Example: Add Polynomials

Find $(3x^2 - 4x + 8) + (2x - 7x^2 - 5)$.

Method 1 Horizontal

$(3x^2 - 4x + 8) + (2x - 7x^2 - 5)$

$= [3x^2 + (-7x^2)] + (-4x + 2x) + [8 + (-5)]$ Group like terms.

$= -4x^2 - 2x + 3$ Add like terms.

Method 2 Vertical

$$\begin{array}{r} 3x^2 - 4x + 8 \\ (+)\ -7x^2 + 2x - 5 \\ \hline -4x^2 - 2x + 3 \end{array}$$

Notice that terms are in descending order with like terms aligned.

Check your progress:

1. $(5x^2 - 3x + 4) + (6x - 3x^2 - 3)$

Because there is an addition between the parentheses, we can drop them:

$$5x^2 - 3x + 4 + 6x - 3x^2 - 3$$

Add like terms: $5x^2 - 3x^2 = 2x^2$ $-3x + 6x = 3x$ $4 - 3 = 1$

$$2x^2 + 3x + 1$$

Example: Subtract Polynomials

Find $(3n^2 + 13n^3 + 5n) - (7n + 4n^3)$.

Method 1 Horizontal

Subtract $7n + 4n^3$ by adding its additive inverse.

$(3n^2 + 13n^3 + 5n) - (7n + 4n^3)$

$= (3n^2 + 13n^3 + 5n) + (-7n - 4n^3)$ The additive inverse of $7n + 4n^3$ is $-7n - 4n^3$.

$= 3n^2 + [13n^3 + (-4n^3)] + [5n + (-7n)]$ Group like terms.

$= 3n^2 + 9n^3 - 2n$ Combine like terms.

Method 2 Vertical

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{r} 3n^2 + 13n^3 + 5n \\ (-)\quad 4n^3 + 7n \\ \hline \end{array} \quad \text{Add the opposite.} \quad \begin{array}{r} 3n^2 + 13n^3 + 5n \\ (+)\quad -4n^3 - 7n \\ \hline 3n^2 + 9n^3 - 2n \end{array}$$

Thus, $(3n^2 + 13n^3 + 5n) - (7n + 4n^3) = 3n^2 + 9n^3 - 2n$ or, arranged in descending order, $9n^3 + 3n^2 - 2n$.

Check your progress:

1. $(4x^3 - 3x^2 + 6x - 4) - (-2x^3 + x^2 - 2)$

Because there is a subtraction between the parentheses, we must distribute the negative through the second parentheses.

$$4x^3 - 3x^2 + 6x + 2x^3 - x^2 + 2$$

Add like terms: $4x^3 + 2x^3 = 6x^3$ $-3x^2 - x^2 = -4x^2$ $6x$ and 2 have no like term

$$6x^3 - 4x^2 + 6x + 2$$

Example: Polynomials

EDUCATION The total number of public school teachers T consists of two groups, elementary E and secondary S. From 1992 through 2003, the number (in thousands) of secondary teachers and total teachers could be modeled by the following equations, where n is the number of years since 1992.

$S = 29n + 949 \quad T = 58n + 2401$

a. Find an equation that models the number of elementary teachers E for this time period.

Subtract the polynomial for S from the polynomial for T.

Total	$58n + 2401$		$58n + 2401$
− Secondary	$(-)\ 29n + 949$	Add the opposite.	$(+)\ -29n - 949$
Elementary			$29n + 1452$

An equation is $E = 29n + 1452$.

b. Use the equation to predict the number of elementary teachers in 2015.

The year 2015 is 2015 − 1992 or 23 years after the year 1992.

If this trend continues, the number of elementary teachers in 2015 would be 29(23) + 1452 or about 2,119,000.

Check your progress:

1. From 1980 through 2003, the female population F and the male population M of the United States (in thousands) are modeled by the following equations, where n is the number of years since 1980.

$$F = 1{,}379n + 115{,}513 \qquad M = 1{,}450n + 108{,}882$$

a) Find an equation that models the total population T of the United States in thousands for this time period.

Total = Female + Male $\quad T = (1379n + 115513) + (1450n + 108882)$

$$T = 2829n + 224395$$

b) If this trend continues, what will the population of the U.S. be in 2010?

$$2010 - 1980 = 30$$

$T = 2829(30) + 224395 = 84870 + 24395 = 309265$ (in thousands)

The population is 2010 would have been 309,265,000 if the trend continued.

Practice

Find each sum or difference.

1) $(4p^2 + 5p) + (-2p^2 + p)$

$2p^2 + 6p$

2) $(5y^2 - 3y + 8) + (4y^2 - 9)$

$9y^2 - 3y - 1$

3) $(8cd - 3d + 4c) + (-6 + 2cd)$

$10cd - 3d + 4c - 6$

4) $(-8xy + 3x^2 - 5y) + (4x^2 - 2y + 6xy)$

$-2xy + 7x^2 - 7y$

5) $(6a^2 + 7a - 9) - (-5a^2 + a - 10)$

$6a^2 + 7a - 9 + 5a^2 - a + 10$

$11a^2 + 6a + 1$

6) $(g^3 - 2g^2 + 5g + 6) - (g^2 + 2g)$

$g^3 - 2g^2 + 5g + 6 - g^2 - 2g$

$g^3 - 3g^2 + 3g + 6$

7) $(3ax^2 - 5x - 3a) - (6a - 8a^2x + 4x)$

$3ax^2 - 5x - 3a - 6a + 8a^2x - 4x$

$3ax^2 - 9x - 9a + 8a^2x$

8) $(4rst - 8r^2s + s^2) - (6rs^2 + 5rst - 2s^2)$

$4rst - 8r^2s + s^2 - 6rs^2 - 5rst + 2s^2$

$-rst - 8r^2s + 3s^2 - 6rs^2$