-114\right)=0$
Divide by 4 on both sides:
$\frac{4\left(x^{2}+35 x-114\right)}{4}=\frac{0}{4}$
$x^{2}+35 x-114=0$
Step 1: $\quad$ Product $=-114 \cdot 1=-114$

$$
\text { Sum }=35
$$

**We need to find numbers that multiply to -114 and add to 35 . Because the product is negative we must have one negative and one positive factor. Since the sum is positive, the larger factor must be positive.

Step 2:
Factors of -114: Sum of factors:
$-1,114 \quad-1+114=113$
-2, 57
$-2+57=55$
-3, 38
$-3+38=35$
-6, 19

$$
-6+19=13
$$

-3 and 38 are the factors that will make a product of -114 and a sum of 35 .

Step 3:

$$
\begin{gathered}
x^{2}+35 x-114=0 \\
x^{2 /-3 x+38 x}-114=0
\end{gathered}
$$

Step 4:
$\left.x^{2}-3 x\right\}+38 x-114=0$
The first two terms are divisible by $x$. The last two terms are divisible by 38 .
$\left.x\left(\frac{x^{2}}{x}-\frac{3 x}{x}\right)\right\}+38\left(\frac{38 x}{38}-\frac{114}{38}\right)=0$
$x(x-3)+38(x-3)=0$
Both terms have an $(x-3)$, so we can factor that out.

$$
\begin{aligned}
& (x-3)\left(\frac{x(x-3)}{x-3}+\frac{38(x-3)}{x-3}\right)=0 \\
& (x-3)(x+38)=0
\end{aligned}
$$

Finally, we use the zero-product property to solve.
Either $x-3=0$ or $x+38=0$.

$$
\begin{array}{rrr}
x-3=0 & \text { or } & x+38=0 \\
+3+3 & & -38-38 \\
x=3 & \text { or } & x=-38
\end{array}
$$

The two solutions are $x=3$ or $x=-38$. Since the negative solution does not makes sense in this situation, we will only consider the positive solution.

Check:
Replace x with 3

$$
\begin{aligned}
& (60+2 x)(10+2 x)=1056 \\
& (60+2 \cdot 3)(10+2 \cdot 3)=1056 \\
& (60+6)(10+6)=1056 \\
& (66)(16)=1056 \\
& 1056=1056
\end{aligned}
$$

Because the equation is true, we know we have the correct solution.
The picture frame is 3 cm wide.

## Guided Practice:

1) Suppose a diver leaps from the edge of a cliff 80 feet above the ocean with an initial upward velocity of 8 feet per second. How long will it take the diver to enter the water below? (Reminder: Vertical motion is modeled by $h=-16 t^{2}+v t+s$ )
$h=-16 t^{2}+v t+s$
$0=-16 t^{2}+8 t+80$
Step 1: $\quad$ Product $=-16 \cdot 80=-1280$

$$
\text { Sum }=8
$$

**We need to find numbers that multiply to -1280 and add to 8 . Because the product is negative we must have one negative and one positive factor. Since the sum is positive, the larger factor must be positive.

Step 2:
Factors of -1280: Sum of factors:
-1, 1280

$$
-1+1280=1279
$$

-2, 640
$-2+640=638$
-4, 320
$-4+320=316$
$-8,160$
$-8+160=152$
-10, 128
$-10+128=118$
-16, 80
$-16+80=64$
-20, 64
$-20+64=44$
-32, 40
$-32+40=8$
-32 and 40 are the factors that will make a product of -1280 and a sum of 8 .

Step 3:

$$
\begin{gathered}
0=-16 t^{2}+8 t+80 \\
0=-16 t^{2}\ulcorner-32 t+40 t+80
\end{gathered}
$$

Step 4:
$\left.0=-16 t^{2}-32 t\right\}+40 t+80$
The first two terms are divisible by $-16 t$. The last two terms are divisible by 40 .
$\left.0=-16 t\left(\frac{-16 t^{2}}{-16 t}+\frac{-32 t}{-16 t}\right)\right\}+40\left(\frac{40 t}{40}+\frac{80}{40}\right)$
$0=-16 t(t+2)+40(t+2)$
Both terms have a $(t+2)$, so we can factor that out.
$0=(t+2)\left(\frac{-16 t(t+2)}{t+2}+\frac{40(t+2)}{t+2}\right)$
$0=(t+2)(-16 t+40)$
The last binomial has a common factor of 8 , so let's factor that out.
$0=8(t+2)\left(\frac{-16 t}{8}+\frac{40}{8}\right)$
$0=8(t+2)(-2 t+5)$

Finally, we use the zero-product property to solve.
Since $0 \neq 8$, we can ignore that part.
Either $t+2=0$ or $-2 t+5=0$.

$$
\begin{aligned}
& t+2=0 \quad \text { or } \quad-2 t+5=0 \\
& \begin{array}{llll}
-2 & -2 & -5 & -5
\end{array} \\
& t=-2 \quad-2 t=-5 \\
& \frac{-2 t}{-2}=\frac{-5}{-2} \\
& t=-2 \quad \text { or } \quad t=\frac{5}{2}=2 \frac{1}{2}
\end{aligned}
$$

The two solutions are $t=-2$ or $t=2 \frac{1}{2}$. Since the negative solution does not makes sense in this situation, we will only consider the positive solution.

Check:
Replace x with $21 / 2$

$$
\begin{aligned}
& 0=-16 t^{2}+8 t+80 \\
& 0=-16(2.5)^{2}+8(2.5)+80 \\
& 0=-16(6.25)+20+80 \\
& 0=-100+100 \\
& 0=0
\end{aligned}
$$

Because the equation is true, we know we have the correct solution.
The diver will reach the water $21 / 2$ seconds after they leap.
2) A picture is 16 cm long by 20 cm wide. The picture is framed using material that is $x \mathrm{~cm}$ wide. The area of the frame and picture together is 480 square centimeters. What is the width of the framing material?

The length of the frame and picture together is $16+2 x$ since we are adding $x \mathrm{~cm}$ on each side of the picture.

The width of the frame and picture together is $20+2 x$ since we are adding $x \mathrm{~cm}$ on each side of the picture.

The area of the picture and the frame can be found by multiplying length times width.
$(16+2 x)(20+2 x)=480$
$320+32 x+40 x+4 x^{2}=480$
$320+72 x+4 x^{2}=480$
We need to get one side equal to zero.
$320+72 x+4 x^{2}=480$
$-480 \quad-480$
$-160+72 x+4 x^{2}=0$
$4 x^{2}+72 x-160=0$
All of the above terms are divisible by 4 .
$4\left(x^{2}+18 x-40\right)=0$
Divide by 4 on both sides:
$\frac{4\left(x^{2}+18 x-40\right)}{4}=\frac{0}{4}$
$x^{2}+18 x-40=0$
Step 1: $\quad$ Product $=-40 \cdot 1=-40$

$$
\text { Sum }=18
$$

**We need to find numbers that multiply to -40 and add to 18 . Because the product is negative we must have one negative and one positive factor. Since the sum is positive, the larger factor must be positive.

Step 2:
Factors of -40: Sum of factors:
-1, 40
$-1+40=39$
$-2,20$
$-2+20=18$
$-4,10$
$-4+10=6$
$-5,8$
$-5+8=3$
-2 and 20 are the factors that will make a product of -40 and a sum of 18 .

Step 3:

$$
\begin{gathered}
x^{2}+18 x-40=0 \\
x^{2} \overbrace{-2 x+20 x}-40=0
\end{gathered}
$$

Step 4:
$x^{2}-2 x\{+20 x-40=0$
The first two terms are divisible by $x$. The last two terms are divisible by 20 .
$\left.x\left(\frac{x^{2}}{x}-\frac{2 x}{x}\right)\right\}+20\left(\frac{20 x}{20}-\frac{40}{20}\right)=0$
$x(x-2)+20(x-2)=0$
Both terms have an $(x-2)$, so we can factor that out.

$$
\begin{aligned}
& (x-2)\left(\frac{x(x-2)}{x-2}+\frac{20(x-2)}{x-2}\right)=0 \\
& (x-2)(x+20)=0
\end{aligned}
$$

Finally, we use the zero-product property to solve.
Either $x-2=0$ or $x+20=0$.

$$
\begin{array}{rrr}
x-2=0 & \text { or } & x+20=0 \\
+2+2 & & -20-20 \\
x=2 & \text { or } & x=-20
\end{array}
$$

The two solutions are $x=2$ or $x=-20$. Since the negative solution does not makes sense in this situation, we will only consider the positive solution.

Check:
Replace x with 2

$$
\begin{aligned}
& (16+2 x)(20+2 x)=480 \\
& (16+2 \cdot 2)(20+2 \cdot 2)=480 \\
& (16+4)(20+4)=480 \\
& (20)(24)=480 \\
& 480=480
\end{aligned}
$$

Because the equation is true, we know we have the correct solution.
The picture frame is 2 cm wide.

