Definition of a Polynomial:

A polynomial is a monomial or a sum of monomials.

Special names of polynomials:

- A binomial is the sum of two monomials.
- A trinomial is the sum of three monomials.

Monomial	Binomial	Trinomial
7	3+4y	x + y + z
4ab ³ c ²	$7pqr + pq^2$	$3v^2 - 2w + ab^3$

Example: State whether each expression is a polynomial. If it is a polynomial, identify it as a monomial, binomial, or trinomial.

	Expression	Polynomial?	Monomial, Binomial, or Trinomial?
a.	2x - 3yz	Yes, $2x - 3yz = 2x + (-3yz)$, the sum of two monomials.	binomial
b.	$8n^3 + 5n^{-2}$	No. $5n^{-2} = \frac{5}{n^2}$, which is not a monomial.	none of these
c.	-8	Yes, —8 is a real number.	monomial
d.	$4a^2 + 5a + a + 9$	Yes, the expression simplifies to $4a^2 + 6a + 9$, so it is the sum of three monomials.	trinomial

Check your progress:

Expression	Polynomial?	Monomial, Binomial, or Trinomial?
x	YES	Monomial
$-3y^2-2y+4y-1$	YES	Trinomial
5rs + 7tuv	YES	Binomial
$10x^{-4} - 8x^3$	NO	$10x^{-4}$ is not a monomial, so none of these.

Example: Write a polynomial.

GEOMETRY Write a polynomial to represent the area of the shaded region.





Check your progress:

1) Write a polynomial to represent the area of the shaded region.



$$A = (2x)(4x) - (2x)(x) = 8x^2 - 2x^2 = 6x^2$$

Degree of a Monomial:

The degree of a monomial is the sum of the exponents of the variables.

Monomial	Degree
8y ⁴	4
<u>3</u> a	1
$-2xy^2z^3$	1 + 2 + 3 or 6
7	0

Degree of a Polynomial:

The degree of a polynomial is the greatest degree of any term in the polynomial. To find the degree of a polynomial, you must find the degree of each term.

Example: Find the degree of each polynomial.

	Polynomial	Terms	Degree of Each Term	Degree of Polynomial
a.	5mn ²	5mn ²	3	3
b.	$-4x^2y^2 + 3x^2 + 5$	$-4x^2y^2$, $3x^2$, 5	4, 2, 0	4
C.	$3a + 7ab - 2a^2b + 16$	3a, 7ab, –2a²b, 16	1, 2, 3, 0	3

Find the degree of each polynomial.

Check your progress:

1) $7xy^5z$

There is only one term, it has degree 1 + 5 + 1 = 7, so the polynomial has degree 7.

- 2) $12m^3n^2 8mn^2 + 3$ The terms have degree 5, 3, and 0, respectively. The polynomial has degree 5.
- 3) $2rs 3rs^2 7r^2x^2 13$ The terms have degree 2, 3, 4, and 0, respectively. The polynomial has degree 4.

Ascending means increasing order. Descending means decreasing order.

Example: Arrange the polynomials in ascending order.

Arrange the terms of each polynomial so that the powers of *x* are in ascending order.

a. $7x^2 + 2x^4 - 11$ $7x^2 + 2x^4 - 11 = 7x^2 + 2x^4 - 11x^0$ $x^0 = 1$ $= -11 + 7x^2 + 2x^4$ Compare powers of x: 0 < 2 < 4. **b.** $2xy^3 + y^2 + 5x^3 - 3x^2y$ $2xy^3 + y^2 + 5x^3 - 3x^2y$ $= 2x^1y^3 + y^2 + 5x^3 - 3x^2y^1$ $x = x^1$ $= y^2 + 2xy^3 - 3x^2y + 5x^3$ Compare powers of x: 0 < 1 < 2 < 3.

Check your progress:

- 1) $3x^2y^4 + 2x^4y^2 4x^3y + x^5 y^2$ $-y^2 + 3x^2y^4 - 4x^3y + 2x^4y^2$
- 2) $7x^3 4xy^4 + 3x^2y^3 11x^6y$ $-4xy^4 + 3x^2y^3 + 7x^3 - 11x^6y$

Example: Arrange the polynomials is descending order

Arrange the terms of each polynomial so that the powers of *x* are in descending order.

a.
$$6x^2 + 5 - 8x - 2x^3$$

 $6x^2 + 5 - 8x - 2x^3 = 6x^2 + 5x^0 - 8x^1 - 2x^3$ $x^0 = 1$ and $x = x^1$
 $= -2x^3 + 6x^2 - 8x + 5$ $3 > 2 > 1 > 0$

b.
$$3a^{3}x^{2} - a^{4} + 4ax^{5} + 9a^{2}x$$

 $3a^{3}x^{2} - a^{4} + 4ax^{5} + 9a^{2}x$
 $= 3a^{3}x^{2} - a^{4}x^{0} + 4a^{1}x^{5} + 9a^{2}x^{1}$ $a = a^{1}, x^{0} = 1, \text{ and } x = x^{1}$
 $= 4ax^{5} + 3a^{3}x^{2} + 9a^{2}x - a^{4}$ $5 > 2 > 1 > 0.$

Check your progress:

- 1) $4x^2 + 2x^3y + 5 x$ $2x^3y + 4x^2 - x + 5$
- 2) $x + 2x^7y 5x^4y^8 x^2y^2 + 3$ $2x^7y - 5x^4y^8 - x^2y^2 + x + 3$

Practice

State whether each expression is a polynomial. If the expression is a polynomial, identify it as a monomial, a binomial, or trinomial.

- 5x 3xy + 2x Is a polynomial. Binomial.
 2z/5 Is a polynomial. Monomial.
- 3) $9a^2 + 7a 5$ Is a polynomial. Trinomial.

Write a polynomial to represent the area of the shaded region.



Find the degree of each polynomial.

5)	1	Degree: 0
6)	3x + 2	Degree: 1
7)	$2x^2y^3 + 6x^4$	Degree: 5

Arrange the terms of the polynomial so that the powers of *x* are in ascending order.

8) $6x^3 - 12 + 5x$ $-12 + 5x + 6x^3$ 9) $-7a^2x^3 + 4x^2 - 2ax^5 + 2a$ $2a + 4x^2 - 7a^2x^3 - 2ax^5$

Arrange the terms of the polynomial so that the powers of x are in descending order.

$$10) 2c^{5} + 9cx^{2} + 3x 9cx^{2} + 3x + 2c^{5} 11) y^{3} + x^{3} + 3x^{2}y + 3xy^{2} x^{3} + 3x^{2}y + 3xy^{2} + y^{3}$$