

Example: Solve an Equation by Factoring**Solve  $x^2 + 5x - 6 = 0$ . Check the solutions.**

$$x^2 + 5x - 6 = 0 \quad \text{Original equation}$$

$$(x - 1)(x + 6) = 0 \quad \text{Factor.}$$

$$x - 1 = 0 \quad \text{or} \quad x + 6 = 0 \quad \text{Zero Product Property}$$

$$x = 1 \quad x = -6 \quad \text{Solve each equation.}$$

The roots are 1 and  $-6$ . Check by substituting 1 and  $-6$  for  $x$  in the original equation.

Check your progress:

$$1) \quad -9x + x^2 = 22$$

We need to get one side of the equation equal to zero.

$$-9x + x^2 = 22$$

$$-22 \quad -22$$

$$-9x + x^2 - 22 = 0$$

Now, we need to put the polynomial on the left in standard form so that it is factorable.

$$x^2 - 9x - 22 = 0$$

$$\text{Factor: } x^2 - 9x - 22 = 0$$

$$\text{Step 1:} \quad \text{Product} = 1 \cdot -22 = -22$$

$$\text{Sum} = -9$$

**\*\*We need to find numbers that multiply to  $-22$  and add to  $-9$ . Because the product is negative, one of the two factors must be negative. Because the sum is also negative, the larger factor must be negative.**

Step 2:

Factors of  $-22$ :                  Sum of factors:

$$1, -22 \quad 1 + -22 = -21$$

$$2, -11 \quad 2 + -11 = -9$$

2 and  $-11$  are the factors that will make a product of  $-22$  and a sum of  $-9$ .

Step 3:

$$x^2 - 9x - 22 = 0$$

$$x^2 + \overbrace{2x - 11x} - 22 = 0$$

Step 4:

$$x^2 + 2x \left\{ -11x - 22 = 0 \right.$$

The first two terms are divisible by  $x$ . The last two terms are divisible by  $-11$ .

$$x \left( \frac{x^2}{x} + \frac{2x}{x} \right) \left\{ -11 \left( \frac{-11x}{-11} + \frac{-22}{-11} \right) = 0 \right.$$

$$x(x + 2) - 11(x + 2) = 0$$

Both terms have an  $(x + 2)$ , so we can factor that out.

$$(x + 2) \left( \frac{x(x + 2)}{x + 2} - \frac{11(x + 2)}{x + 2} \right) = 0$$

$$(x + 2)(x - 11) = 0$$

Finally, we use the zero-product property to solve.

$$(x + 2)(x - 11) = 0$$

Either  $x + 2 = 0$  or  $x - 11 = 0$ .

$$\begin{array}{ccc} x + 2 = 0 & \text{or} & x - 11 = 0 \\ -2 & -2 & +11 & +11 \\ x = -2 & \text{or} & x = 11 \end{array}$$

The two solutions are  $x = -2$  or  $x = 11$ . \*\*These are our solutions, but it is always a good idea to double-check our solutions and see if we have the correct ones.

Check:

Replace  $x$  with  $-2$

$$-9x + x^2 = 22$$

$$-9(-2) + (-2)^2 = 22$$

$$18 + 4 = 22$$

$$22 = 22$$

Replace  $x$  with  $11$

$$-9x + x^2 = 22$$

$$-9(11) + (11)^2 = 22$$

$$-99 + 121 = 22$$

$$22 = 22$$

Because both equations are true, we know we have found the correct solutions.

$$2) \quad x^2 + 9 = 10x$$

We need to get one side of the equation equal to zero.

$$x^2 + 9 = 10x$$

$$-10x \quad -10x$$

$$x^2 - 10x + 9 = 0$$

\*\*Put the -10x between the other two terms so that the polynomial is in standard form.

Factor:  $x^2 - 10x + 9 = 0$

Step 1:            Product =  $1 \cdot 9 = 9$

Sum =  $-10$

\*\*We need to find numbers that multiply to 9 and add to -10. Because the product is positive and the sum is negative, both factors must be negative.

Step 2:

Factors of -22:            Sum of factors:

-1, -9                       $-1 + -9 = -10$

-3, -3                       $-3 + -3 = -6$

-1 and -9 are the factors that will make a product of 9 and a sum of -10.

Step 3:

$$x^2 - 10x + 9 = 0$$

$$x^2 \overbrace{-1x - 9x} + 9 = 0$$

Step 4:

$$x^2 - 1x \left\{ -9x + 9 = 0 \right.$$

The first two terms are divisible by  $x$ . The last two terms are divisible by -9.

$$x \left( \frac{x^2}{x} - \frac{1x}{x} \right) \left\{ -9 \left( \frac{-9x}{-9} + \frac{9}{-9} \right) = 0 \right.$$

$$x(x - 1) - 9(x - 1) = 0$$

Both terms have an  $(x - 1)$ , so we can factor that out.

$$(x - 1) \left( \frac{x(x - 1)}{x - 1} - \frac{9(x - 1)}{x - 1} \right) = 0$$

$$(x - 1)(x - 9) = 0$$

Finally, we use the zero-product property to solve.

$$(x - 1)(x - 9) = 0$$

Either  $x - 1 = 0$  or  $x - 9 = 0$ .

$$x - 1 = 0 \quad \text{or} \quad x - 9 = 0$$

$$+1 \quad +1 \quad \quad \quad +9 \quad +9$$

$$x = 1 \quad \text{or} \quad x = 9$$

The two solutions are  $x = 1$  or  $x = 9$ .

\*\*These are our solutions, but it is always a good idea to double-check our solutions and see if we have the correct ones.

Check:

Replace x with 1

$$x^2 + 9 = 10x$$

$$(1)^2 + 9 = 10(1)$$

$$1 + 9 = 10$$

$$10 = 10$$

Replace x with 9

$$x^2 + 9 = 10x$$

$$(9)^2 + 9 = 10(9)$$

$$81 + 9 = 90$$

$$90 = 90$$

Because both equations are true, we know we have found the correct solutions.

Example: Solve an Equation by Factoring

**Solve  $8a^2 - 9a - 5 = 4 - 3a$ . Check the solutions.**

$$8a^2 - 9a - 5 = 4 - 3a$$

Write the equation.

$$8a^2 - 6a - 9 = 0$$

Rewrite so that one side equals 0.

$$(4a + 3)(2a - 3) = 0$$

Factor the left side.

$$4a + 3 = 0 \quad \text{or} \quad 2a - 3 = 0$$

Zero Product Property

$$4a = -3$$

$$2a = 3$$

Solve each equation.

$$a = -\frac{3}{4}$$

$$a = \frac{3}{2}$$

The roots are  $-\frac{3}{4}$  and  $\frac{3}{2}$ .

**CHECK** Check each solution in the original equation.

$$8a^2 - 9a - 5 = 4 - 3a$$

$$8a^2 - 9a - 5 = 4 - 3a$$

$$8\left(-\frac{3}{4}\right)^2 - 9\left(-\frac{3}{4}\right) - 5 \stackrel{?}{=} 4 - 3\left(-\frac{3}{4}\right) \quad 8\left(\frac{3}{2}\right)^2 - 9\left(\frac{3}{2}\right) - 5 \stackrel{?}{=} 4 - 3\left(\frac{3}{2}\right)$$

$$\frac{9}{2} + \frac{27}{4} - 5 \stackrel{?}{=} 4 + \frac{9}{4}$$

$$18 - \frac{27}{2} - 5 \stackrel{?}{=} 4 - \frac{9}{2}$$

$$\frac{25}{4} = \frac{25}{4} \quad \checkmark$$

$$-\frac{1}{2} = -\frac{1}{2} \quad \checkmark$$

Check your progress:

$$1) \quad 6x^2 - 7x = 7x + 12$$

We need to get one side of the equation equal to zero.

$$6x^2 - 7x = 7x + 12$$

$$-7x \quad -7x$$

$$6x^2 - 14x = 12$$

$$-12 \quad -12$$

$$6x^2 - 14x - 12 = 0$$

$$\text{Factor: } 6x^2 - 14x - 12 = 0$$

$$\text{Step 1:} \quad \text{Product} = 6 \cdot -12 = -72$$

$$\text{Sum} = -14$$

\*\*We need to find numbers that multiply to -72 and add to -14. Because the product is negative, one of the factors must be negative. Because the sum is negative, the larger factor must be negative.

Step 2:

Factors of -72: Sum of factors:

$$1, -72 \quad 1 + -72 = -71$$

$$2, -36 \quad 2 + -36 = -34$$

$$3, -24 \quad 3 + -24 = -21$$

$$4, -18 \quad 4 + -18 = -14$$

$$6, -12 \quad 6 + -12 = -6$$

$$8, -9 \quad 8 + -9 = -1$$

4 and -18 are the factors that will make a product of -72 and a sum of -14.

Step 3:

$$6x^2 - 14x - 12 = 0$$

$$6x^2 + \overbrace{4x - 18x} - 20 = 0$$

Step 4:

$$6x^2 + 4x \left\{ -18x - 12 = 0 \right.$$

The first two terms are divisible by  $2x$ . The last two terms are divisible by -6.

$$2x \left( \frac{6x^2}{2x} + \frac{4x}{2x} \right) \left\{ -6 \left( \frac{-18x}{-6} + \frac{-12}{-6} \right) = 0 \right.$$

$$2x(3x + 2) - 6(3x + 2) = 0$$

Both terms have an  $(3x + 2)$ , so we can factor that out.

$$(3x + 2) \left( \frac{2x(3x + 2)}{3x + 2} - \frac{6(3x + 2)}{3x + 2} \right) = 0$$

$$(3x + 2)(2x - 6) = 0$$

At this point, we should recognize that the second binomial,  $2x - 6$ , has a common factor of 2. That means both terms are divisible by 2. So, we should divide that out.

$$2(3x + 2)\left(\frac{2x}{2} - \frac{6}{2}\right) = 0$$

$$2(3x + 2)(x - 3) = 0$$

Finally, we use the zero-product property to solve.

$$2(3x + 2)(x - 3) = 0$$

Either  $3x + 2 = 0$  or  $x - 3 = 0$ . The 2 out front does not affect the problem since 2 cannot be equal to zero. So, we ignore the 2 out front.

$$3x + 2 = 0 \quad \text{or} \quad x - 3 = 0$$

$$\begin{array}{cc} -2 & -2 \\ 3x = -2 & \text{or} \end{array} \quad \begin{array}{cc} +3 & +3 \\ x = 3 \end{array}$$

$$3x = -2 \quad \text{or} \quad x = 3$$

$$\frac{3x}{3} = \frac{-2}{3}$$

$$x = -\frac{2}{3} \quad \text{or} \quad x = 3$$

The two solutions are  $x = -\frac{2}{3}$  or  $x = 3$ . \*\*These are our solutions, but it is always a good idea to double-check our solutions and see if we have the correct ones.

Check:

Replace x with  $-\frac{2}{3}$

$$6x^2 - 14x = 12$$

$$6\left(-\frac{2}{3}\right)^2 - 14\left(-\frac{2}{3}\right) = 12$$

$$6\left(\frac{4}{9}\right) + \frac{28}{3} = 12$$

$$\frac{24}{9} + 9\frac{1}{3} = 12$$

$$\frac{8}{3} + 9\frac{1}{3} = 12$$

$$2\frac{2}{3} + 9\frac{1}{3} = 12$$

$$12 = 12$$

Replace x with 3

$$6x^2 - 14x = 12$$

$$6(3)^2 - 14(3) = 12$$

$$6(9) - 42 = 12$$

$$54 - 42 = 12$$

$$12 = 12$$

Because both equations are true, we know we have found the correct solutions.

$$2) -19x + 10x^2 = -7$$

We need to get one side of the equation equal to zero.

$$-19x + 10x^2 = -7$$

$$+7 \quad +7$$

$$-19x + 10x^2 + 7 = 0$$

Now, we need to put the polynomial on the left in standard form so that it is factorable.

$$10x^2 - 19x + 7 = 0$$

$$\text{Factor: } 10x^2 - 19x + 7 = 0$$

$$\text{Step 1:} \quad \text{Product} = 10 \cdot 7 = 70$$

$$\text{Sum} = -19$$

\*\*We need to find numbers that multiply to 70 and add to -19. Because the product is positive and the sum is negative, both factors must be negative.

Step 2:

Factors of 70:

Sum of factors:

$$-1, -70$$

$$-1 + -70 = -71$$

$$-2, -35$$

$$-2 + -35 = -37$$

$$-5, -14$$

$$-5 + -14 = -19$$

$$-7, -10$$

$$-7 + -10 = -17$$

-5 and -14 are the factors that will make a product of 70 and a sum of -19.

Step 3:

$$10x^2 - 19x + 7 = 0$$

$$10x^2 - \overbrace{5x - 14x} + 7 = 0$$

Step 4:

$$10x^2 - 5x \left\{ -14x + 7 = 0 \right.$$

The first two terms are divisible by  $5x$ . The last two terms are divisible by  $-7$ .



$$5x \left( \frac{10x^2}{5x} - \frac{5x}{5x} \right) \left\{ -7 \left( \frac{-14x}{-7} + \frac{7}{-7} \right) \right\} = 0$$

$$5x(2x - 1) - 7(2x - 1) = 0$$

Both terms have an  $(2x - 1)$ , so we can factor that out.

$$(2x - 1) \left( \frac{5x(2x - 1)}{2x - 1} - \frac{7(2x - 1)}{2x - 1} \right) = 0$$

$$(2x - 1)(5x - 7) = 0$$

Finally, we use the zero-product property to solve.

$$(2x - 1)(5x - 7) = 0$$

Either  $2x - 1 = 0$  or  $5x - 7 = 0$ .

$$2x - 1 = 0 \quad \text{or} \quad 5x - 7 = 0$$

$$+1 \quad +1 \qquad \qquad +7 \quad +7$$

$$2x = 1 \quad \text{or} \quad 5x = 7$$

$$\frac{2x}{2} = \frac{1}{2} \quad \text{or} \quad \frac{5x}{5} = \frac{7}{5}$$

$$x = \frac{1}{2} \quad \text{or} \quad x = \frac{7}{5}$$

The two solutions are  $x = \frac{1}{2}$  or  $x = \frac{7}{5}$ . \*\*These are our solutions, but it is always a good idea to double-check our solutions and see if we have the correct ones.

Check:

Replace x with 1/2

$$-19x + 10x^2 = -7$$

$$-19\left(\frac{1}{2}\right) + 10\left(\frac{1}{2}\right)^2 = -7$$

$$-9\frac{19}{2} + 10\left(\frac{1}{4}\right) = -7$$

$$-9\frac{1}{2} + \frac{10}{4} = -7$$

$$-9\frac{1}{2} + \frac{5}{2} = -7$$

Replace x with 7/5

$$-19x + 10x^2 = -7$$

$$-19\left(\frac{7}{5}\right) + 10\left(\frac{7}{5}\right)^2 = -7$$

$$-\frac{133}{5} + 10\left(\frac{49}{25}\right) = -7$$

$$-26\frac{3}{5} + \frac{490}{25} = -7$$

$$-26\frac{3}{5} + \frac{98}{5} = -7$$

$$-9\frac{1}{2} + 2\frac{1}{2} = -7$$

$$-7 = -7$$

$$-26\frac{3}{5} + 19\frac{3}{5} = -7$$

$$-7 = -7$$

Because both equations are true, we know we have found the correct solutions.