Module 4 Lesson 3 Notes ELIMINATION

Hot air balloons

Balloon 1 is ten meters above the ground and rising at 15 meters per minute. Balloon 2 is 150 meters above the ground, descending at twenty meters per minute. In how many minutes will the balloons be at the same height? How high will the balloons be at that time?

We need to start by writing two equations. We know that the slope is the rate of change of the balloon (meters per minute). A descending balloon is decreasing in altitude and should have a negative slope. I decided to call the x-axis time (in minutes), and the y-axis height (in meters).

$y = 15x + 10$

$y = -20x + 150$

The y-terms both have the same coefficients, but we need the signs to be different so that the y-terms will disappear when we add them together. In order to get the signs to be different, we can multiply one of the equations by -1. It does not matter which equation we choose.

$y = 15x + 10$
$-1(y = -20x + 150)$

$y = 15x + 10$
$-y = 20x - 150$

$$\begin{array}{rrcl} & y & = & 15x + 10 \\ + & -y & = & 20x - 150 \\ \hline & 0 & = & 35x - 140 \\ & +140 & & +140 \end{array}$$

$140 = 35x$

$\frac{140}{35} = \frac{35x}{35}$

$4 = x$

So, we know that the balloons are at the same height after 4 minutes. Now that we have that value solved for, we can plug this back into one of the two original equations and solve for y (the height). I am going to pick the first equation, but you could pick the second equation if you wanted to.

$y = 15x + 10$

$y = 15(4) + 10$

$y = 60 + 10$

$y = 70$

The balloons are at the same height after 4 minutes and that height is 70 meters.

Module 4 Lesson 3 Notes ELIMINATION

Skydiving

Two skydivers jumped from different airplanes. Skydiver A was at an altitude of 9,976 feet after 1 second and reached the ground after 5.75 minutes (345 seconds). Skydiver B jumped from an altitude of 10,005 feet and reached an altitude of 9,686 feet after 11 seconds. When will the skydivers be at the same height?

We need to start by writing two equations. We are not given the slope and will need to find that. I decided to call the x-axis time (in seconds), and the y-axis height (in feet).

x	y
1	9976
345	0

$+344$ -9976

$\frac{\Delta y}{\Delta x} = \frac{-9976}{344} = -29$

x	y
0	10005
11	9686

$+11$ -319

$\frac{\Delta y}{\Delta x} = \frac{-319}{11} = -29$

$b = 10{,}005$

$y - 9976 = -29(x - 1)$

$y - 9976 = -29x + 29$

$y = -29x + 10{,}005$

The y-terms both have the same coefficients, but we need the signs to be different so that the y-terms will disappear when we add them together. In order to get the signs to be different, we can multiply one of the equations by -1. It does not matter which equation we choose.

$-1(y - 9976 = -29x + 29)$
$y = -29x + 10{,}005$

$$\begin{array}{rl} -y + 9976 &= 29x - 29 \\ +\quad y \qquad\quad &= -29x + 10{,}005 \\ \hline 9976 &= \quad 9976 \end{array}$$

Because the variable disappeared and what remains is a true statement, the system has Infinitely Many Solutions.

This means that the skydivers are always at the same height.

Pizza and Soda

Alexis bought pizza and soda for the ski club meeting. For one meeting she bought 4 pizzas and 10 sodas for $63. The next meeting she bought 3 pizzas and 8 sodas for $48. What is the cost of one pizza? What is the cost of one soda?

Let's start by defining our variables. I chose to use the cost (in dollars) of a pizza on the x-axis, and the cost (in dollars) of a soda on the y-axis

In order to complete this problem, we have to split the information and write an equation from each piece of the problem.

$4x + 10y = 63$
$3x + 8y = 48$

Since none of the coefficients are similar, we must multiply to make the coefficients similar. I am choosing to multiply to eliminate the x-terms. The least common multiple of 4 and 3 (the coefficients on the x-terms) is 12. To get the coefficient of the first equation to be a 12, I need to multiply by 3. To get the coefficient on the x-term of the second equation to be a 12, I need to multiply by 4. The signs need to different, so I will need to multiply one of the equations by a negative. It does not matter which one I choose to make a negative.

$-3(4x + 10y = 63)$
$4(3x + 8y = 48)$

$$\begin{array}{rl} & -12x - 30y = -189 \\ + & \underline{\ \ 12x + 32y = 192} \end{array}$$

$2y = 3$

$\frac{2y}{2} = \frac{3}{2}$

$y = \frac{3}{2} = 1.5$

So, we know that the cost of a soda is \$1.50. Now that we have that value solved for, we can plug this back into one of the two original equations and solve for x (the cost of a pizza). I am going to pick the second equation.

$3x + 8y = 48$

$3x + 8(1.5) = 48$

$3x + 12 = 48$

$\quad -12 \quad -12$

$3x = 36$

$\frac{3x}{3} = \frac{36}{3}$

$x = 12$

The cost of a pizza is \$12, and the cost of a soda is \$1.50.

Vans and Buses

The senior classes at High School A and High school B planned separate trips to the county fair. The senior class at High School A rented and filled 2 vans and 8 buses with 442 students. High School B rented and filled a van and 4 buses with 224 students. Every van had the same number of students in it as did the buses. Find the number of students in each van and in each bus.

Let's start by defining our variables. I chose to use the number of students in a van on the x-axis, and the number of students on a bus on the y-axis

In order to complete this problem, we have to split the information and write an equation from each piece of the problem.

$2x + 8y = 442$

$x + 4y = 224$

Since none of the coefficients are similar, we must multiply to make the coefficients similar. I am choosing to multiply to eliminate the x-terms. The least common multiple of 2 and 1 (the coefficients on the x-terms) is 2. Since the first equation already has a coefficient of 2, I will only need to multiply the second equation. To get the coefficient on the x-term of the second equation to be a 2, I need to multiply by 2. I need to ensure that the signs differ so I will need to multiply by a negative two.

$2x + 8y = 442$
$-2(x + 4y = 224)$

$$\begin{array}{rl} & 2x + 8y = 442 \\ + & \underline{-2x - 8y = -448} \\ & 0 = -6 \end{array}$$

Because the variable disappeared and what remains is a false statement, the system has No Solution.

This means that there is no way that the way this situation was described can happen.

Boat Travel

A boat traveled 189 miles each way downstream and back. The trip downstream took 9 hours. The trip back took 21 hours. What is the speed of the boat in still water? What is the speed of the current?

This problem relies on using the formula for distance: distance = rate • time. (Rate is another word for speed)
Let's start by defining our variables. I chose to use speed of the boat on the x-axis, and the speed of the current on the y-axis
We will write two equations, one for the trip upstream and one for the trip downstream.
Downstream:
As the coal barge goes downstream its rate is comprised of two parts: the speed of the coal barge and the speed of the current. When going downstream the current increases the speed of the coal barge because the current is working with the barge. The rate of travel downstream is $x + y$.
This means the equation for the upstream trip is $189 = (x + y) \cdot 9$ or $189 = 9x + 9y$.

Upstream:
As the coal barge goes upstream its rate is comprised of two parts: the speed of the coal barge and the speed of the current. When going upstream the current actually slows the coal barge down because the current is working against the speed of the barge. The rate of travel upstream is $x - y$.
This means the equation for the upstream trip is $189 = (x - y) \cdot 21$ or $189 = 21x - 21y$.

$189 = 9x + 9y$

$189 = 21x - 21y$

In this situation, I am going to decide to divide the first equation by 9, and divide the second equation by 21.

$$\frac{189 = 9x + 9y}{9}$$

$$\frac{189 = 21x - 21y}{21}$$

$21 = x + y$
$9 = x - y$

Since the coefficients on the y-terms are the similar but the signs differ, we can add the two equations together and the y-terms will disappear.

$$\begin{array}{r} 21 = x + y \\ \underline{+\ \ 9 = x - y} \end{array}$$

$$30 = 2x$$

$$\frac{30}{2} = \frac{2x}{2}$$

$$15 = x$$

Now we know that the boat is traveling at 15 miles per hour. We need to plug this back into one of the original equations and solve for x.

$$189 = 21x - 21y$$

$$189 = 21(15) - 21y$$

$$189 = 315 - 21y$$

$$-315 \quad -315$$

$$-126 = -21y$$

$$\frac{-126}{-21} = \frac{-21y}{-21}$$

$$6 = y$$

The speed of the boat is 15 miles per hour. The speed of the current is 6 miles per hour.

Example #1: Elimination Using Addition

Use elimination to solve the system of equations.

$3x - 5y = -16$

$2x + 5y = 31$

Since the coefficients on the y-terms are the similar but the signs differ, we can add the two equations together and the y-terms will disappear.

$$\begin{array}{rcl} 3x - 5y &=& -16 \\ 2x + 5y &=& 31 \\ \hline 5x &=& 15 \end{array}$$

$$\frac{5x}{5} = \frac{15}{5}$$

$$x = 3$$

Now that we have the x-value solved for, we can plug this back into one of the two original equations and solve for y. I am going to pick the first equation, but you could pick the second equation if you wanted to.

$$3(3) - 5y = -16$$
$$9 - 5y = -16$$
$$-9 \qquad -9$$

$-5y = -25$

$\frac{-5y}{-5} = \frac{-25}{-5}$

$y = 5$

The solution of the system of equations is $(3, 5)$.

Example #2: Elimination Using Subtraction

Use elimination to solve the system of equations.

$5s + 2t = 6$
$9s + 2t = 22$

The t-terms both have the same coefficients, but we need the signs to be different so that the t-terms will disappear. In order to get the signs to be different, we can multiply one of the equations by -1. It does not matter which equation we choose.

$-1(5s + 2t = 6)$
$9s + 2t = 22$

$-5s - 2t = -6$
$9s + 2t = 22$

Since the coefficients on the t-terms are now similar but the signs differ, we can add the two equations together and the t-terms will disappear.

$$\begin{array}{r} -5s - 2t = -6 \\ 9s + 2t = 22 \\ \hline 4s \qquad = 16 \end{array}$$

$\frac{4s}{4} = \frac{16}{4}$

$s = 4$

Now that we have the s-value solved for, we can plug this back into one of the two original equations and solve for t. I am going to pick the first equation.

$5(4) + 2t = 6$
$20 + 2t = 6$
$-20 \qquad - 20$
$2t = -14$
$\frac{2t}{2} = \frac{-14}{2}$

$t = -7$

We always put our solution in alphabetical order. The solution of the system of equations is $(4, -7)$.

Example #3: Multiply One Equation to Eliminate

Use elimination to solve the system of equations.

$$3x + 4y = 6$$
$$5x + 2y = -4$$

Since none of the coefficients are similar, we must multiply to make the coefficients similar. I am choosing to multiply to eliminate the y-terms. The least common multiple of 4 and 2 (the coefficients on the y-terms) is 4. Since the first equation already has a coefficient of 4, I will only need to multiply the second equation. To get the coefficient on the y-term of the second equation to be a 4, I need to multiply by 2. I need to ensure that the signs differ so I will need to multiply by a negative two.

$$3x + 4y = 6$$
$$-2(5x + 2y = -4)$$

$$3x + 4y = 6$$
$$-10x - 4y = 8$$

Since the coefficients on the y-terms are now similar but the signs differ, we can add the two equations together and the y-terms will disappear.

$$3x + 4y = 6$$
$$-10x - 4y = 8$$
$$-7x \quad = 14$$

$$\frac{-7x}{-7} = \frac{14}{-7}$$

$$x = -2$$

Now that we have the x-value solved for, we can plug this back into one of the two original equations and solve for y. I am going to pick the first equation.

$$3(-2) + 4y = 6$$

$$-6 + 4y = 6$$
$$+6 \quad + 6$$
$$4y = 12$$

$$\frac{4y}{4} = \frac{12}{4}$$

$y = 3$ The solution of the system of equations is $(-2, 3)$.

Example #4: Multiply Both Equations to Eliminate

Use elimination to solve the system of equations.

$$3x + 4y = -25$$
$$2x - 3y = 6$$

Since none of the coefficients are similar, we must multiply to make the coefficients similar. I am choosing to multiply to eliminate the y-terms. The least common multiple of 4 and 3 (the coefficients on the y-terms) is 12. To get the coefficient of the first equation to be a 12, I need to multiply by 3. To get the coefficient on the y-term of the second equation to be a 12, I need to multiply by 4. The signs are already different, so I do not need to multiply by a negative.

$3(3x + 4y = -25)$
$4(2x - 3y = 6)$

$9x + 12y = -75$
$8x - 12y = 24$

Since the coefficients on the y-terms are now similar but the signs differ, we can add the two equations together and the y-terms will disappear.

$9x + 12y = -75$
$8x - 12y = 24$
$17x \quad = -51$

$$\frac{17x}{17} = \frac{-51}{17}$$

$x = -3$

Now that we have the x-value solved for, we can plug this back into one of the two original equations and solve for y. I am going to pick the first equation.

$3(-3) + 4y = -25$

$-9 + 4y = -25$
$+9 \qquad +9$
$4y = -16$

$$\frac{4y}{4} = \frac{-16}{4}$$

$y = -4$

The solution of the system of equations is $(-3, -4)$.

Example #5: Special Case

Use elimination to solve the system of equations.
$4x - 2y = -18$
$2x - y = -9$
I am choosing to multiply to eliminate the y-terms. The least common multiple of 2 and 1 (the coefficients on the y-terms) is 2. The first equation already has a coefficient of 2. To get the coefficient on the y-term of the second equation to be a 2, I need to multiply by 2. I need to ensure that the signs differ so I will need to multiply by a negative two.

$4x - 2y = -18$
$-2(2x - y = -9)$

$4x - 2y = -18$
$-4x + 2y = 18$

Since the coefficients on the y-terms are now similar but the signs differ, we can add the two equations together and the y-terms will disappear.

$$\begin{array}{r} 4x - 2y = -18 \\ \underline{-4x + 2y = 18} \\ 0 = 0 \end{array}$$

Since all of the variables have disappeared and what we are left with is a true statement (0 does equal 0), we have lines that are on top of each other and there are INFINITELY MANY SOLUTIONS to the system of equations.

<u>Example #6: Special Case</u>

Use elimination to solve the system of equations.

$6x - y = 9$
$6x - y = 11$

The y-terms both have the same coefficients, but we need the signs to be different so that the y-terms will disappear. In order to get the signs to be different, we can multiply one of the equations by -1.

$$\begin{array}{r} 6x - y = 9 \\ -1(6x - y = 11) \end{array}$$

$$\begin{array}{r} 6x - y = 9 \\ -6x + y = -11 \end{array}$$

Since the coefficients on the y-terms are now similar but the signs differ, we can add the two equations together and the y-terms will disappear.

$$\begin{array}{r} 6x - y = 9 \\ \underline{-6x + y = -11} \\ 0 = -2 \end{array}$$

Since all of the variables have disappeared and what we are left with is a false statement (0 does not equal -2), we have parallel lines and there is NO SOLUTION to the system of equations.