## Example: Use the Distributive Property

## Use the Distributive Property to factor each polynomial.

a. $12 a^{2}+16 a$

First, find the GCF of $12 a^{2}$ and $16 a$.
$12 a^{2}=(2) \cdot(2) \cdot 3 \cdot(a) \cdot a \quad$ Fattor each monomial.
$16 a=(2) \cdot(2) \cdot 2 \cdot 2 \cdot$ (a) Cirde the common prime factors.
GCF: $2 \cdot 2 \cdot a$ or $4 a$
Write each term as the product of the GCF and its remaining factors.
Then use the Distributive Property to factor out the GCF.

$$
\begin{aligned}
12 a^{2}+16 a & =4 a(3 \cdot a)+4 a(2 \cdot 2) & & \text { Rewrite each term using the GCF. } \\
& =4 a(3 a)+4 a(4) & & \text { Simplify remaining factors. } \\
& =4 a(3 a+4) & & \text { Distributive Property }
\end{aligned}
$$

Thus, the completely factored form of $12 a^{2}+16 a$ is $4 a(3 a+4)$.
b. $18 c d^{2}+12 c^{2} d+9 c d$
$18 c d^{2}=2 \cdot$ (3) $\cdot 3 \cdot$ (c) (d) $\cdot d \quad$ Factor each monomial.
$12 c^{2} d=2 \cdot 2 \cdot$ (3). (c) $\cdot c$.(d) Circle the common prime factors.

$$
9 c d=(3) \cdot 3 \cdot(c \cdot(d)
$$

GCF: $3 \cdot c \cdot d$ or $3 c d$

$$
\begin{aligned}
18 c d^{2}+12 c^{2} d+9 c d & =3 c d(6 d)+3 c d(4 c)+3 c d(3) & & \text { Rewrite each term using the GCF. } \\
& =3 c d(6 d+4 c+3) & & \text { Distributive Property }
\end{aligned}
$$

## Check your progress:

1) $9 x^{2}+36 x$

We need to think about the biggest factor that both $9 x^{2}$ and $36 x$ can be divided by. 9 and 36 are both divisible (can be divided) by 3 and 9 . Since 9 is the biggest of those numbers, we will use 4. $x^{2}$ and $a$ are both divisible by $x$. So, we will factor out a $9 x$.

We start by writing what we are factoring out outside of a set of parentheses.
$9 x($
Then we divide each term of the polynomial by what we are factoring out.
$9 x\left(\frac{9 x^{2}}{9 x}+\frac{36 x}{9 x}\right)$
Now, we simplify. $\frac{9}{9}=1, \frac{x^{2}}{x}=x, \frac{36}{9}=4, \frac{x}{x}=1$.
$9 x(x+4)$

## 2) $16 x z-40 x z^{2}$

We need to think about the biggest factor that both $16 x z$ and $40 x z^{2}$ can be divided by. 16 and 40 are both divisible by 2,4 , and 8 . Since 8 is the biggest of those numbers, we will use 8. $x z$ and $x z^{2}$ are both divisible by $x z$. So, we will factor out a $8 x z$.

We start by writing what we are factoring out outside of a set of parentheses.
$8 x z($
Then we divide each term of the polynomial by what we are factoring out.
$8 x z\left(\frac{16 x z}{8 x z}-\frac{40 x z^{2}}{8 x z}\right)$
Now, we simplify. $\frac{16}{8}=2, \frac{x z}{x z}=1, \frac{40}{8}=5, \frac{x z^{2}}{x z}=z$.
$8 x z(2-5 z)$

## 3) $24 m^{2} n p^{2}+36 m^{2} n^{2} p$

We need to think about the biggest factor that both $24 m^{2} n p^{2}$ and $36 m^{2} n^{2} p$ can be divided by. 24 and 36 are both divisible by $2,3,4,6$, and 12 . Since 12 is the biggest of those numbers, we will use $12 . m^{2} n p^{2}$ and $m^{2} n^{2} p$ are both divisible by $m^{2} n p$. So, we will factor out a $12 m^{2} n p$.

We start by writing what we are factoring out outside of a set of parentheses.
$12 m^{2} n p($
Then we divide each term of the polynomial by what we are factoring out.
$12 m^{2} n p\left(\frac{24 m^{2} n p^{2}}{12 m^{2} n p}+\frac{36 m^{2} n^{2} p}{12 m^{2} n p}\right)$
Now, we simplify. $\frac{24}{12}=2, \frac{m^{2} n p^{2}}{m^{2} n p}=p, \frac{36}{12}=3, \frac{m^{2} n^{2} p}{m^{2} n p}=n$.
$12 m^{2} n p(2 p+3 n)$

## 4) $2 a^{3} b^{2}+8 a b+16 a^{2} b^{3}$

When we have three terms, we have to determine the largest thing that divides all three terms.
2,8 , and 16 are all divisible by 2 , but there is nothing else that ALL three numbers are divisible by.
$a^{3} b^{2}, a b$, and $a^{2} b^{3}$ are divisible by $a b$.

So, we will factor out a $2 a b$.
$2 a b($
Then we divide each term of the polynomial by what we are factoring out.
$2 a b\left(\frac{2 a^{3} b^{2}}{2 a b}+\frac{8 a b}{2 a b}+\frac{16 a^{2} b^{3}}{2 a b}\right)$
Now, we simplify.
$2 a b\left(a^{2} b+4+8 a b^{2}\right)$

## Example: Use Grouping

## Factor $4 a b+8 b+3 a+6$.

$$
\begin{aligned}
4 a b & +8 b+3 a+6 & & \\
& =(4 a b+8 b)+(3 a+6) & & \text { Group terms with common factors. } \\
& =4 b(a+2)+3(a+2) & & \text { Factor the GCF from each grouping. } \\
& =(a+2)(4 b+3) & & \text { Distributive Property }
\end{aligned}
$$

## Check your progress:

1) $5 y^{2}-15 y+4 y-12$

When we have four terms we split the polynomial in half and treat it like two binomials we are factoring separately.
$\left.5 y^{2}-15 y\right\}_{\}}^{\}}+4 y-12$
We start with the first two terms and decide what $5 y^{2}$ and $-15 y$ are both divisible by (5y).
$\left.5 y\left(\frac{5 y^{2}}{5 y}-\frac{15 y}{5 y}\right)\right\}+4 y-12$
$5 y(y-3)^{3}+4 y-12$
Then we decide what the last two terms are both divisible by and we determine that $4 y$ and -12 are divisible by 4 .
$5 y(y-3)\}^{3} 4\left(\frac{4 y}{4}-\frac{12}{4}\right)$
$5 y(y-3)+4(y-3)$
Now, we should notice that both terms have a $(y-3)$, so we can factor that out.
$(y-3)\left(\frac{5 y(y-3)}{y-3}+\frac{4(y-3)}{y-3}\right)$
$(y-3)(5 y+4)$
2) $6 x^{2}-15 x-8 x+20$
$6 x^{2}-15 x^{\}}-8 x+20$
We start with the first two terms and decide what $6 x^{2}$ and $-15 x$ are both divisible by ( $3 x$ ).
$\left.3 x\left(\frac{6 x^{2}}{3 x}-\frac{15 x}{3 x}\right)\right\}-8 x+20$
$3 x(2 x-5\}-8 x+20$
Then we decide what the last two terms are both divisible by and we determine that $-8 y$ and 20 are divisible by 4 . However, because the $-8 x$ is the first term in that binomial and it is negative, we must factor out a -4 .
$3 x(2 x-5)\}_{\}}^{3} 4\left(\frac{-8 x}{-4}+\frac{20}{-4}\right)$
$3 x(2 x-5)-4(2 x-5)$
Now, we should notice that both terms have a $(2 x-5)$, so we can factor that out.
$(2 x-5)\left(\frac{3 x(2 x-5)}{2 x-5}-\frac{4(2 x-5)}{2 x-5}\right)$
$(2 x-5)(3 x-4)$

## 3) $r s+5 s-r-5$

$r s+5\}-r-5$
We start with the first two terms and decide what $r s$ and $5 s$ are both divisible by $(s)$.
$\left.s\left(\frac{r s}{s}+\frac{5 s}{s}\right)\right\}_{?} r-5$
$s(r+5) \sum_{r}^{b} r-5$
Then we decide what the last two terms are both divisible by and we determine that $-r$ and $-s$ are not divisible by the same thing. However, everything is always divisible by 1 . Because the $-r$ is the first term in that binomial and it is negative, we must factor out a -1 .
$s(r+5)\}_{\gamma}^{3}-1\left(\frac{-r}{-1}-\frac{5}{-1}\right)$
$s(r+5)-1(r+5)$
Now, we should notice that both terms have a $(r+5)$, so we can factor that out.

$$
\begin{aligned}
& (r+5)\left(\frac{s(r+5)}{r+5}-\frac{1(r+5)}{r+5}\right) \\
& (r+5)(s-1)
\end{aligned}
$$

## Example: Using the Additive Inverse Property

Factor $35 x-5 x y+3 y-21$.

$$
\begin{aligned}
35 x-5 x y+3 y-21 & =(35 x-5 x y)+(3 y-21) & & \text { Group terms with common factors. } \\
& =5 x(7-y)+3(y-7) & & \text { Factor the GCF from each grouping. } \\
& =5 x(-1)(y-7)+3(y-7) & & 7-y=-1(y-7) \\
& =-5 x(y-7)+3(y-7) & & 5 x(-1)=-5 x \\
& =(y-7)(-5 x+3) & & \text { Distributive Property }
\end{aligned}
$$

## Check your progress:

1) $c-2 c d+8 d-4$
$c-2 c d\}+8 d-4$
$c$ and $2 c d$ are both divisible by $c$.
$\left.c\left(\frac{c}{c}-\frac{2 c d}{c}\right)\right\} 8 d-4$
$c(1-2 d)\}^{3}+8 d-4 \quad * *$ Remember that $\frac{c}{c}=1$.
$8 d$ and -4 are divisible by 4 .
$c(1-2 d\}^{3}+4\left(\frac{8 d}{4}-\frac{4}{4}\right)$
$c(1-2 d)+4(2 d-1)$
We notice that $(1-2 d)$ is not the same as $(2 d-1)$. We can make them look the same if we factor a negative one out of either one. So, let's take a negative one out of the second one this time.
$c(1-2 d)+4 \cdot-1\left(\frac{2 d}{-1}-\frac{1}{-1}\right)$

$$
\begin{aligned}
& c(1-2 d)-4(-2 d+1) \\
& c(1-2 d)-4(1-2 d)
\end{aligned}
$$

Now both terms have a $(1-2 d)$, so we can factor that out.

$$
\begin{aligned}
& (1-2 d)\left(\frac{c(1-2 d)}{1-2 d}-\frac{4(1-2 d)}{1-2 d}\right) \\
& (1-2 d)(c-4)
\end{aligned}
$$

## Practice

1) $15 y^{3}-45 y^{2}$

$$
\begin{array}{r}
15 y^{2}(y-3) \quad \text { 4) } 5 c-10 c^{2}+2 d-4 c d \\
5 c(1-c)+2 d(1-c)
\end{array}
$$

2) $4 r^{2}+8 r s+28 r$

$$
4 r(r+2 s+7)
$$

$$
\text { 5) } 3 p-2 p^{2}-18 p+27
$$

3) $5 y^{2}-15 y+4 y-12$

$$
\begin{gathered}
5 y(y-3)+4(y-3) \\
(y-3)(5 y+4)
\end{gathered}
$$

$$
\begin{gathered}
p(3-2 p)-9(2 p-3) \\
p(3-2 p)+9(3-2 p) \\
(3-2 p)(p+9)
\end{gathered}
$$

