Module 4 Lesson 2 Notes SUBSTITUTION

Hot air balloons

Balloon 1 is ten meters above the ground and rising at 15 meters per minute. Balloon 2 is 150 meters above the ground, descending at twenty meters per minute. In how many minutes will the balloons be at the same height? How high will the balloons be at that time?

We need to start by writing two equations. We know that the slope is the rate of change of the balloon (meters per minute). A descending balloon is decreasing in altitude and should have a negative slope. I decided to call the x-axis time (in minutes), and the y-axis height (in meters).

$y = 15x + 10$

$y = -20x + 150$

Since we know that $y = 15x + 10$ in the first equation, we can replace y in the second equation with $15x + 10$.

$15x + 10 = -20x + 150$

$+20x \qquad + 20x$

$35x + 10 = 150$

$\quad -10 \qquad -10$

$35x = 140$

$\frac{35x}{35} = \frac{140}{35}$

$x = 4$

$y = 15x + 10$

$y = 15(4) + 10$

$y = 60 + 10$

$y = 70$

Since we called the x-axis time (in minutes), and the y-axis height (in meters), the balloons are at the same height after 4 minutes and that height is 70 meters.

Skydiving

Two skydivers jumped from different airplanes. Skydiver A was at an altitude of 9,976 feet after 1 second and reached the ground after 5.75 minutes (345 seconds). Skydiver B jumped from an altitude of 10,005 feet and reached an altitude of 9,686 feet after 11 seconds. When will the skydivers be at the same height?

We need to start by writing two equations. We are not given the slope and will need to find that. I decided to call the x-axis time (in seconds), and the y-axis height (in feet).

	x	y	
$+344$	1	9976	-9976
	345	0	

$\frac{\Delta y}{\Delta x} = \frac{-9976}{344} = -29$

	x	y	
$+11$	0	10005	-319
	11	9686	

$\frac{\Delta y}{\Delta x} = \frac{-31}{11} = -29$

$b = 10{,}005$

$y - 9976 = -29(x - 1)$

$y = -29x + 10{,}005$

Since we know that $y = -29x + 10{,}005$ in the second equation, we can replace y in the first equation with $-29x + 10{,}005$.

$-29x + 10{,}005 - 9976 = -29(x - 1)$

$-29x + 29 = -29x + 29$

$+29x \qquad + 29x$

$29 = 29$

Because the variable disappeared and what remains is a true statement, the system has Infinitely Many Solutions.

This means that the skydivers are always at the same height.

Pizza and Soda

Alexis bought pizza and soda for the ski club meeting. For one meeting she bought 4 pizzas and 10 sodas for $63. The next meeting she bought 3 pizzas and 8 sodas for $48. What is the cost of one pizza? What is the cost of one soda?

Let's start by defining our variables. I chose to use the cost (in dollars) of a pizza on the x-axis, and the cost (in dollars) of a soda on the y-axis
In order to complete this problem, we have to split the information and write an equation from each piece of the problem.

$4x + 10y = 63 \qquad\qquad 3x + 8y = 48$

The next step is pick a variable in one equation to solve for. I'm choosing to solve the green equation for x.

$3x + 8y = 48$
$\quad -8y \quad -8y$

$3x = -8y + 48$

$\frac{3x}{3} = \frac{-8y}{3} + \frac{48}{3}$

$x = -\frac{8}{3}y + 16$

Since we know that $x = -\frac{8}{3}y + 16$ in the second equation, we can replace x in the first equation with $-\frac{8}{3}y + 16$.

$4\left(-\frac{8}{3}y + 16\right) + 10y = 63$

$-10\frac{2}{3}y + 64 + 10y = 63$

$-\frac{2}{3}y + 64 = 63$
$\quad -64 \quad -64$

$-\frac{2}{3}y = -1$

$y = \frac{-1}{-\frac{2}{3}} = \frac{3}{2} = 1.5$

$3x + 8y = 48$

$3x + 8(1.5) = 48$

$3x + 12 = 48$
$\quad -12 \quad -12$

$3x = 36$

$\frac{3x}{3} = \frac{36}{3}$

$x = 12$

The cost of a pizza is $12, and the cost of a soda is $1.50.

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Vans and Buses

The senior classes at High School A and High school B planned separate trips to the county fair. The senior class at High School A rented and filled 2 vans and 8 buses with 442 students. High School B rented and filled a van and 4 buses with 224 students. Every van had the same number of students in it as did the buses. Find the number of students in each van and in each bus.

Let's start by defining our variables. I chose to use the number of students in a van on the x-axis, and the number of students on a bus on the y-axis

In order to complete this problem, we have to split the information and write an equation from each piece of the problem.

$2x + 8y = 442$ $x + 4y = 224$

The next step is pick a variable in one equation to solve for. I'm choosing to solve the green equation for x.

$x + 4y = 224$
$\quad -4y \quad -4y$

$x = -4y + 224$

Since we know that $x = -4y + 224$ in the second equation, we can replace x in the first equation with $-4y + 224$.

$2(-4y + 224) + 8y = 442$

$-8y + 448 + 8y = 442$

$448 = 442$

Because the variable disappeared and what remains is a false statement, the system has No Solution.

This means that there is no way that the way this situation was described can happen.

Boat Travel

A boat traveled 189 miles each way downstream and back. The trip downstream took 9 hours. The trip back took 21 hours. What is the speed of the boat in still water? What is the speed of the current?

This problem relies on using the formula for distance: distance = rate • time. (Rate is another word for speed)
Let's start by defining our variables. I chose to use speed of the boat on the x-axis, and the speed of the current on the y-axis
We will write two equations, one for the trip upstream and one for the trip downstream.
Downstream:
As the coal barge goes downstream its rate is comprised of two parts: the speed of the coal barge and the speed of the current. When going downstream the current increases the speed of the coal barge because the current is working with the barge. The rate of travel downstream is $x + y$.
This means the equation for the upstream trip is $189 = (x + y) \cdot 9$ or $189 = 9x + 9y$.

Upstream:
As the coal barge goes upstream its rate is comprised of two parts: the speed of the coal barge and the speed of the current. When going upstream the current actually slows the coal barge down because the current is working against the speed of the barge. The rate of travel upstream is $x - y$.
This means the equation for the upstream trip is $189 = (x - y) \cdot 21$ or $189 = 21x - 21y$.

The next step is pick a variable in one equation to solve for. I'm choosing to solve the purple equation for y.

$9x + 9y = 189$

$-9x \qquad -9x$

$9y = -9x + 189$

$\frac{9y}{9} = \frac{-9x}{9} + \frac{189}{9}$

$y = -x + 21$

Since we know that $y = -x + 21$ in the first equation, we can replace y in the second equation with $-x + 21$.

$189 = 21x - 21(-x + 21)$

$189 = 21x + 21x - 441$

$189 = 42x - 441$

$+441 \qquad +441$

$630 = 42x$

$\frac{630}{42} = \frac{42x}{42}$

$15 = x$

$189 = 21x - 21y$

$189 = 21(15) - 21y$

$189 = 315 - 21y$

$-315 \quad -315$

$-126 = -21y$

$\frac{-126}{-21} = \frac{-21y}{-21}$

$6 = y$

The speed of the boat is 15 miles per hour. The speed of the current is 6 miles per hour.

Use substitution to solve each system of equations. If the system does not have exactly one solution, state whether it has *no* solution or *infinitely many* solutions.

1) $2x + 7y = 3$
$x = 1 - 4y$

$x = 1 - 4y$

$2x + 7y = 3$

$2(1 - 4y) + 7y = 3$

$2 - 8y + 7y = 3$

$2 - y = 3$

$-2 \qquad -2$

$-y = 1$

$y = -1$

$x = 1 - 4y$

$x = 1 - 4(-1)$

$x = 1 + 4$

$x = 5$

One Solution: $(5, -1)$

2) $6x - 2y = -4$
$y = 3x + 2$

$y = 3x + 2$

$6x - 2y = -4$

$6x - 2(3x + 2) = -4$

$6x - 6x - 4 = -4$

$-4 = -4$

Because the variable disappeared and what remains is a true statement, the system has Infinitely Many Solutions.

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3) $y = \frac{3}{5}x$
$3x - 5y = 15$

$y = \frac{3}{5}x$

$3x - 5y = 15$

$3x - 5\left(\frac{3}{5}x\right) = 15$

$3x - 3x = 15$

$0 = 15$

Because the variable disappeared and what remains is a false statement, the system has No Solution.

4) $x + 3y = 12$
$x - y = 8$

$x - y = 8$
$+y \quad +y$
$x = y + 8$

$x = y + 8$

$x + 3y = 12$

$(y + 8) + 3y = 12$

$y + 8 + 3y = 12$

$4y + 8 = 12$

$-8 \quad -8$

$4y = 4$

$y = 1$

$x = y + 8$

$x = 1 + 8$

$x = 9$

One Solution: $(9, 1)$