

MODULE 6 LESSON 3 NOTES

FACTORING TRINOMIALS

In order to factor trinomials, we need to turn them into polynomials with four terms and factor in parts. We will follow a special set of rules to do this:

- 1) Find Product and Sum values
- 2) Determine which values satisfy the product and sum values
- 3) Rewrite the trinomial as a polynomial in four terms
- 4) Factor the polynomial in parts

Example 1: b and c Are Positive

Factor: $x^2 + 6x + 8$

** The "Product" value comes from multiplying the leading coefficient with the last term coefficient. The "Sum" value comes from the middle term.

Step 1: Product = $1 \cdot 8 = 8$

$$\text{Sum} = 6$$

**Product indicates multiplication. Sum indicates addition. So, we need to find numbers that multiply to 8 and add to 6. We do this by listing the factors (things that multiply to the number) of the "product" (in this case 8) and finding their sum.

Step 2:

Factors of 8: Sum of factors:

$$1, 8 \qquad 1 + 8 = 9$$

$$2, 4 \qquad 2 + 4 = 6$$

So, we can see that 2 and 4 are the appropriate factors that will make a product of 8 and a sum of 6.

Step 3:

$$x^2 + 6x + 8$$

$$x^2 + \underbrace{2x + 4x} + 8$$

**Notice that I haven't actually changed the polynomial. The $+2x + 4x$ is equal to $6x$. I've just split the middle term into two terms that can be combined to recreate the original trinomial.

Step 4:

$$x^2 + 2x + 4x + 8$$

Remember that to factor a polynomial with four terms, we split the polynomial in half and treat it like two binomials we are factoring separately.

$$x^2 + 2x + 4x + 8$$

Remember to start with the first two terms and decide what x^2 and $2x$ are both divisible by (x).

$$x \left(\frac{x^2}{x} + \frac{2x}{x} \right) + 4x + 8$$

$$x(x + 2) + 4x + 8$$

The last two terms are both divisible by 4.

$$x(x + 2) + 4 \left(\frac{4x}{4} + \frac{8}{4} \right)$$

$$x(x + 2) + 4(x + 2)$$

The last step is to notice that both terms have an $(x + 2)$, so we can factor that out.

$$(x + 2) \left(\frac{x(x + 2)}{x + 2} + \frac{4(x + 2)}{x + 2} \right)$$

$$(x + 2)(x + 4) \quad \text{**This is our solution}$$

Example 2: b and c Are Positive

$$\text{Factor: } x^2 + 11x + 24$$

** The “Product” value comes from multiplying the leading coefficient with the last term coefficient. The “Sum” value comes from the middle term.

$$\text{Step 1: Product} = 1 \cdot 24 = 24$$

$$\text{Sum} = 11$$

**We need to find numbers that multiply to 24 and add to 11. We do this by listing the factors (things that multiply to the number) of the “product” (in this case 24) and finding their sum.

Step 2:

Factors of 24: Sum of factors:

$$1, 24 \qquad 1 + 24 = 25$$

$$2, 12 \qquad 2 + 12 = 14$$

$$3, 8 \qquad 3 + 8 = 11$$

$$4, 6 \qquad 4 + 6 = 10$$

So, we can see that 3 and 8 are the factors that will make a product of 24 and a sum of 8.

Step 3:

$$\begin{array}{l} x^2 + 11x + 24 \\ x^2 + \overbrace{3x + 8x} + 24 \end{array}$$

Step 4:

$$x^2 + 3x \left\{ + 8x + 24 \right.$$

Remember that to factor a polynomial with four terms, we split the polynomial in half and treat it like two binomials we are factoring separately.

$$x^2 + 3x \left\{ + 8x + 24 \right.$$

This time we will do both factors in the same step. The first two terms are divisible by x . The last two terms are divisible by 8.

$$x \left(\frac{x^2}{x} + \frac{3x}{x} \right) + 8 \left(\frac{8x}{8} + \frac{24}{8} \right)$$

$$x(x + 3) + 8(x + 3)$$

Both terms have an $(x + 3)$, so we can factor that out.

$$(x + 3) \left(\frac{x(x + 3)}{x + 3} + \frac{8(x + 3)}{x + 3} \right)$$

$$(x + 3)(x + 8) \quad \text{**This is our solution}$$

Example 3: b and c Are Positive

Factor: $6x^2 + 17x + 5$

** The “Product” value comes from multiplying the leading coefficient with the last term coefficient. The “Sum” value comes from the middle term.

Step 1: Product = $6 \cdot 5 = 30$

$$\text{Sum} = 17$$

**Product indicates multiplication. Sum indicates addition. So, we need to find numbers that multiply to 30 and add to 17. We need to list factors of 30. Since both 30 and 17 are positive, both factors should also be positive.

Step 2:

Factors of 30:	Sum of factors:
1, 30	$1 + 30 = 31$
2, 15	$2 + 15 = 17$
3, 10	$3 + 10 = 13$
5, 6	$5 + 6 = 11$

So, we can see that 2 and 15 are the appropriate factors that will make a product of 30 and a sum of 17.

Step 3:

$$6x^2 + 17x + 5$$

$$6x^2 + \overbrace{2x + 15x} + 5$$

Step 4:

$$6x^2 + 2x \left\{ + 15x + 5 \right.$$

The first two terms are divisible by 2x. The last two terms are divisible by 5.

$$2x \left(\frac{6x^2}{2x} + \frac{2x}{2x} \right) + 5 \left(\frac{15x}{5} + \frac{5}{5} \right)$$

$$2x(3x + 1) + 5(3x + 1)$$

The last step is to notice that both terms have an $(3x + 1)$, so we can factor that out.

$$(3x + 1) \left(\frac{2x(3x + 1)}{3x + 1} + \frac{5(3x + 1)}{3x + 1} \right)$$

$$(3x + 1)(2x + 5) \quad \text{**This is our solution}$$

Example 4: b and c are positive with a common factor

Factor: $2x^2 + 14x + 12$

Step 1: Product = $2 \cdot 12 = 24$

$$\text{Sum} = 14$$

**We need to find numbers that multiply to 24 and add to 14. We do this by listing the factors of the “product” (in this case -35) and finding their sum. Because both the product and the sum are positive, both factors will be positive.

Step 2:

Factors of 24:	Sum of factors:
1, 24	$1 + 24 = 25$
2, 12	$2 + 12 = 14$
3, 8	$3 + 8 = 11$
4, 6	$4 + 6 = 10$

2 and 12 are the factors that will make a product of 24 and a sum of 14.

Step 3:

$$2x^2 + 14x + 12$$

$$2x^2 + 2x + 12x + 12$$

Step 4:

$$2x^2 + 2x + 12x + 12$$

The first two terms are divisible by $2x$. The last two terms are divisible by 12.

$$2x \left(\frac{2x^2}{2x} + \frac{2x}{2x} \right) + 12 \left(\frac{12x}{12} + \frac{12}{12} \right)$$

$$2x(x + 1) + 12(x + 1)$$

Both terms have an $(x + 1)$, so we can factor that out.

$$(x + 1) \left(\frac{2x(x + 1)}{x + 1} + \frac{12(x + 1)}{x + 1} \right)$$

$$(x + 1)(2x + 12)$$

At this point, I should always check and make sure that I did not miss a common factor. I notice that the last binomial has terms that are both divisible by 2. So, we should factor that out.

$$2(x + 1) \left(\frac{2x}{2} + \frac{12}{2} \right)$$

$$2(x + 1)(x + 6)$$

**This is our solution

Example 5: b Is Negative and c Is Positive

Factor: $x^2 - 10x + 16$

** The “Product” value comes from multiplying the leading coefficient with the last term coefficient. The “Sum” value comes from the middle term.

Step 1: Product = $1 \cdot 16 = 16$

$$\text{Sum} = -10$$

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**We need to find numbers that multiply to 16 and add to -10. We do this by listing the factors of the “product” (in this case 16) and finding their sum. Because the product is positive and the sum is negative, that means both factors must be negative.

Step 2:

Factors of 16:	Sum of factors:
-1, -16	$-1 + -16 = -17$
-2, -8	$-2 + -8 = -10$
-4, -4	$-4 + -4 = -8$

-2 and -8 are the factors that will make a product of 16 and a sum of -10.

Step 3:

$$x^2 - 10x + 16$$

$$x^2 \overbrace{-2x - 8x} + 16$$

Step 4:

$$x^2 - 2x \left\{ -8x + 16 \right.$$

The first two terms are divisible by x . The last two terms are divisible by -8. Note that I’m choosing a negative eight for the last two terms because the first term in that binomial is negative.

$$x \left(\frac{x^2}{x} - \frac{2x}{x} \right) \left\{ 8 \left(\frac{-8x}{-8} + \frac{16}{-8} \right) \right.$$

$$x(x - 2) - 8(x - 2)$$

Both terms have an $(x - 2)$, so we can factor that out.

$$(x - 2) \left(\frac{x(x - 2)}{x - 2} - \frac{8(x - 2)}{x - 2} \right)$$

$$(x - 2)(x - 8) \quad \text{**This is our solution}$$

Example 6: b Is Negative and c Is Positive

Factor: $x^2 - 3x + 2$

** The “Product” value comes from multiplying the leading coefficient with the last term coefficient. The “Sum” value comes from the middle term.

Step 1: Product = $1 \cdot 2 = 2$

$$\text{Sum} = -3$$

**We need to find numbers that multiply to 2 and add to -3. We do this by listing the factors of the “product” (in this case 2) and finding their sum. Because the product is positive and the sum is negative, that means both factors must be negative.

Step 2:

Factors of 2:	Sum of factors:
-1, -2	$-1 + -2 = -3$

-1 and -2 are the factors that will make a product of 2 and a sum of -3.

Step 3:

$$\begin{array}{c} x^2 - 3x + 2 \\ \underbrace{}_{x^2 - 1x - 2x + 2} \end{array}$$

Step 4:

$$x^2 - 1x \left\{ -2x + 2 \right.$$

The first two terms are divisible by x . The last two terms are divisible by -2 . Note that I'm choosing a negative two for the last two terms because the first term in that binomial is negative.

$$x \left(\frac{x^2}{x} - \frac{1x}{x} \right) \left\{ -2 \left(\frac{-2x}{-2} + \frac{2}{-2} \right) \right.$$

$$x(x - 1) - 2(x - 1)$$

Both terms have an $(x - 1)$, so we can factor that out.

$$(x - 1) \left(\frac{x(x - 1)}{x - 1} - \frac{2(x - 1)}{x - 1} \right)$$

$$(x - 1)(x - 2)$$

**This is our solution

Example 7: b Is Negative and c Is Positive

$$\text{Factor: } 10x^2 - 43x + 28$$

** The "Product" value comes from multiplying the leading coefficient with the last term coefficient. The "Sum" value comes from the middle term.

$$\text{Step 1: Product} = 10 \cdot 28 = 280$$

$$\text{Sum} = -43$$

We need to find numbers that multiply to 280 and add to -43. Because the product is positive and the sum is negative, both factors must be negative.

Step 2:

Factors of 280:

Sum of factors:

$$-1, -280$$

$$-1 + -280 = -281$$

$$-2, -140$$

$$-2 + -140 = -142$$

$$-4, -70$$

$$-4 + -70 = -74$$

$$-5, -56$$

$$-5 + -56 = -61$$

$$-7, -40$$

$$-7 + -40 = -47$$

$$-8, -35$$

$$-8 + -35 = -43$$

$$-10, -28$$

$$-10 + -28 = -38$$

$$-14, -20$$

$$-14 + -20 = -34$$

-8 and -35 are the factors that will make a product of 280 and a sum of -43.

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Step 3:

$$10x^2 - 43x + 28$$

$$10x^2 - 8x - 35x + 28$$

Step 4:

$$10x^2 - 8x \left\{ -35x + 28 \right.$$

The first two terms are divisible by $2x$. The last two terms are divisible by -7 . Note that I'm choosing a negative seven for the last two terms because the first term in that binomial is negative.

$$2x \left(\frac{10x^2}{2x} - \frac{8x}{2x} \right) - 7 \left(\frac{-35x}{-7} + \frac{28}{-7} \right)$$

$$2x(5x - 4) - 7(5x - 4)$$

Both terms have an $(5x - 4)$, so we can factor that out.

$$(5x - 4) \left(\frac{2x(5x - 4)}{5x - 4} - \frac{7(5x - 4)}{5x - 4} \right)$$

$$(5x - 4)(2x - 7)$$

**This is our solution

Example 8: b Is Positive and c Is Negative

Factor: $x^2 + x - 12$

** The "Product" value comes from multiplying the leading coefficient with the last term coefficient. The "Sum" value comes from the middle term.

Step 1: Product = $1 \cdot -12 = -12$

$$\text{Sum} = 1$$

We need to find numbers that multiply to -12 and add to 1 . We do this by listing the factors of the "product" (in this case -12) and finding their sum. Because the product is negative, **one of the factors must be negative. Because sum is positive, that means the larger factor must be positive.

Step 2:

Factors of -12 : Sum of factors:

$-1, 12$ $-1 + 12 = 11$

$-2, 6$ $-2 + 6 = 4$

$-3, 4$ $-3 + 4 = 1$

-3 and 4 are the factors that will make a product of -12 and a sum of 1 .

Step 3:

$$x^2 + x - 12$$

$$x^2 - 3x + 4x - 12$$

Step 4:

$$x^2 - 3x \left\{ + 4x - 12 \right.$$

The first two terms are divisible by x . The last two terms are divisible by 4. Note that I'm choosing a positive four for the last two terms because the first term in that binomial is positive.

$$x \left(\frac{x^2}{x} - \frac{3x}{x} \right) + 4 \left(\frac{4x}{4} - \frac{12}{4} \right)$$

$$x(x - 3) + 4(x - 3)$$

Both terms have an $(x - 3)$, so we can factor that out.

$$(x - 3) \left(\frac{x(x - 3)}{x - 3} + \frac{4(x - 3)}{x - 3} \right)$$

$$(x - 3)(x + 4) \quad \text{**This is our solution}$$

Example 9: b Is Positive and c Is Negative

Factor: $x^2 + 13x - 48$

Step 1: Product = $1 \cdot -48 = -48$

$$\text{Sum} = 13$$

**We need to find numbers that multiply to -48 and add to 13. We do this by listing the factors of the "product" (in this case -48) and finding their sum. Because the product is negative, *one* of the factors must be negative. Because sum is positive, that means the larger factor must be positive.

Step 2:

Factors of -48:	Sum of factors:
-1, 48	$-1 + 48 = 47$
-2, 24	$-2 + 24 = 22$
-3, 16	$-3 + 16 = 13$
-4, 12	$-4 + 12 = 8$
-6, 8	$-6 + 8 = 2$

-3 and 16 are the factors that will make a product of -48 and a sum of 13.

Step 3:

$$\begin{array}{c} x^2 + 13x - 48 \\ \underbrace{}_{x^2 - 3x + 16x - 48} \end{array}$$

Step 4:

$$x^2 - 3x \left\{ + 16x - 48 \right.$$

The first two terms are divisible by x . The last two terms are divisible by 16. Note that I'm choosing a positive sixteen for the last two terms because the first term in that binomial is positive.

$$x \left(\frac{x^2}{x} - \frac{3x}{x} \right) + 16 \left(\frac{16x}{16} - \frac{48}{16} \right)$$

$$x(x - 3) + 16(x - 3)$$

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Both terms have an $(x - 3)$, so we can factor that out.

$$(x - 3) \left(\frac{x(x - 3)}{x - 3} + \frac{16(x - 3)}{x - 3} \right)$$

$$(x - 3)(x + 16) \quad \text{**This is our solution}$$

**I will do Example 10 twice and show two different ways of handling the common factor. Please look over both and use the one that works for you.

Example 10 (One Way of Handling the Common Factor): b Is Positive and c Is Negative and a Common Factor Exists

Factor: $6x^2 + 15x - 9$

This problem has a common factor for all three terms in the trinomial. 6, 15, and -9 are all divisible by 3, so we should factor that out first.

$$3 \left(\frac{6x^2}{3} + \frac{15x}{3} - \frac{9}{3} \right)$$

$$3(2x^2 + 5x - 3)$$

Now, we kind of ignore the 3 out front when going through our four step process.

Step 1: Product = $2 \cdot -3 = -6$

Sum = 5

We need to find numbers that multiply to -6 and add to 5. Because the product is negative, **one of the factors must be negative. Because sum is positive, that means the larger factor must be positive.

Step 2:

Factors of -6: Sum of factors:

-1, 6 $-1 + 6 = 5$

-2, 3 $-2 + 3 = 1$

-1 and 6 are the factors that will make a product of -6 and a sum of 5.

Step 3:

$$3(2x^2 + 5x - 3)$$

$$3(2x^2 - 1x + 6x - 3)$$

Step 4:

$$3(2x^2 - 1x) + 6x - 3$$

The first two terms are divisible by x . The last two terms are divisible by 3. Note that I'm choosing a positive three for the last two terms because the first term in that binomial is positive.

$$3 \left(x \left(\frac{2x^2}{x} - \frac{1x}{x} \right) + 3 \left(\frac{6x}{3} - \frac{3}{3} \right) \right)$$

$$3(x(2x - 1) + 3(2x - 1))$$

Both terms have an $(x - 3)$, so we can factor that out.

$$3 \left((2x - 1) \left(\frac{x(2x - 1)}{2x - 1} + \frac{3(2x - 1)}{2x - 1} \right) \right)$$

$$3(2x - 1)(x + 3)$$

**This is our solution

Example 10 (Another Way of Handling the Common Factor): b Is Positive and c Is Negative and a Common Factor Exists

Factor: $6x^2 + 15x - 9$

This time we will leave the common factor in.

Step 1: Product = $6 \cdot -9 = -54$

Sum = 15

**We need to find numbers that multiply to -54 and add to 15. Because the product is negative, *one* of the factors must be negative. Because sum is positive, that means the larger factor must be positive.

Step 2:

Factors of -54:

Sum of factors:

-1, 54

$-1 + 54 = 53$

-2, 27

$-2 + 27 = 25$

-3, 18

$-3 + 18 = 15$

-6, 9

$-6 + 9 = 3$

-3 and 18 are the factors that will make a product of -54 and a sum of 15.

Step 3:

$$6x^2 + 15x - 9$$

$$6x^2 - 3x + 18x - 9$$

Step 4:

$$6x^2 - 3x + 18x - 9$$

The first two terms are divisible by $3x$. The last two terms are divisible by 9. Note that I'm choosing a positive nine for the last two terms because the first term in that binomial is positive.

$$3x \left(\frac{6x^2}{3x} - \frac{3x}{3x} \right) + 9 \left(\frac{18x}{9} - \frac{9}{9} \right)$$

$$3x(2x - 1) + 9(2x - 1)$$

Both terms have an $(2x - 1)$, so we can factor that out.

$$(2x - 1) \left(\frac{3x(2x - 1)}{2x - 1} + \frac{9(2x - 1)}{2x - 1} \right)$$

$$(2x - 1)(3x + 9)$$

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At this point, we should recognize that the second binomial, $3x + 9$, has a common factor of 3. That means both terms are divisible by 3. So, we should divide that out. We put the 3 that we are dividing out at the front of the problem.

$$3(2x - 1)\left(\frac{3x}{3} + \frac{9}{3}\right)$$

$$3(2x - 1)(x + 3)$$

**This is our solution. Notice that this is the same answer.

Example 11: b Is Negative and c Is Negative

Factor: $x^2 - 7x - 18$

Step 1: Product = $1 \cdot -18 = -18$

$$\text{Sum} = -7$$

**We need to find numbers that multiply to -18 and add to -7. We do this by listing the factors of the “product” (in this case -18) and finding their sum. Because the product is negative, *one* of the factors must be negative. Because sum is negative, that means the larger factor must be negative.

Step 2:

Factors of -18:	Sum of factors:
1, -18	$1 + -18 = -17$
2, -9	$2 + -9 = -7$
3, -6	$3 + -6 = -3$

3 and -9 are the factors that will make a product of -18 and a sum of -7.

Step 3:

$$\begin{array}{l} x^2 - 7x - 18 \\ x^2 + 2x - 9x - 18 \end{array}$$

Step 4:

$$x^2 + 2x \left\{ \begin{array}{l} 9x - 18 \end{array} \right.$$

The first two terms are divisible by x . The last two terms are divisible by -9. Note that I’m choosing a negative nine for the last two terms because the first term in that binomial is negative.

$$x\left(\frac{x^2}{x} + \frac{2x}{x}\right) - 9\left(\frac{-9x}{-9} + \frac{-18}{-9}\right)$$

$$x(x + 2) - 9(x + 2)$$

Both terms have an $(x + 2)$, so we can factor that out.

$$(x + 2)\left(\frac{x(x + 2)}{x + 2} - \frac{9(x + 2)}{x + 2}\right)$$

$$(x + 2)(x - 9)$$

**This is our solution

Example 12: b Is Negative and c Is Negative

Factor: $x^2 - 2x - 35$

Step 1: Product = $1 \cdot -35 = -35$

Sum = -2

**We need to find numbers that multiply to -35 and add to -2. We do this by listing the factors of the “product” (in this case -35) and finding their sum. Because the product is negative, *one* of the factors must be negative. Because sum is negative, that means the larger factor must be negative.

Step 2:

Factors of -35:	Sum of factors:
1, -35	$1 + -35 = -34$
5, -7	$5 + -7 = -2$

5 and -7 are the factors that will make a product of -35 and a sum of -2.

Step 3:

$$x^2 - 2x - 35$$

$$x^2 + 5x - 7x - 35$$

Step 4:

$$x^2 + 5x \quad \left\{ \begin{array}{l} 7x - 35 \end{array} \right.$$

The first two terms are divisible by x . The last two terms are divisible by -7. Note that I’m choosing a negative seven for the last two terms because the first term in that binomial is negative.

$$x \left(\frac{x^2}{x} + \frac{5x}{x} \right) \left\{ -7 \left(\frac{-7x}{-7} + \frac{-35}{-7} \right) \right.$$

$$x(x + 5) - 7(x + 5)$$

Both terms have an $(x + 5)$, so we can factor that out.

$$(x + 5) \left(\frac{x(x + 5)}{x + 5} - \frac{7(x + 5)}{x + 5} \right)$$

$(x + 5)(x - 7)$ **This is our solution

Example 13: b Is Negative and c Is Negative

Factor: $4x^2 - 4x - 35$

Step 1: Product = $4 \cdot -35 = -140$

Sum = -4

**We need to find numbers that multiply to -140 and add to -4. Because the product is negative, *one* of the factors must be negative. Because sum is negative, that means the larger factor must be negative.

Step 2:

Factors of -140:	Sum of factors:
1, -140	$1 + -140 = -139$
2, -70	$2 + -70 = -68$
4, -35	$4 + -35 = -31$

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5, -28 $5 + -28 = -23$

7, -20 $7 + -20 = -13$

10, -14 $10 + -14 = -4$

10 and -14 are the factors that will make a product of -140 and a sum of -4.

Step 3:

$$4x^2 - 4x - 35$$

$$4x^2 + 10x - 14x - 35$$

Step 4:

$$4x^2 + 10x \left\{ \begin{array}{l} \\ \\ \end{array} \right. 14x - 35$$

The first two terms are divisible by $2x$. The last two terms are divisible by -7 . Note that I'm choosing a negative seven for the last two terms because the first term in that binomial is negative.

$$2x \left(\frac{4x^2}{2x} + \frac{10x}{2x} \right) - 7 \left(\frac{-14x}{-7} + \frac{-35}{-7} \right)$$

$$2x(2x + 5) - 7(2x + 5)$$

Both terms have an $(2x + 5)$, so we can factor that out.

$$(2x + 5) \left(\frac{2x(2x + 5)}{2x + 5} - \frac{7(2x + 5)}{2x + 5} \right)$$

$$(2x + 5)(2x - 7)$$

**This is our solution

Example 14: Factoring a Trinomial Not In Standard Form

Factor: $-72 + 6x + x^2$

Step 1: Product = $-72 \cdot 1 = -72$

Sum = 6

We need to find numbers that multiply to -72 and add to 6. Because the product is negative, **one of the factors must be negative. The larger factor must be positive since the sum is positive.

Step 2:

Factors of -72:

Sum of factors:

-1, 72

$-1 + 72 = 71$

-2, 36

$-2 + 36 = 34$

-3, 24

$-3 + 24 = 21$

-4, 18

$-4 + 18 = 14$

-6, 12

$-6 + 12 = 6$

-8, 9

$-8 + 9 = 1$

-6 and 12 are the factors that will make a product of -72 and a sum of 6.

Step 3:



$$-72 + 6x + x^2$$

$$-72 - 6x + 12x + x^2$$

Step 4:

$$-72 - 6x \left\} + 12x + x^2$$

The first two terms are divisible by -6 . Note that I'm choosing a negative six for the first two terms because the first term in that binomial is negative. The last two terms are divisible by x .

$$-6 \left(\frac{-72}{-6} + \frac{-6x}{-6} \right) \left\} + x \left(\frac{12x}{x} + \frac{x^2}{x} \right)$$

$$-6(12 + x) + x(12 + x)$$

Both terms have a $(12 + x)$, so we can factor that out.

$$(12 + x) \left(\frac{-6(12 + x)}{12 + x} + \frac{x(12 + x)}{12 + x} \right)$$

$$(12 + x)(-6 + x)$$

**This is our solution

Example 15: Factoring a Trinomial with Two Variables

Factor: $x^2 - 4xy + 3y^2$

Step 1: Product = $1 \cdot 3 = 3$

Sum = -4

**We need to find numbers that multiply to 3 and add to -4 . Because the product is positive and the sum is negative, that means both factors must be negative.

Step 2:

Factors of 3:

Sum of factors:

$-1, -3$

$-1 + -3 = -4$

-1 and -3 are the factors that will make a product of 3 and a sum of -4 .

Step 3:

$$x^2 - 4xy + 3y^2$$

$$x^2 - xy - 3xy + 3y^2$$

**Remember that both middle terms should have the same variable set as the middle term in the original trinomial.

Step 4:

$$x^2 - xy \left\} - 3xy + 3y^2$$

The first two terms are divisible by x . The last two terms are divisible by $-3y$. Note that I'm choosing a negative $3y$ for the last two terms because the first term in that binomial is negative.

$$x \left(\frac{x^2}{x} - \frac{xy}{x} \right) \left\} - 3y \left(\frac{-3xy}{-3y} + \frac{3y^2}{-3y} \right)$$

$$x(x - y) - 3y(x - y)$$

Both terms have an $(x - y)$, so we can factor that out.

$$(x - y) \left(\frac{x(x - y)}{x - y} - \frac{3y(x - y)}{x - y} \right)$$

$$(x - y)(x - 3y)$$

**This is our solution

Example 16: Factoring a Trinomial with Two Variables

Factor: $36x^2 + 9xy - 10y^2$

Step 1: Product = $36 \cdot -10 = -360$

Sum = 9

**We need to find numbers that multiply to -360 and add to 9. Because the product is negative, *one* of the factors must be negative. Because sum is positive, that means the larger factor must be positive.

Step 2:

Factors of -360: Sum of factors:

-1, 360 $-1 + 360 = 359$

-2, 180 $-2 + 180 = 178$

-3, 120 $-3 + 120 = 117$

-4, 90 $-4 + 90 = 86$

-5, 72 $-5 + 72 = 67$

-6, 60 $-6 + 60 = 54$

-8, 45 $-8 + 45 = 37$

-9, 40 $-9 + 40 = 31$

-10, 36 $-10 + 36 = 26$

-12, 30 $-12 + 30 = 18$

-15, 24 $-15 + 24 = 9$

-18, 20 $-18 + 20 = 2$

-18 and 20 are the factors that will make a product of -360 and a sum of 9.

Step 3:

$$36x^2 + 9xy - 10y^2$$

$$36x^2 - 15xy + 24xy - 10y^2$$

**Remember that both middle terms should have the same variable set as the middle term in the original trinomial.

Step 4:

$$36x^2 - 15xy \left\{ + 24xy - 10y^2 \right.$$

The first two terms are divisible by $3x$. The last two terms are divisible by $2y$.

$$3x \left(\frac{36x^2}{3x} - \frac{15xy}{3x} \right) + 2y \left(\frac{24xy}{2y} - \frac{10y^2}{2y} \right)$$

$$3x(12x - 5y) + 2y(12x - 5y)$$

Both terms have an $(12x - 5y)$, so we can factor that out.

$$(12x - 5y) \left(\frac{3x(12x - 5y)}{12x - 5y} + \frac{2y(12x - 5y)}{12x - 5y} \right)$$

$$(12x - 5y)(3x + 2y)$$

**This is our solution

Example 17: Prime Trinomials

$$\text{Factor: } 2x^2 + 5x - 2$$

$$\text{Step 1: Product} = 2 \cdot -2 = -4$$

$$\text{Sum} = 5$$

We need to find numbers that multiply to -4 and add to 5. Because the product is negative, **one of the factors must be negative. The larger factor must be positive since the sum is positive.

Step 2:

Factors of -4: Sum of factors:

$$-1, 4 \qquad -1 + 4 = 3$$

$$-2, 2 \qquad -2 + 2 = 0$$

We have found all factors of -4 and none of them add to 5, so the polynomial is prime and cannot be factored.

Prime

**This is our solution