MODULE 6 LESSON 1 NOTES FACTORING USING THE DISTRIBUTIVE PROPERTY

Example: Use the Distributive Property

Use the Distributive Property to factor each polynomial.

a. $12a^2 + 16a$

First, find the GCF of $12a^2$ and $16a$.

$12a^2 = 2 \cdot 2 \cdot 3 \cdot a \cdot a$ Factor each monomial.

$16a = 2 \cdot 2 \cdot 2 \cdot 2 \cdot a$ Circle the common prime factors.

GCF: $2 \cdot 2 \cdot a$ or $4a$

Write each term as the product of the GCF and its remaining factors. Then use the Distributive Property to factor out the GCF.

$12a^2 + 16a = 4a(3 \cdot a) + 4a(2 \cdot 2)$ Rewrite each term using the GCF.

$= 4a(3a) + 4a(4)$ Simplify remaining factors.

$= 4a(3a + 4)$ Distributive Property

Thus, the completely factored form of $12a^2 + 16a$ is $4a(3a + 4)$.

b. $18cd^2 + 12c^2d + 9cd$

$18cd^2 = 2 \cdot 3 \cdot 3 \cdot c \cdot d \cdot d$ Factor each monomial.

$12c^2d = 2 \cdot 2 \cdot 3 \cdot c \cdot c \cdot d$ Circle the common prime factors.

$9cd = 3 \cdot 3 \cdot c \cdot d$

GCF: $3 \cdot c \cdot d$ or $3cd$

$18cd^2 + 12c^2d + 9cd = 3cd(6d) + 3cd(4c) + 3cd(3)$ Rewrite each term using the GCF.

$= 3cd(6d + 4c + 3)$ Distributive Property

Check your progress:

1) $9x^2 + 36x$

We need to think about the biggest factor that both $9x^2$ and $36x$ can be divided by. 9 and 36 are both divisible (can be divided) by 3 and 9. Since 9 is the biggest of those numbers, we will use 4. x^2 and a are both divisible by x. So, we will factor out a $9x$.

We start by writing what we are factoring out outside of a set of parentheses.

$9x($

Then we divide each term of the polynomial by what we are factoring out.

$$9x\left(\frac{9x^2}{9x} + \frac{36x}{9x}\right)$$

Now, we simplify. $\frac{9}{9} = 1, \frac{x^2}{x} = x, \frac{36}{9} = 4, \frac{x}{x} = 1.$

$9x(x + 4)$

2) $16xz - 40xz^2$

We need to think about the biggest factor that both $16xz$ and $40xz^2$ can be divided by. 16 and 40 are both divisible by 2, 4, and 8. Since 8 is the biggest of those numbers, we will use 8. xz and xz^2 are both divisible by xz. So, we will factor out a $8xz$.

We start by writing what we are factoring out outside of a set of parentheses.

$8xz($

Then we divide each term of the polynomial by what we are factoring out.

$$8xz\left(\frac{16xz}{8xz} - \frac{40xz^2}{8xz}\right)$$

Now, we simplify. $\frac{16}{8} = 2, \frac{xz}{xz} = 1, \frac{40}{8} = 5, \frac{xz^2}{xz} = z$.

$8xz(2 - 5z)$

3) $24m^2np^2 + 36m^2n^2p$

We need to think about the biggest factor that both $24m^2np^2$ and $36m^2n^2p$ can be divided by. 24 and 36 are both divisible by 2, 3, 4, 6, and 12. Since 12 is the biggest of those numbers, we will use 12. m^2np^2 and m^2n^2p are both divisible by m^2np. So, we will factor out a $12m^2np$.

We start by writing what we are factoring out outside of a set of parentheses.

$12m^2np($

Then we divide each term of the polynomial by what we are factoring out.

$$12m^2np\left(\frac{24m^2np^2}{12m^2np} + \frac{36m^2n^2p}{12m^2np}\right)$$

Now, we simplify. $\frac{24}{12} = 2, \frac{m^2np^2}{m^2np} = p, \frac{36}{12} = 3, \frac{m^2n^2p}{m^2np} = n$.

$12m^2np(2p + 3n)$

4) $2a^3b^2 + 8ab + 16a^2b^3$

When we have three terms, we have to determine the largest thing that divides all three terms.

2, 8, and 16 are all divisible by 2, but there is nothing else that ALL three numbers are divisible by.

a^3b^2, ab, and a^2b^3 are divisible by ab.

So, we will factor out a $2ab$.

$2ab($

Then we divide each term of the polynomial by what we are factoring out.

$$2ab\left(\frac{2a^3b^2}{2ab}+\frac{8ab}{2ab}+\frac{16a^2b^3}{2ab}\right)$$

Now, we simplify.

$2ab(a^2b+4+8ab^2)$

Example: Use Grouping

Factor $4ab + 8b + 3a + 6$.

$4ab + 8b + 3a + 6$

$= (4ab + 8b) + (3a + 6)$ Group terms with common factors.

$= 4b(a + 2) + 3(a + 2)$ Factor the GCF from each grouping.

$= (a + 2)(4b + 3)$ Distributive Property

Check your progress:

1) $5y^2 - 15y + 4y - 12$

When we have four terms we split the polynomial in half and treat it like two binomials we are factoring separately.

$5y^2 - 15y + 4y - 12$

We start with the first two terms and decide what $5y^2$ and $-15y$ are both divisible by $(5y)$.

$$5y\left(\frac{5y^2}{5y}-\frac{15y}{5y}\right)+4y-12$$

$5y(y-3)+4y-12$

Then we decide what the last two terms are both divisible by and we determine that $4y$ and -12 are divisible by 4.

$$5y(y-3)+4\left(\frac{4y}{4}-\frac{12}{4}\right)$$

$5y(y-3)+4(y-3)$

Now, we should notice that both terms have a $(y-3)$, so we can factor that out.

$$(y-3)\left(\frac{5y(y-3)}{y-3}+\frac{4(y-3)}{y-3}\right)$$

$$(y-3)(5y+4)$$

2) $6x^2-15x-8x+20$

$$6x^2-15x-8x+20$$

We start with the first two terms and decide what $6x^2$ and $-15x$ are both divisible by $(3x)$.

$$3x\left(\frac{6x^2}{3x}-\frac{15x}{3x}\right)-8x+20$$

$$3x(2x-5)-8x+20$$

Then we decide what the last two terms are both divisible by and we determine that $-8y$ and 20 are divisible by 4. However, because the $-8x$ is the first term in that binomial and it is negative, we must factor out a -4.

$$3x(2x-5)-4\left(\frac{-8x}{-4}+\frac{20}{-4}\right)$$

$$3x(2x-5)-4(2x-5)$$

Now, we should notice that both terms have a $(2x-5)$, so we can factor that out.

$$(2x-5)\left(\frac{3x(2x-5)}{2x-5}-\frac{4(2x-5)}{2x-5}\right)$$

$$(2x-5)(3x-4)$$

3) $rs+5s-r-5$

$$rs+5s-r-5$$

We start with the first two terms and decide what rs and $5s$ are both divisible by (s).

$$s\left(\frac{rs}{s}+\frac{5s}{s}\right)-r-5$$

$$s(r+5)-r-5$$

Then we decide what the last two terms are both divisible by and we determine that $-r$ and $-s$ are not divisible by the same thing. However, everything is always divisible by 1. Because the $-r$ is the first term in that binomial and it is negative, we must factor out a -1.

$s(r+5) - 1\left(\frac{-r}{-1} - \frac{5}{-1}\right)$

$x(r+5) - 1(r+5)$

Now, we should notice that both terms have a $(r+5)$, so we can factor that out.

$(r+5)\left(\frac{x(r+5)}{r+5} - \frac{1(r+5)}{r+5}\right)$

$(r+5)(x-1)$

Example: Using the Additive Inverse Property

Factor $35x - 5xy + 3y - 21$.

$35x - 5xy + 3y - 21$	$= (35x - 5xy) + (3y - 21)$	Group terms with common factors.
	$= 5x(7 - y) + 3(y - 7)$	Factor the GCF from each grouping.
	$= 5x(-1)(y - 7) + 3(y - 7)$	$7 - y = -1(y - 7)$
	$= -5x(y - 7) + 3(y - 7)$	$5x(-1) = -5x$
	$= (y - 7)(-5x + 3)$	Distributive Property

Check your progress:

1) $c - 2cd + 8d - 4$

$c - 2cd + 8d - 4$

c and $2cd$ are both divisible by c.

$c\left(\frac{c}{c} - \frac{2cd}{c}\right) + 8d - 4$

$c(1-2d) + 8d - 4$ **Remember that $\frac{c}{c} = 1$.

$8d$ and -4 are divisible by 4.

$c(1-2d) + 4\left(\frac{8d}{4} - \frac{4}{4}\right)$

$c(1-2d) + 4(2d-1)$

We notice that $(1-2d)$ is not the same as $(2d-1)$. We can make them look the same if we factor a negative one out of either one. So, let's take a negative one out of the second one this time.

$c(1-2d) + 4 \cdot -1\left(\frac{2d}{-1} - \frac{1}{-1}\right)$

$c(1-2d)-4(-2d+1)$

$c(1-2d)-4(1-2d)$

Now both terms have a $(1-2d)$, so we can factor that out.

$$(1-2d)\left(\frac{c(1-2d)}{1-2d}-\frac{4(1-2d)}{1-2d}\right)$$

$(1-2d)(c-4)$

Practice

1) $15y^3-45y^2$

    $15y^2(y-3)$

2) $4r^2+8rs+28r$

    $4r(r+2s+7)$

3) $5y^2-15y+4y-12$

    $5y(y-3)+4(y-3)$

    $(y-3)(5y+4)$

4) $5c-10c^2+2d-4cd$

    $5c(1-c)+2d(1-c)$

    $(1-c)(5c+2d)$

5) $3p-2p^2-18p+27$

    $p(3-2p)-9(2p-3)$

    $p(3-2p)+9(3-2p)$

    $(3-2p)(p+9)$