KEY CONCEPT — *Zero Product Property*

Word If the product of two factors is 0, then at least one of the factors must be 0.

Symbols For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal zero.

This property should make sense to you. If we have two things multiplied together and the result is zero, then one or the other of those things has to be equal to zero.

Example: Solve an Equation

Solve each equation. Check the solutions.

a. $(d-5)(3d+4)=0$

If $(d-5)(3d+4)=0$, then according to the Zero Product Property either $d-5=0$ or $3d+4=0$.

$(d-5)(3d+4)=0$ Original equation

$d-5=0$ or $3d+4=0$ Set each factor equal to zero.

$d=5$ $\quad 3d=-4$ Solve each equation.

$d=-\frac{4}{3}$

The roots are 5 and $-\frac{4}{3}$.

CHECK Substitute 5 and $-\frac{4}{3}$ for d in the original equation.

$$(d-5)(3d+4)=0 \qquad (d-5)(3d+4)=0$$

$$(5-5)[3(5)+4] \stackrel{?}{=} 0 \qquad \left[\left(-\frac{4}{3}\right)-5\right]\left[3\left(-\frac{4}{3}\right)+4\right] \stackrel{?}{=} 0$$

$$(0)(19) \stackrel{?}{=} 0 \qquad \left(-\frac{19}{3}\right)(0) \stackrel{?}{=} 0$$

$$0=0\ \checkmark \qquad 0=0\ \checkmark$$

b. $x^2=7x$

Write the equation so that it is of the form $ab=0$.

$x^2=7x$ Original equation

$x^2-7x=0$ Subtract $7x$ from each side.

$x(x-7)=0$ Factor using the GCF of x^2 and $-7x$, which is x.

$x=0$ or $x-7=0$ Zero Product Property

$x=7$ Solve each equation.

The roots are 0 and 7. Check by substituting 0 and 7 for x in the original equation.

Check your progress:

1) $h(h+5)=0$

We have two things (h and $(h+5)$) that are multiplied together and the product is zero. That means either $h=0$ or $h+5=0$.

So, we write both of those equations down and solve.

$h=0$ or $\quad h+5=0$

$\quad\quad -5 \quad -5$

$h=0$ or $\quad h=-5$

The two solutions are $h=0$ or $h=-5$.

2) $(n-4)(n+2)=0$

We have two things ($(n-4)$ and $(n+2)$) that are multiplied together and the product is zero. That means either $n-4=0$ or $n+2=0$.

$n-4=0$ or $\quad n+2=0$

$+4 \quad +4 \quad\quad -2 \quad -2$

$n=4$ or $\quad n=-2$

The two solutions are $n=4$ or $n=-2$.

3) $5m=3m^2$

In order to solve any equation where we have a variable with a power higher than one, we must get one side of the equation equal to zero.

$5m=3m^2$

$-3m^2 \quad -3m^2$

$5m-3m^2=0$ **Don't forget that $5m$ and $-3m^2$ are not like terms and cannot be combined.

In order to be able to use our zero product property, we must have two things multiplied together to equal zero. So, we have to factor out the left-hand side of the equation.

Both $5m$ and $-3m^2$ are divisible by m. So, we will factor that out.

$5m-3m^2=0$

$m(5-3m)=0$

We have two things (m and $(5-3m)$) that are multiplied together and the product is zero. That means either $m = 0$ or $5 - 3m = 0$.

$m = 0$ or $\quad 5 - 3m = 0$

$\quad -5 \quad -5$

$m = 0$ or $\quad -3m = -5$

$\frac{-3m}{-3} = \frac{-5}{-3}$

$m = \frac{5}{3}$

The two solutions are $m = 0$ or $m = \frac{5}{3}$.

Practice

Solve each equation. Check the solutions.

1) $3n(n+2) = 0$

$3n = 0$ or $n + 2 = 0$

$n = 0$ or $n = -2$

2) $7d^2 - 35d = 0$

$7d(d-5) = 0$

$7d = 0$ or $d - 5 = 0$

$d = 0$ or $d = 5$

3) $x^2 = -10x$

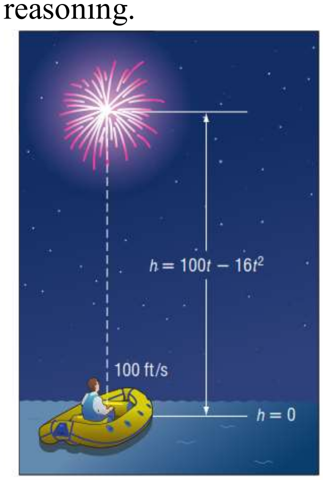
$x^2 + 10x = 0$

$x(x+10) = 0$

$x = 0$ or $x + 10 = 0$

$x = 0$ or $x = -10$

4) A flare is launched from a life raft. The height h of the flare in feet above the sea is modeled by the formula $h = 100t - 16t^2$, where t is the time in seconds after the flare is launched. Let $h = 0$ and solve $0 = 100t - 16t^2$ for t. How many seconds will it take for the flare to return to the sea? Explain your reasoning.



$0 = 100t - 16t^2$

$0 = 4t(25 - 4t)$

$0 = 4t$ or $0 = 25 - 4t$

$0 = t$ or $-25 = -4t$

$\frac{25}{4} = t$

The flare is at a height of zero at time zero (when it is launched) and again at time $6\frac{1}{4}$ seconds (when it returns to sea). So, it takes $6\frac{1}{4}$ seconds to return to the sea.