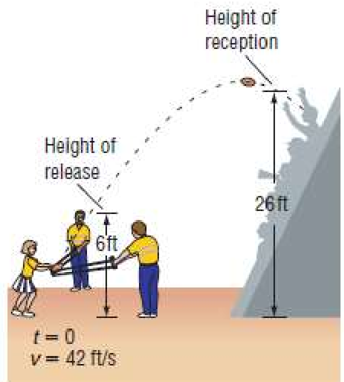
Vertical Motion Model:

A model for the vertical motion of a projected object is given by the equation $h = -16t^2 + vt + s$, where h is the height in feet, t is the time in seconds, v is the initial upward velocity in feet per second, and s is the initial height of the object in feet.

Example: Using the Vertical Motion Model:

PEP RALLY At a pep rally, small foam footballs are launched by cheerleaders using a sling-shot. How long is a football in the air if a student catches it on its way down 26 feet above the gym floor?



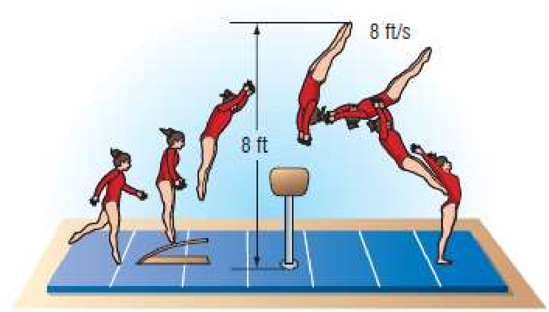
$h = -16t^2 + vt + s$	Vertical motion model
$26 = -16t^2 + 42t + 6$	$h = 26, v = 42, s = 6$
$0 = -16t^2 + 42t - 20$	Subtract 26 from each side.
$0 = -2(8t^2 - 21t + 10)$	Factor out -2.
$0 = 8t^2 - 21t + 10$	Divide each side by -2.
$0 = (8t - 5)(t - 2)$	Factor $8t^2 - 21t + 10$.
$8t - 5 = 0$ or $t - 2 = 0$	Zero Product Property
$8t = 5$ $\quad t = 2$	Solve each equation.
$t = \frac{5}{8}$	

The solutions are $\frac{5}{8}$ second and 2 seconds.

The first time represents how long it takes the football to reach a height of 26 feet on its way up. The later time represents how long it takes the ball to reach a height of 26 feet again on its way down. Thus, the football will be in the air for 2 seconds before the student catches it.

Check your progress:

The feet of a gymnast making a vault leave the horse at a height of 8 feet with an initial upward velocity of 8 feet per second. Use the model for vertical motion to find the time t in seconds it take for the gymnast's feet to reach the mat.



$h = -16t^2 + vt + s$

$0 = -16t^2 + 8t + 8$

Step 1: Product $= -16 \cdot 8 = -128$

Sum $= 8$

**We need to find numbers that multiply to -128 and add to 8. Because the product is negative we must have one negative and one positive factor. Since the sum is positive, the larger factor must be positive.

Step 2:

Factors of -128:	Sum of factors:
-1, 128	$-1 + 128 = 127$
-2, 64	$-2 + 64 = 62$
-4, 32	$-4 + 32 = 28$
-8, 16	$-8 + 16 = 8$

-8 and 16 are the factors that will make a product of -128 and a sum of 8.

Step 3:

$$0 = -16t^2 + \underbrace{8t}_{} + 8$$

$$0 = -16t^2 \overbrace{- 8t + 16t}^{} + 8$$

Step 4:

$$0 = -16t^2 - 8t + 16t + 8$$

The first two terms are divisible by $-8t$. The last two terms are divisible by 8.

$$0 = -8t\left(\frac{-16t^2}{-8t} + \frac{-8t}{-8t}\right) + 8\left(\frac{16t}{8} + \frac{8}{8}\right)$$

$$0 = -8t(2t + 1) + 8(2t + 1)$$

Both terms have a $(2t + 1)$, so we can factor that out.

$$0 = (2t + 1)\left(\frac{-8t(2t + 1)}{2t + 1} + \frac{8(2t + 1)}{2t + 1}\right)$$

$$0 = (2t + 1)(-8t + 8)$$

The last binomial has a common factor of 8, so let's factor that out.

$$0 = 8(2t + 1)\left(\frac{-8t}{8} + \frac{8}{8}\right)$$

$$0 = 8(2t + 1)(-t + 1)$$

Finally, we use the zero-product property to solve.

Since $0 \neq 8$, we can ignore that part.

Either $2t + 1 = 0$ or $-t + 1 = 0$.

$2t + 1 = 0$ or $-t + 1 = 0$

$-1 \quad -1$ $\qquad$ $-1 \quad -1$

$2t = -1$ $\qquad$ $-t = -1$

$\frac{2t}{2} = \frac{-1}{2}$ $\qquad$ $\frac{-t}{-1} = \frac{-1}{-1}$

$t = -\frac{1}{2}$ or $t = 1$

The two solutions are $t = -\frac{1}{2}$ or $t = 1$. Since the negative solution does not makes sense in this situation, we will only consider the positive solution.

Check:

Replace x with 1

$0 = -16t^2 + 8t + 8$

$0 = -16(1)^2 + 8(1) + 8$

$0 = -16(1) + 8 + 8$

$0 = -16 + 16$

$0 = 0$

Because the equation is true, we know we have the correct solution.

The gymnast's feet reach the mat 1 second after she leaves the horse.