Example: Solve an Equation by Factoring

Solve $x^2 + 5x - 6 = 0$. Check the solutions.

$x^2 + 5x - 6 = 0$	Original equation
$(x - 1)(x + 6) = 0$	Factor.
$x - 1 = 0$ or $x + 6 = 0$	Zero Product Property
$x = 1$ $\quad$ $x = -6$	Solve each equation.

The roots are 1 and −6. Check by substituting 1 and −6 for x in the original equation.

Check your progress:

1) $-9x + x^2 = 22$

We need to get one side of the equation equal to zero.

$-9x + x^2 = 22$

$\quad -22 \quad -22$

$-9x + x^2 - 22 = 0$

Now, we need to put the polynomial on the left in standard form so that it is factorable.

$x^2 - 9x - 22 = 0$

Factor: $x^2 - 9x - 22 = 0$

Step 1: Product $= 1 \cdot -22 = -22$

Sum $= -9$

**We need to find numbers that multiply to -22 and add to -2. Because the product is negative, one of the two factors must be negative. Because the sum is also negative, the larger factor must be negative.

Step 2:

Factors of -22:	Sum of factors:
1, -22	$1 + -22 = -21$
2, -11	$2 + -11 = -9$

2 and -11 are the factors that will make a product of -22 and a sum of -9.

Step 3:

$x^2 - 9x - 22 = 0$

$x^2 + 2x - 11x - 22 = 0$

Step 4:

$x^2 + 2x - 11x - 22 = 0$

The first two terms are divisible by x. The last two terms are divisible by -11.

$x\left(\frac{x^2}{x} + \frac{2x}{x}\right) - 11\left(\frac{-11x}{-11} + \frac{-22}{-11}\right) = 0$

$x(x+2) - 11(x+2) = 0$

Both terms have an $(x+2)$, so we can factor that out.

$(x+2)\left(\frac{x(x+2)}{x+2} - \frac{11(x+2)}{x+2}\right) = 0$

$(x+2)(x-11) = 0$

Finally, we use the zero-product property to solve.

$(x+2)(x-11) = 0$

Either $x + 2 = 0$ or $x - 11 = 0$.

$x + 2 = 0$ or $x - 11 = 0$

$-2 \quad -2 \qquad +11 \quad +11$

$x = -2$ or $x = 11$

The two solutions are $x = -2$ or $x = 11$. **These are our solutions, but it is always a good idea to double-check our solutions and see if we have the correct ones.

Check:

Replace x with -2	Replace x with 11
$-9x + x^2 = 22$	$-9x + x^2 = 22$
$-9(-2) + (-2)^2 = 22$	$-9(11) + (11)^2 = 22$
$18 + 4 = 22$	$-99 + 121 = 22$
$22 = 22$	$22 = 22$

Because both equations are true, we know we have found the correct solutions.

2) $x^2 + 9 = 10x$

We need to get one side of the equation equal to zero.

$x^2 + 9 = 10x$

$\quad -10x \quad -10x$

$x^2 - 10x + 9 = 0$ **Put the -10x between the other two terms so that the polynomial is in standard form.

Factor: $x^2 - 10x + 9 = 0$

Step 1: $\text{Product} = 1 \cdot 9 = 9$

$\text{Sum} = -10$

**We need to find numbers that multiply to 9 and add to -10. Because the product is positive and the sum is negative, both factors must be negative.

Step 2:

Factors of -22:	Sum of factors:
-1, -9	$-1 + -9 = -10$
-3, -3	$-3 + -3 = -6$

-1 and -9 are the factors that will make a product of 9 and a sum of -10.

Step 3:

$x^2 - 10x + 9 = 0$

$x^2 - 1x - 9x + 9 = 0$

Step 4:

$x^2 - 1x - 9x + 9 = 0$

The first two terms are divisible by x. The last two terms are divisible by -9.

$$x\left(\frac{x^2}{x} - \frac{1x}{x}\right) - 9\left(\frac{-9x}{-9} + \frac{9}{-9}\right) = 0$$

$x(x - 1) - 9(x - 1) = 0$

Both terms have an $(x - 1)$, so we can factor that out.

$(x-1)\left(\frac{x(x-1)}{x-1}-\frac{9(x-1)}{x-1}\right)=0$

$(x-1)(x-9)=0$

Finally, we use the zero-product property to solve.

$(x-1)(x-9)=0$

Either $x-1=0$ or $x-9=0$.

$x-1=0$ or $x-9=0$

$+1 \quad +1$ $\qquad +9 \quad +9$

$x=1$ or $x=9$

The two solutions are $x=1$ or $x=9$.

**These are our solutions, but it is always a good idea to double-check our solutions and see if we have the correct ones.

Check:

Replace x with 1	Replace x with 9
$x^2+9=10x$	$x^2+9=10x$
$(1)^2+9=10(1)$	$(9)^2+9=10(9)$
$1+9=10$	$81+9=90$
$10=10$	$90=90$

Because both equations are true, we know we have found the correct solutions.

Example: Solve an Equation by Factoring

Solve $8a^2 - 9a - 5 = 4 - 3a$. Check the solutions.

$8a^2 - 9a - 5 = 4 - 3a$	Write the equation.
$8a^2 - 6a - 9 = 0$	Rewrite so that one side equals 0.
$(4a + 3)(2a - 3) = 0$	Factor the left side.
$4a + 3 = 0$ or $2a - 3 = 0$	Zero Product Property
$4a = -3$ $\quad$ $2a = 3$	Solve each equation.
$a = -\frac{3}{4}$ $\quad$ $a = \frac{3}{2}$	

The roots are $-\frac{3}{4}$ and $\frac{3}{2}$.

CHECK Check each solution in the original equation.

$$8a^2 - 9a - 5 = 4 - 3a \qquad 8a^2 - 9a - 5 = 4 - 3a$$

$$8\left(-\frac{3}{4}\right)^2 - 9\left(-\frac{3}{4}\right) - 5 \stackrel{?}{=} 4 - 3\left(-\frac{3}{4}\right) \qquad 8\left(\frac{3}{2}\right)^2 - 9\left(\frac{3}{2}\right) - 5 \stackrel{?}{=} 4 - 3\left(\frac{3}{2}\right)$$

$$\frac{9}{2} + \frac{27}{4} - 5 \stackrel{?}{=} 4 + \frac{9}{4} \qquad 18 - \frac{27}{2} - 5 \stackrel{?}{=} 4 - \frac{9}{2}$$

$$\frac{25}{4} = \frac{25}{4} \checkmark \qquad -\frac{1}{2} = -\frac{1}{2} \checkmark$$

Check your progress:

1) $6x^2 - 7x = 7x + 12$

We need to get one side of the equation equal to zero.

$6x^2 - 7x = 7x + 12$

$\quad -7x \quad -7x$

$6x^2 - 14x = 12$

$\quad -12 \quad -12$

$6x^2 - 14x - 12 = 0$

Factor: $6x^2 - 14x - 12 = 0$

Step 1: $\quad$ Product $= 6 \cdot -12 = -72$

$\quad$ Sum $= -14$

**We need to find numbers that multiply to -72 and add to -14. Because the product is negative, one of the factors must be negative. Because the sum is negative, the larger factor must be negative.

Step 2:

Factors of -72:	Sum of factors:
1, -72	$1 + -72 = -71$
2, -36	$2 + -36 = -34$
3, -24	$3 + -24 = -21$
4, -18	$4 + -18 = -14$
6, -12	$6 + -12 = -6$
8, -9	$8 + -9 = -1$

4 and -18 are the factors that will make a product of -72 and a sum of -14.

Step 3:

$$6x^2 - 14x - 12 = 0$$

$$6x^2 \overbrace{+ 4x - 18x} - 20 = 0$$

Step 4:

$$6x^2 + 4x - 18x - 12 = 0$$

The first two terms are divisible by $2x$. The last two terms are divisible by -6.

$$2x\left(\frac{6x^2}{2x} + \frac{4x}{2x}\right) - 6\left(\frac{-18x}{-6} + \frac{-12}{-6}\right) = 0$$

$$2x(3x + 2) - 6(3x + 2) = 0$$

Both terms have an $(3x + 2)$, so we can factor that out.

$$(3x + 2)\left(\frac{2x(3x + 2)}{3x + 2} - \frac{6(3x + 2)}{3x + 2}\right) = 0$$

$$(3x + 2)(2x - 6) = 0$$

At this point, we should recognize that the second binomial, $2x - 6$, has a common factor of 2. That means both terms are divisible by 2. So, we should divide that out.

$2(3x+2)\left(\frac{2x}{2}-\frac{6}{2}\right)=0$

$2(3x+2)(x-3)=0$

Finally, we use the zero-product property to solve.

$2(3x+2)(x-3)=0$

Either $3x+2=0$ or $x-3=0$. The 2 out front does not affect the problem since 2 cannot be equal to zero. So, we ignore the 2 out front.

$3x+2=0$ or $x-3=0$

$-2 \quad -2 \qquad\qquad +3 \quad +3$

$3x=-2$ or $x=3$

$\frac{3x}{3}=\frac{-2}{3}$

$x=-\frac{2}{3}$ or $x=3$

The two solutions are $x=-\frac{2}{3}$ or $x=3$. **These are our solutions, but it is always a good idea to double-check our solutions and see if we have the correct ones.

Check:

Replace x with -2/3

$6x^2-14x=12$

$6\left(-\frac{2}{3}\right)^2-14\left(-\frac{2}{3}\right)=12$

$6\left(\frac{4}{9}\right)+\frac{28}{3}=12$

$\frac{24}{9}+9\frac{1}{3}=12$

$\frac{8}{3}+9\frac{1}{3}=12$

$2\frac{2}{3}+9\frac{1}{3}=12$

$12=12$

Replace x with 3

$6x^2-14x=12$

$6(3)^2-14(3)=12$

$6(9)-42=12$

$54-42=12$

$12=12$

Because both equations are true, we know we have found the correct solutions.

2) $-19x + 10x^2 = -7$

We need to get one side of the equation equal to zero.

$-19x + 10x^2 = -7$

$+7 \quad +7$

$-19x + 10x^2 + 7 = 0$

Now, we need to put the polynomial on the left in standard form so that it is factorable.

$10x^2 - 19x + 7 = 0$

Factor: $10x^2 - 19x + 7 = 0$

Step 1: Product $= 10 \cdot 7 = 70$

Sum $= -19$

**We need to find numbers that multiply to 70 and add to -19. Because the product is positive and the sum is negative, both factors must be negative.

Step 2:

Factors of 70:	Sum of factors:
-1, -70	$-1 + -70 = -71$
-2, -35	$-2 + -35 = -37$
-5, -14	$-5 + -14 = -19$
-7, -10	$-7 + -10 = -17$

-5 and -14 are the factors that will make a product of 70 and a sum of -19.

Step 3:

$10x^2 - 19x + 7 = 0$

$10x^2 \overbrace{- 5x - 14x} + 7 = 0$

Step 4:

$10x^2 - 5x - 14x + 7 = 0$

The first two terms are divisible by $5x$. The last two terms are divisible by -7.

$$5x\left(\frac{10x^2}{5x}-\frac{5x}{5x}\right)-7\left(\frac{-14x}{-7}+\frac{7}{-7}\right)=0$$

$$5x(2x-1)-7(2x-1)=0$$

Both terms have an $(2x-1)$, so we can factor that out.

$$(2x-1)\left(\frac{5x(2x-1)}{2x-1}-\frac{7(2x-1)}{2x-1}\right)=0$$

$$(2x-1)(5x-7)=0$$

Finally, we use the zero-product property to solve.

$$(2x-1)(5x-7)=0$$

Either $2x-1=0$ or $5x-7=0$.

$2x-1=0$	or	$5x-7=0$
$+1 \quad +1$		$+7 \quad +7$
$2x=1$	or	$5x=7$
$\frac{2x}{2}=\frac{1}{2}$	or	$\frac{5x}{5}=\frac{7}{5}$
$x=\frac{1}{2}$	or	$x=\frac{7}{5}$

The two solutions are $x=\frac{1}{2}$ or $x=\frac{7}{5}$. **These are our solutions, but it is always a good idea to double-check our solutions and see if we have the correct ones.

Check:

Replace x with 1/2	Replace x with 7/5
$-19x+10x^2=-7$	$-19x+10x^2=-7$
$-19\left(\frac{1}{2}\right)+10\left(\frac{1}{2}\right)^2=-7$	$-19\left(\frac{7}{5}\right)+10\left(\frac{7}{5}\right)^2=-7$
$-\frac{19}{2}+10\left(\frac{1}{4}\right)=-7$	$-\frac{133}{5}+10\left(\frac{49}{25}\right)=-7$
$-9\frac{1}{2}+\frac{10}{4}=-7$	$-26\frac{3}{5}+\frac{490}{25}=-7$
$-9\frac{1}{2}+\frac{5}{2}=-7$	$-26\frac{3}{5}+\frac{98}{5}=-7$

$-9\frac{1}{2} + 2\frac{1}{2} = -7$

$-7 = -7$

$-26\frac{3}{5} + 19\frac{3}{5} = -7$

$-7 = -7$

Because both equations are true, we know we have found the correct solutions.