Definition of a Monomial: A monomial is a number, a variable, or a product of a number and one or more variables

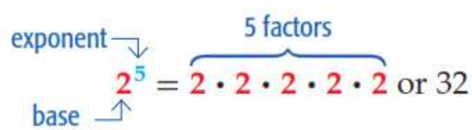
**Monomials that are real numbers are called constants.

Expression	Monomial?	Reason
-5	yes	-5 is a real number and an example of a constant.
$p+q$	no	The expression involves the addition, not the product, of two variables.
x	yes	Single variables are monomials.

Check your progress:

Expression	Monomial?	Reason
$-x+5$	NO	The expression involves the addition, no the product, of two variables.
$23abcd^2$	YES	The expression involves the product of a number and multiple variables.
$\frac{xyz^3}{2}$	YES	The expression involves the product of a number (dividing by two is just multiplying by $\frac{1}{2}$) and multiple variables.
$\frac{ab}{c}$	NO	The expression involves the quotient of variables.

Exponent and Base:



PROPERTIES OF EXPONENTS:

Name	Words	Symbols	Example	Justification
Product of Powers	To multiply two powers that have the same base, add their exponents.	$a^m \cdot a^n$ $= a^{m+n}$	$a^4 \cdot a^{12}$ $= a^{4+12}$ $= a^{16}$	$2^3\cdot 2^5 = \underbrace{\overbrace{2\cdot2\cdot2}^{3\text{ factors}}\cdot\overbrace{2\cdot2\cdot2\cdot2\cdot2}^{5\text{ factors}}}_{3+5\text{ or }8\text{ factors}}$ or 2^8 $3^2\cdot 3^4 = \underbrace{\overbrace{3\cdot3}^{2\text{ factors}}\cdot\overbrace{3\cdot3\cdot3\cdot3}^{4\text{ factors}}}_{2+4\text{ or }6\text{ factors}}$ or 3^6
Power of a Power	To find the power of a power, multiply the exponents.	$(a^m)^n$ $= a^{m\cdot n}$	$(k^5)^9 = k^{5\cdot 9}$ $= k^{45}$	$(4^2)^5 = \overbrace{(4^2)(4^2)(4^2)(4^2)(4^2)}^{5\text{ factors}}$ $= 4^{2+2+2+2+2}$ $= 4^{10}$

				$(z^8)^3 = \overbrace{(z^8)(z^8)(z^8)}^{\text{3 factors}}$ $\longrightarrow = z^{8+8+8}$ $= z^{24}$
Power of a Product	To find the power of a product, find the power of each factor and multiply.	$(ab)^m$ $= a^m b^m$	$(-2xy)^3$ $= (-2)^3 x^3 y^3$ $= -8x^3y^3$	$(xy)^4 = (xy)(xy)(xy)(xy)$ $= (x \cdot x \cdot x \cdot x)(y \cdot y \cdot y \cdot y)$ $= x^4y^4$ $(6ab)^3 = (6ab)(6ab)(6ab)$ $= (6 \cdot 6 \cdot 6)(a \cdot a \cdot a)(b \cdot b \cdot b)$ $= 6^3a^3b^3$ or $216a^3b^3$

Example: Product of Powers

Simplify each expression.

a. $(5x^7)(x^6)$

$(5x^7)(x^6) = (5)(1)(x^7)(x^6)$ Group the coefficients and the variables.

$= (5 \cdot 1)(x^{7+6})$ Product of Powers

$= 5x^{13}$ Simplify.

b. $(4ab^6)(-7a^2b^3)$

$(4ab^6)(-7a^2b^3) = (4)(-7)(a \cdot a^2)(b^6 \cdot b^3)$ Group the coefficients and the variables.

$= -28(a^{1+2})(b^{6+3})$ Product of Powers

$= -28a^3b^9$ Simplify.

Check your progress:

1) $(3y^4)(7y^5)$

$(3 \cdot 7)(y^4 \cdot y^5) = (21)(y^{4+5}) = 21y^9$

2) $(-4rs^2t^3)(-6r^5s^2t^3)$

$(-4 \cdot -6)(r \cdot r^5)(s^2 \cdot s^2)(t^3 \cdot t^3) = (24)(r^{1+5})(s^{2+2})(t^{3+3}) = 24r^6s^4t^6$

Example: Power of a Power

Simplify $[(3^2)^3]^2$.

$[(3^2)^3]^2 = (3^{2 \cdot 3})^2$ Power of a Power

$= (3^6)^2$ Simplify.

$= 3^{6 \cdot 2}$ Power of a Power

$= 3^{12}$ or 531,441 Simplify.

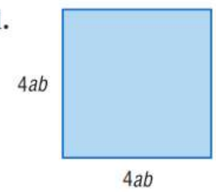
Check your progress:

1) $[(2^2)^2]^4$

$[2^{2 \cdot 2}]^4 = [2^4]^4 = 2^{4 \cdot 4} = 2^{16}$

Example: Power of a Product

GEOMETRY **Express the area of the square as a monomial.**



Area $= s^2$ Formula for the area of a square

$= (4ab)^2$ Replace s with $4ab$.

$= 4^2a^2b^2$ Power of a Product

$= 16a^2b^2$ Simplify.

The area of the square is $16a^2b^2$ square units.

Check your progress:

1) Express the area of a square with sides of length $2xy^2$ as a monomial.

$A = s^2$

$A = (2xy^2)^2$

$A = 2^2x^2(y^2)^2 = 4x^2y^{2\cdot 2} = 4x^2y^4$

To simplify an expression involving monomials, write an equivalent expression in which:

- each base appears exactly once,
- there are no powers of powers, and
- all fractions are in simplest form.

Example: Simplifying Expressions

Simplify $(3xy^4)^2[(-2y)^2]^3$.

$(3xy^4)^2[(-2y)^2]^3 = (3xy^4)^2(-2y)^6$ Power of a Power

$= (3)^2x^2(y^4)^2(-2)^6y^6$ Power of a Product

$= 9x^2y^8(64)y^6$ Power of a Power

$= 9(64)x^2 \cdot y^8 \cdot y^6$ Commutative Property

$= 576x^2y^{14}$ Product of Powers

Check your progress:

1) $\left(\frac{1}{2}a^2b^2\right)^3[(-4b)^2]^2$

$\left(\frac{1}{2}\right)^3(a^2)^3(b^2)^3(-4b)^{2\cdot 2}$

$= \left(\frac{1}{8}\right)(a^{2\cdot 3})(b^{2\cdot 3})(-4b)^4$

$= \left(\frac{1}{8}\right)(a^6)(b^6)(-4)^4(b^4) = \left(\frac{1}{8}\right)(a^6)(b^6)(256)(b^4)$

$= \left(\frac{1}{8}\cdot 256\right)(a^6)(b^6\cdot b^4) = 32a^6b^{6+4} = 32a^6b^{10}$

Practice

Determine whether the expression is a monomial. Write *yes* or *no*. Explain.

1) $5 - 7d$ No. The expression involves the difference, not the product.

2) $\frac{4a}{3b}$ No. The expression involves a quotient of variables.

3) n Yes. A single variable is a monomial.

Simplify.

4) $x(x^4)(x^6)$

$$x^{1+4+6} = x^{11}$$

5) $(4a^4b)(9a^2b^3)$

$$(4 \cdot 9)(a^4 \cdot a^2)(b \cdot b^3) = 36(a^{4+2})(b^{1+3})$$

$$= 36a^6b^4$$

6) $[(2^3)^2]^3$

$$[2^{3\cdot 2}]^3 = [2^6]^3 = 2^{6\cdot 3}$$

$$= 2^{18}$$

7) $[(3^2)^2]^2$

$$[3^{2\cdot 2}]^2 = [3^4]^2 = 3^{4\cdot 2}$$

$$= 3^8$$

8) $(3y^5z)^2$

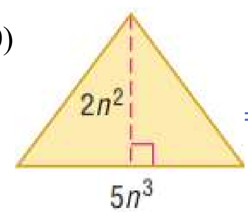
$$(3^2)(y^{5\cdot 2})(z^2) = 9y^{10}z^2$$

9) $(-2f^2g)^3$

$$(-2)^3(f^{2\cdot 3})(g^3) = -8f^6g^3$$

Express the area of the triangle as a monomial. Reminder: area of a triangle is $A = \frac{1}{2}bh$.

10)



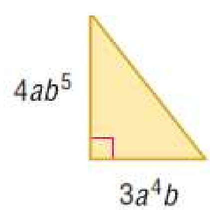
$$A = \frac{1}{2}(5n^3)(2n^2)$$

$$= \frac{1}{2}(5 \cdot 2)(n^3 \cdot n^2)$$

$$= \frac{1}{2}(10)(n^{3+2})$$

$$= 5n^5$$

11)



$$= \frac{1}{2}(3a^4b)(4ab^5)$$

$$A = \frac{1}{2}(3 \cdot 4)(a^4 \cdot a)(b \cdot b^5)$$

$$= \frac{1}{2}(12)(a^{4+1})(b^{1+5})$$

$$= 6a^5b^6$$

Simplify.

12) $(-2v^3w^4)^3(-3vw^3)^2$

$$(-2)^3(v^3)^3(w^4)^3(-3)^2(v)^2(w^3)^2$$

$$= -8(v^{3\cdot 3})(w^{4\cdot 3})(9)(v^2)(w^{3\cdot 2})$$

$$= (-8)(v^9)(w^{12})(9)(v^2)(w^6)$$

$$= (-8 \cdot 9)(v^9 \cdot v^2)(w^{12} \cdot w^6)$$

$$= (-72)(v^{9+2})(w^{12+6})$$

$$= -72v^{11}w^{18}$$

13) $(5x^2y)^2(2xy^3z)^3(4xyz)$

$$(5)^2(x^2)^2(y)^2(2)^3(x)^3(y^3)^3(z)^3(4xyz)$$

$$= (25)(x^{2\cdot 2})(y^2)(8)(x^3)(y^{3\cdot 3})(z^3)(4xyz)$$

$$= (25)(x^4)(y^2)(8)(x^3)(y^9)(z^3)(4xyz)$$

$$= (25 \cdot 8 \cdot 4)(x^4 \cdot x^3 \cdot x)(y^2 \cdot y^9 \cdot y)(z^3 \cdot z)$$

$$= 800(x^{4+3+1})(y^{2+9+1})(z^{3+1})$$

$$= 800x^8y^{12}z^4$$