PROPERTIES OF EXPONENTS:

Name	Words	Symbols	Example	Justification
Product of Powers	To multiply two powers that have the same base, add their exponents.	$a^m \cdot a^n$ $= a^{m+n}$	$a^4 \cdot a^{12}$ $= a^{4+12}$ $= a^{16}$	$2^3 \cdot 2^5 = \underbrace{\overbrace{2 \cdot 2 \cdot 2}^{3\text{ factors}} \cdot \overbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}^{5\text{ factors}}}_{3+5\text{ or }8\text{ factors}}$ or 2^8 $3^2 \cdot 3^4 = \underbrace{\overbrace{3 \cdot 3}^{2\text{ factors}} \cdot \overbrace{3 \cdot 3 \cdot 3 \cdot 3}^{4\text{ factors}}}_{2+4\text{ or }6\text{ factors}}$ or 3^6
Power of a Power	To find the power of a power, multiply the exponents.	$(a^m)^n$ $= a^{m \cdot n}$	$(k^5)^9 = k^{5 \cdot 9}$ $= k^{45}$	$(4^2)^5 = \overbrace{(4^2)(4^2)(4^2)(4^2)(4^2)}^{5\text{ factors}}$ $= 4^{2+2+2+2+2}$ $= 4^{10}$ $(z^8)^3 = \overbrace{(z^8)(z^8)(z^8)}^{3\text{ factors}}$ $= z^{8+8+8}$ $= z^{24}$
Power of a Product	To find the power of a product, find the power of each factor and multiply.	$(ab)^m$ $= a^m b^m$	$(-2xy)^3$ $= (-2)^3 x^3 y^3$ $= -8x^3y^3$	$(xy)^4 = (xy)(xy)(xy)(xy)$ $= (x \cdot x \cdot x \cdot x)(y \cdot y \cdot y \cdot y)$ $= x^4y^4$ $(6ab)^3 = (6ab)(6ab)(6ab)$ $= (6 \cdot 6 \cdot 6)(a \cdot a \cdot a)(b \cdot b \cdot b)$ $= 6^3a^3b^3$ or $216a^3b^3$
Quotient of Powers	To divide two powers with the same base, subtract the exponents.	$\frac{a^m}{b^m}$ $= a^{m-n}$	$\frac{b^{15}}{b^7} = b^{15-7}$ $= b^8$	$\frac{4^5}{4^3} = \frac{\overbrace{\overset{1}{\cancel{4}} \cdot \overset{1}{\cancel{4}} \cdot \overset{1}{\cancel{4}} \cdot 4 \cdot 4}^{5\text{ factors}}}{\underbrace{\underset{1}{\cancel{4}} \cdot \underset{1}{\cancel{4}} \cdot \underset{1}{\cancel{4}}}_{3\text{ factors}}} = \underbrace{4 \cdot 4}_{5-3\text{ or }2\text{ factors}}$ or 4^2 $\frac{3^6}{3^2} = \frac{\overbrace{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}} \cdot 3 \cdot 3 \cdot 3 \cdot 3}^{6\text{ factors}}}{\underbrace{\underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{3}}}_{2\text{ factors}}} = \underbrace{3 \cdot 3 \cdot 3 \cdot 3}_{6-2\text{ or }4\text{ factors}}$ or 3^4
Power of a Quotient	To find the power of a quotient, find the power of the numerator and the denominator.	$\left(\frac{a}{b}\right)^m$ $= \frac{a^m}{b^m}$	$\left(\frac{x}{3}\right)^3 = \frac{x^3}{3^3}$ $= \frac{x^3}{27}$	$\left(\frac{2}{5}\right)^3 = \underbrace{\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)}_{3\text{ factors}} = \frac{\overbrace{2 \cdot 2 \cdot 2}^{3\text{ factors}}}{\underbrace{5 \cdot 5 \cdot 5}_{3\text{ factors}}}$ or $\frac{2^3}{5^3}$

Zero Exponent	Any nonzero number raised to the zero power is 1.	$a^0 = 1$	$(-0.25)^0 = 1$	**Method 1** $\frac{2^4}{2^4} = 2^{4-4}$ Quotient of Powers $= 2^0$ Subtract. **Method 2** $\frac{2^4}{2^4} = \frac{\cancel{2}^1 \cdot \cancel{2}^1 \cdot \cancel{2}^1 \cdot \cancel{2}^1}{\cancel{2}_1 \cdot \cancel{2}_1 \cdot \cancel{2}_1 \cdot \cancel{2}_1}$ Definition of powers $= 1$ Simplify. Since $\frac{2^4}{2^4}$ cannot have two different values, we can conclude that $2^0 = 1$.
Negative Exponent	For any nonzero number a and any integer n, a^{-n} is the reciprocal of a^n. In addition, the reciprocal of a^{-n} is a^n.	$a^{-n} = \frac{1}{a^n}$ $\frac{1}{a^{-n}} = a^n$	$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$ $\frac{1}{6^{-3}} = 6^3 = 216$	**Method 1** $\frac{8^2}{8^5} = 8^{2-5}$ Quotient of Powers $= 8^{-3}$ Subtract. **Method 2** $\frac{8^2}{8^5} = \frac{\cancel{8}^1 \cdot \cancel{8}^1}{\cancel{8}_1 \cdot \cancel{8}_1 \cdot 8 \cdot 8 \cdot 8}$ Definition of powers $= \frac{1}{8^3}$ Simplify. Since $\frac{8^2}{8^5}$ cannot have two different values, we can conclude that $8^{-3} = \frac{1}{8^3}$.

<u>Example: Quotient of Powers</u>

Simplify $\frac{a^5b^8}{ab^3}$. Assume that no denominator is equal to zero.

$\frac{a^5b^8}{ab^3} = \left(\frac{a^5}{a}\right)\left(\frac{b^8}{b^3}\right)$ Group powers that have the same base.

$= \left(a^{5-1}\right), \left(b^{8-3}\right)$ or a^4b^5 Quotient of Powers

<u>Check your progress:</u>

1) $\frac{x^3y^4}{x^2y}$

$$\left(\frac{x^3}{x^2}\right)\left(\frac{y^4}{y}\right) = (x^{3-2})(y^{4-1}) = x^1y^3 = xy^3$$

Example: Power of a Quotient

Simplify $\left(\frac{2p^2}{3}\right)^4$.

$\left(\frac{2p^2}{3}\right)^4 = \frac{(2p^2)^4}{3^4}$ Power of a Quotient

$= \frac{2^4(p^2)^4}{3^4}$ Power of a Product

$= \frac{16p^8}{81}$ Power of a Power

Check your progress:

1) $\left(\frac{3x^4}{4}\right)^3$

$$\frac{(3x^4)^3}{(4)^3} = \frac{(3)^3(x^4)^3}{(4)^3} = \frac{27(x^{4\cdot 3})}{64} = \frac{27}{64}x^{12}$$

2) $\left(\frac{5x^5y}{6}\right)^2$

$$\frac{(5x^5y)^2}{(6)^2} = \frac{(5)^2(x^5)^2(y)^2}{(6)^2} = \frac{25(x^{5\cdot 2})y^2}{36} = \frac{25}{36}x^{10}y^2$$

Example: Zero Exponent

Simplify each expression. Assume that no denominator is equal to zero.

a. $\left(-\frac{3x^5y}{8xy^7}\right)^0$

$\left(-\frac{3x^5y}{8xy^7}\right)^0 = 1$ $a^0 = 1$

b. $\frac{t^3s^0}{t}$

$\frac{t^3s^0}{t} = \frac{t^3(1)}{t}$ $a^0 = 1$

$= \frac{t^3}{t}$ Simplify.

$= t^2$ Quotient of Powers

Check your progress:

1) $\frac{x^0y^4}{y^2}$

$$\left(\frac{x^0}{1}\right)\left(\frac{y^4}{y^2}\right) = \left(\frac{1}{1}\right)(y^{4-2}) = (1)(y^2) = y^2$$

2) $\left(\frac{2x^3y^2z^5}{10xy^3z^4}\right)^0$

$$\left(\frac{2x^3y^2z^5}{10xy^3z^4}\right)^0 = 1$$

Example: Negative Exponent

Simplify each expression. Assume that no denominator is equal to zero.

a. $\dfrac{b^{-3}c^2}{d^{-5}}$

$$\frac{b^{-3}c^c}{d^{-5}} = \left(\frac{b^{-3}}{1}\right)\left(\frac{c^2}{1}\right)\left(\frac{1}{d^{-5}}\right) \qquad \text{Write as a product of fractions.}$$

$$= \left(\frac{1}{b^3}\right)\left(\frac{c^2}{1}\right)\left(\frac{d^5}{1}\right) \qquad a^{-n} = \frac{1}{a^n}$$

$$= \frac{c^2d^5}{b^3} \qquad \text{Multiply fractions.}$$

b. $\dfrac{-3a^{-4}b^7}{21a^2b^7c^{-5}}$

$$\frac{-3a^{-4}b^7}{21a^2b^7c^{-5}} = \left(\frac{-3}{21}\right)\left(\frac{a^{-4}}{a^2}\right)\left(\frac{b^7}{b^7}\right)\left(\frac{1}{c^{-5}}\right) \qquad \text{Group powers with the same base.}$$

$$= \frac{-1}{7}\left(a^{-4-2}\right)\left(b^{7-7}\right)\left(c^5\right) \qquad \text{Quotient of Powers and Negative Exponent Properties}$$

$$= \frac{-1}{7}a^{-6}b^0c^5 \qquad \text{Simplify.}$$

$$= \frac{-1}{7}\left(\frac{1}{a^6}\right)(1)c^5 \qquad \text{Negative Exponent and Zero Exponent Properties}$$

$$= -\frac{c^5}{7a^6} \qquad \text{Multiply fractions.}$$

c. $\dfrac{-3q^{-2}rs^4}{-12qr^{-3}s^{-5}}$

$$\frac{-3q^{-2}rs^4}{-12qr^{-3}s^{-5}} = \left(\frac{-3}{-12}\right)\left(\frac{q^{-2}}{q}\right)\left(\frac{r}{r^{-3}}\right)\left(\frac{s^4}{s^{-5}}\right) \qquad \text{Group powers with the same base.}$$

$$= \frac{1}{4}q^{-3}r^4s^9 \qquad \text{Simplify.}$$

$$= \frac{r^4s^9}{4q^3} \qquad \text{Negative Exponent Property}$$

Check your progress:

1) $\dfrac{r^{-5}s^4}{t^{-3}}$

$$\left(\frac{r^{-5}}{1}\right)\left(\frac{s^4}{1}\right)\left(\frac{1}{t^{-3}}\right) = \left(\frac{1}{r^5}\right)\left(\frac{s^4}{1}\right)\left(\frac{t^3}{1}\right) = \frac{s^4t^3}{r^5}$$

2) $\dfrac{24\quad^{-2}y^4}{-6x^{-3}y^{-2}z^{-1}}$

$$\left(\frac{24}{-6}\right)\left(\frac{x^{-2}}{x^{-3}}\right)\left(\frac{y^4}{y^{-2}}\right)\left(\frac{1}{z^{-1}}\right) = (-4)\left(x^{-2--3}\right)\left(y^{4--2}\right)\left(\frac{z^1}{1}\right) = (-4)(x^1)(y^6)(z^1) = -4xy^6z$$

***Do not confuse a negative number with a negative exponent. $3^{-1} = \frac{1}{3}$ $\quad -3 \neq \frac{1}{3}$

Practice

Simplify. Assume that no denominator is equal to zero.

1) $\frac{7^8}{7^2}$

$$(7^{8-2}) = 7^6$$

2) $\frac{x^8y^{12}}{x^2y^7}$

$$\left(\frac{x^8}{x^2}\right)\left(\frac{y^{12}}{y^7}\right) = (x^{8-2})(y^{12-7}) = x^6y^5$$

3) $\frac{5pq^7}{10p^6q^3}$

$$\left(\frac{5}{10}\right)\left(\frac{p}{p^6}\right)\left(\frac{q^7}{q^3}\right) = \left(\frac{1}{2}\right)(p^{1-6})(q^{7-3})$$

$$= \left(\frac{1}{2}\right)(p^{-5})(q^4)$$

$$= \left(\frac{1}{2}\right)\left(\frac{p^{-5}}{1}\right)\left(\frac{q^4}{1}\right) = \left(\frac{1}{2}\right)\left(\frac{1}{p^5}\right)\left(\frac{q^4}{1}\right)$$

$$= \frac{q^4}{2p^5}$$

4) $\left(\frac{2c^3d}{7z^2}\right)^3$

$$\frac{(2c^3d)^3}{(7z^2)^3} = \frac{(2)^3(c^3)^3(d)^3}{(7)^3(z^2)^3} = \frac{(2^3)(c^{3\cdot 3})(d^3)}{(7^3)(z^{2\cdot 3})}$$

$$= \frac{8c^9d^3}{343z^6}$$

5) $\left(\frac{4a^2b}{2c^3}\right)^2$

$$\frac{(4a^2b)^2}{(2c^3)^2} = \frac{(4)^2(a^2)^2(b)^2}{(2)^2(c^3)^2} = \frac{(4^2)(a^{2\cdot 2})(b^2)}{2^2(c^{3\cdot 2})}$$

$$= \frac{16a^4b^2}{4c^6} = \left(\frac{16}{4}\right)\left(\frac{a^4b^2}{c^6}\right) = \frac{4a^4b^2}{c^6}$$

6) $\left(\frac{3mn^3}{6n^2}\right)^2$

$$\frac{(3mn^3)^2}{(6n^2)^2} = \frac{(3)^2(m)^2(n^3)^2}{(6)^2(n^2)^2} = \frac{(3^2)(m^2)(n^{3\cdot 2})}{(6^2)(n^{2\cdot 2})}$$

$$= \frac{9m^2n^6}{36n^4} = \left(\frac{9}{36}\right)\left(\frac{m^2}{1}\right)\left(\frac{n^6}{n^4}\right)$$

$$= \left(\frac{1}{4}\right)\left(\frac{m^2}{1}\right)\left(\frac{n^{6-4}}{1}\right)$$

$$= \left(\frac{1}{4}\right)\left(\frac{m^2}{1}\right)\left(\frac{n^2}{1}\right) = \frac{m^2n^2}{4}$$

7) $y^0(y^5)(y^{-9})$

$$y^{0+5+-9} = y^{-4} = \frac{y^{-4}}{1} = \frac{1}{y^4}$$

8) $\frac{(4m^{-3}n^5)^0}{mn}$

$$\frac{1}{mn}$$

9) $\frac{(3x^2y^5)^0}{(21\ \ {}^5y^2)^0}$

$$1$$

10) 13^{-2}

$$\frac{13^{-2}}{1} = \frac{1}{13^2} = \frac{1}{169}$$

11) $\frac{c^{-5}}{d^3g^{-8}}$

$$\left(\frac{c^{-5}}{1}\right)\left(\frac{1}{d^3}\right)\left(\frac{1}{g^{-8}}\right) = \left(\frac{1}{c^5}\right)\left(\frac{1}{d^3}\right)\left(\frac{g^8}{1}\right)$$

$$= \frac{g^8}{c^5d^3}$$

12) $\frac{(cd^{-2})^3}{(c^4d^9)^{-2}}$

$$\frac{(c)^3(d^{-2})^3}{(c^4)^{-2}(d^9)^{-2}} = \frac{(c^3)(d^{-2\cdot 3})}{(c^{4\cdot -2})(d^{9\cdot -2})}$$

$$= \frac{c^3d^{-6}}{c^{-8}d^{-18}} = \left(\frac{c^3}{c^{-8}}\right)\left(\frac{d^{-6}}{d^{-18}}\right)$$

$$= (c^{3--8})(d^{-6--18}) = c^{11}d^{12}$$