

Definition of a Polynomial:

A polynomial is a monomial or a sum of monomials.

Special names of polynomials:

- A binomial is the sum of two monomials.
- A trinomial is the sum of three monomials.

Monomial	Binomial	Trinomial
7	$3 + 4y$	$x + y + z$
$4ab^3c^2$	$7pqr + pq^2$	$3v^2 - 2w + ab^3$

Example: State whether each expression is a polynomial. If it is a polynomial, identify it as a monomial, binomial, or trinomial.

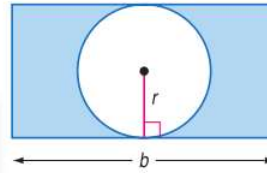
	Expression	Polynomial?	Monomial, Binomial, or Trinomial?
a.	$2x - 3yz$	Yes, $2x - 3yz = 2x + (-3yz)$, the sum of two monomials.	binomial
b.	$8n^3 + 5n^{-2}$	No. $5n^{-2} = \frac{5}{n^2}$, which is not a monomial.	none of these
c.	-8	Yes, -8 is a real number.	monomial
d.	$4a^2 + 5a + a + 9$	Yes, the expression simplifies to $4a^2 + 6a + 9$, so it is the sum of three monomials.	trinomial

Check your progress:

Expression	Polynomial?	Monomial, Binomial, or Trinomial?
x	YES	Monomial
$-3y^2 - 2y + 4y - 1$	YES	Trinomial
$5rs + 7tuv$	YES	Binomial
$10x^{-4} - 8x^3$	NO	$10x^{-4}$ is not a monomial, so none of these.

Example: Write a polynomial.

GEOMETRY Write a polynomial to represent the area of the shaded region.

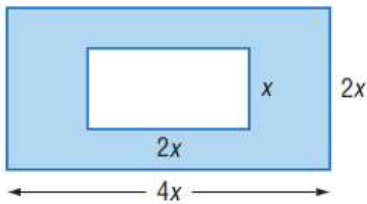


Words	The area of the shaded region is the area of the rectangle minus the area of the circle.		
Variables	area of shaded region = A width of rectangle = $2r$ rectangle area = $b(2r)$ circle area = πr^2		
	Area of shaded region = rectangle area - circle area.		
Equation	A	$=$	$b(2r) - \pi r^2$
	A	$=$	$2br - \pi r^2$

The polynomial representing the area of the shaded region is $2br - \pi r^2$.

Check your progress:

- Write a polynomial to represent the area of the shaded region.



$$A = (2x)(4x) - (2x)(x) = 8x^2 - 2x^2 = 6x^2$$

Degree of a Monomial:

The degree of a monomial is the sum of the exponents of the variables.

Monomial	Degree
$8y^4$	4
$3a$	1
$-2xy^2z^3$	1 + 2 + 3 or 6
7	0

Degree of a Polynomial:

The degree of a polynomial is the greatest degree of any term in the polynomial. To find the degree of a polynomial, you must find the degree of each term.

Example: Find the degree of each polynomial.

Find the degree of each polynomial.

	Polynomial	Terms	Degree of Each Term	Degree of Polynomial
a.	$5mn^2$	$5mn^2$	3	3
b.	$-4x^2y^2 + 3x^2 + 5$	$-4x^2y^2, 3x^2, 5$	4, 2, 0	4
c.	$3a + 7ab - 2a^2b + 16$	$3a, 7ab, -2a^2b, 16$	1, 2, 3, 0	3

Check your progress:

1) $7xy^5z$

There is only one term, it has degree $1 + 5 + 1 = 7$, so the polynomial has degree 7.

2) $12m^3n^2 - 8mn^2 + 3$

The terms have degree 5, 3, and 0, respectively. The polynomial has degree 5.

3) $2rs - 3rs^2 - 7r^2x^2 - 13$

The terms have degree 2, 3, 4, and 0, respectively. The polynomial has degree 4.

Ascending means increasing order. Descending means decreasing order.

Example: Arrange the polynomials in ascending order.

Arrange the terms of each polynomial so that the powers of x are in ascending order.

a. $7x^2 + 2x^4 - 11$

$$\begin{aligned} 7x^2 + 2x^4 - 11 &= 7x^2 + 2x^4 - 11x^0 \quad x^0 = 1 \\ &= -11 + 7x^2 + 2x^4 \quad \text{Compare powers of } x: 0 < 2 < 4. \end{aligned}$$

b. $2xy^3 + y^2 + 5x^3 - 3x^2y$

$$\begin{aligned} 2xy^3 + y^2 + 5x^3 - 3x^2y &= 2x^1y^3 + y^2 + 5x^3 - 3x^2y^1 \quad x = x^1 \\ &= y^2 + 2xy^3 - 3x^2y + 5x^3 \quad \text{Compare powers of } x: 0 < 1 < 2 < 3. \end{aligned}$$

Check your progress:

1) $3x^2y^4 + 2x^4y^2 - 4x^3y + x^5 - y^2$

$$-y^2 + 3x^2y^4 - 4x^3y + 2x^4y^2$$

2) $7x^3 - 4xy^4 + 3x^2y^3 - 11x^6y$

$$-4xy^4 + 3x^2y^3 + 7x^3 - 11x^6y$$

Example: Arrange the polynomials in descending order

Arrange the terms of each polynomial so that the powers of x are in descending order.

a. $6x^2 + 5 - 8x - 2x^3$

$$6x^2 + 5 - 8x - 2x^3 = 6x^2 + 5x^0 - 8x^1 - 2x^3 \quad x^0 = 1 \text{ and } x = x^1$$

$$= -2x^3 + 6x^2 - 8x + 5 \quad 3 > 2 > 1 > 0$$

b. $3a^3x^2 - a^4 + 4ax^5 + 9a^2x$

$$3a^3x^2 - a^4 + 4ax^5 + 9a^2x$$

$$= 3a^3x^2 - a^4x^0 + 4a^1x^5 + 9a^2x^1 \quad a = a^1, x^0 = 1, \text{ and } x = x^1$$

$$= 4ax^5 + 3a^3x^2 + 9a^2x - a^4 \quad 5 > 2 > 1 > 0.$$

Check your progress:

1) $4x^2 + 2x^3y + 5 - x$
 $2x^3y + 4x^2 - x + 5$

2) $x + 2x^7y - 5x^4y^8 - x^2y^2 + 3$
 $2x^7y - 5x^4y^8 - x^2y^2 + x + 3$

Practice

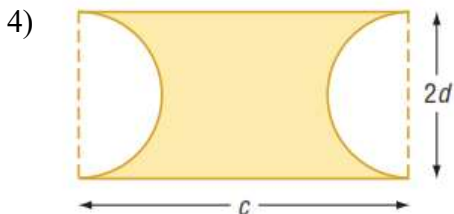
State whether each expression is a polynomial. If the expression is a polynomial, identify it as a monomial, a binomial, or trinomial.

1) $5x - 3xy + 2x$
 Is a polynomial. Binomial.

2) $\frac{2z}{5}$
 Is a polynomial. Monomial.

3) $9a^2 + 7a - 5$
 Is a polynomial. Trinomial.

Write a polynomial to represent the area of the shaded region.



$$A = (c)(2d) - \pi(d^2) = 2cd - \pi d^2$$

Find the degree of each polynomial.

5) 1 Degree: 0

6) $3x + 2$ Degree: 1

7) $2x^2y^3 + 6x^4$ Degree: 5

Arrange the terms of the polynomial so that the powers of x are in ascending order.

8) $6x^3 - 12 + 5x$
 $-12 + 5x + 6x^3$

9) $-7a^2x^3 + 4x^2 - 2ax^5 + 2a$
 $2a + 4x^2 - 7a^2x^3 - 2ax^5$

Arrange the terms of the polynomial so that the powers of x are in descending order.

10) $2c^5 + 9cx^2 + 3x$
 $9cx^2 + 3x + 2c^5$

11) $y^3 + x^3 + 3x^2y + 3xy^2$
 $x^3 + 3x^2y + 3xy^2 + y^3$