Module 4 Lesson 1 Notes GRAPHING SYSTEMS OF EQUATIONS

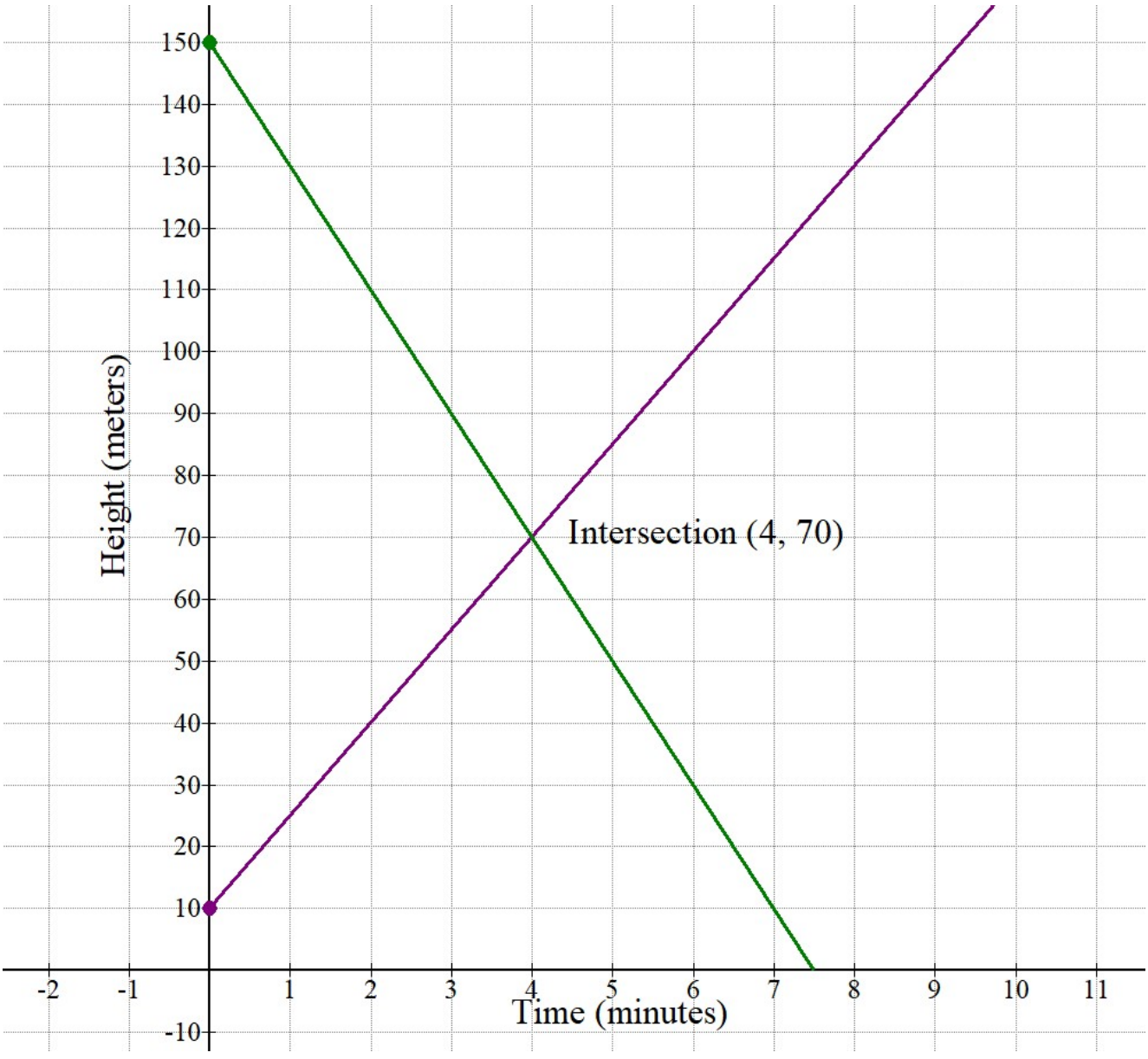
Hot air balloons

Balloon 1 is ten meters above the ground and rising at 15 meters per minute. Balloon 2 is 150 meters above the ground, descending at twenty meters per minute. In how many minutes will the balloons be at the same height? How high will the balloons be at that time?

We need to start by writing two equations. We know that the slope is the rate of change of the balloon (meters per minute). A descending balloon is decreasing in altitude and should have a negative slope. I decided to call the x-axis time (in minutes), and the y-axis height (in meters).

$y = 15x + 10$ Start at 10 meters and go up 15 for each minute.

$y = -20x + 150$ Start at 150 and go down 20 meters per minute.



We can see that the lines cross at one point, so our system has a solution.

The balloons are at the same height after 4 minutes. That height is 70 meters.

Module 4 Lesson 1 Notes GRAPHING SYSTEMS OF EQUATIONS

Skydiving

Two skydivers jumped from different airplanes. Skydiver A was at an altitude of 9,976 feet after 1 second and reached the ground after 5.75 minutes (345 seconds). Skydiver B jumped from an altitude of 10,005 feet and reached an altitude of 9,686 feet after 11 seconds. When will the skydivers be at the same height?

We need to start by writing two equations. We are not given the slope and will need to find that. I decided to call the x-axis time (in seconds), and the y-axis height (in feet).

x	y
1	9976
345	0

$+344$ -9976

$\frac{\Delta y}{\Delta x} = \frac{-99}{344} = -29$

$0 = -29(345) + b$

$0 = -10005 + b$

$10005 = b$

$y = -29x + 10{,}005$ Start at 10,005 feet and go down 29 feet for each second.

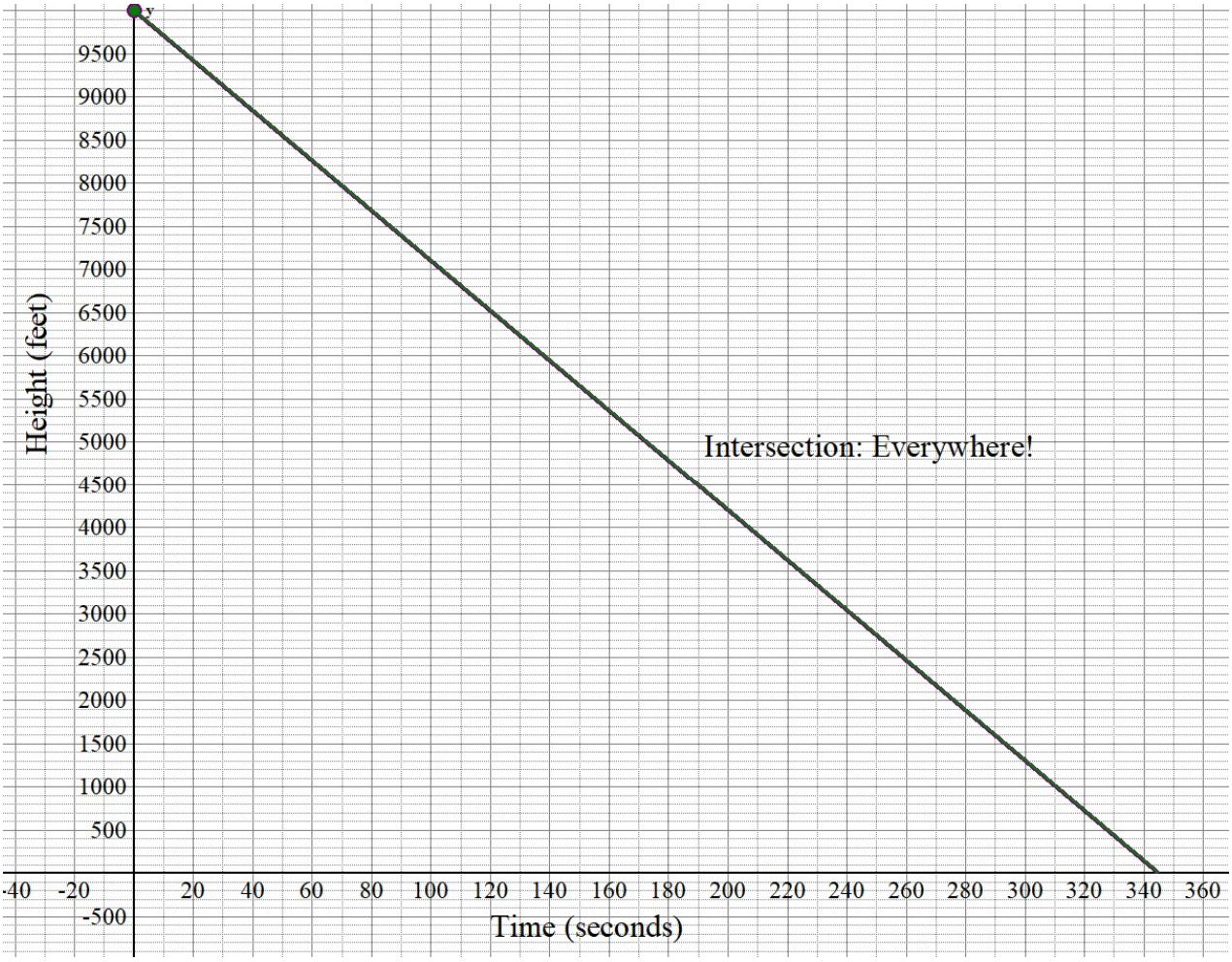
x	y
0	10005
11	9686

$+11$ -319

$\frac{\Delta y}{\Delta x} = \frac{-319}{11} = -29$

$b = 10005$

$y = -29x + 10{,}005$ Start at 10,005 feet and go down 29 feet for each second.



The skydivers are always at the same height. This is what we would call infinitely many solutions.

Module 4 Lesson 1 Notes GRAPHING SYSTEMS OF EQUATIONS

Pizza and Soda

Alexis bought pizza and soda for the ski club meeting. For one meeting she bought 4 pizzas and 10 sodas for \$63. The next meeting she bought 3 pizzas and 8 sodas for \$48. What is the cost of one pizza? What is the cost of one soda?

Let's start by defining our variables. I chose to use the cost (in dollars) of a pizza on the x-axis, and the cost (in dollars) of a soda on the y-axis

In order to complete this problem, we have to split the information and write an equation from each piece of the problem.

$4x + 10y = 63$ $3x + 8y = 48$

The next step is to solve both equations for y.

$4x + 10y = 63$ $3x + 8y = 48$

$-4x \quad -4x$ $-3x \quad -3x$

$10y = -4x + 63$ $8y = -3x + 48$

$\frac{10}{10} = -\frac{4x}{10} + \frac{63}{10}$ $\frac{8y}{8} = -\frac{3x}{8} + \frac{48}{8}$

$y = -\frac{2}{5}x + 6\frac{3}{10}$ $y = -\frac{3}{8}x + 6$

Start at $6\frac{3}{10}$ dollars for a soda and go down $\frac{2}{5}$ dollar for each pizza (or we can go down \$2 for every \$5 per pizza).

Start at 6 dollars for a soda and go down $\frac{3}{8}$ dollar for each pizza dollar (or we can go down \$3 per soda and over \$8 per pizza).

The cost of a pizza is \$12, and the cost of a soda is \$1.50.

Module 4 Lesson 1 Notes GRAPHING SYSTEMS OF EQUATIONS

Vans and Buses

The senior classes at High School A and High school B planned separate trips to the county fair. The senior class at High School A rented and filled 2 vans and 8 buses with 442 students. High School B rented and filled a van and 4 buses with 224 students. Every van had the same number of students in it as did the buses. Find the number of students in each van and in each bus.

Let's start by defining our variables. I chose to use the number of students in a van on the x-axis, and the number of students on a bus on the y-axis

In order to complete this problem, we have to split the information and write an equation from each piece of the problem.

$2x + 8y = 442$ $x + 4y = 224$

The next step is to solve both equations for y.

$2x + 8y = 442$ $x + 4y = 224$

$-2x \quad -2x$ $-x \quad -x$

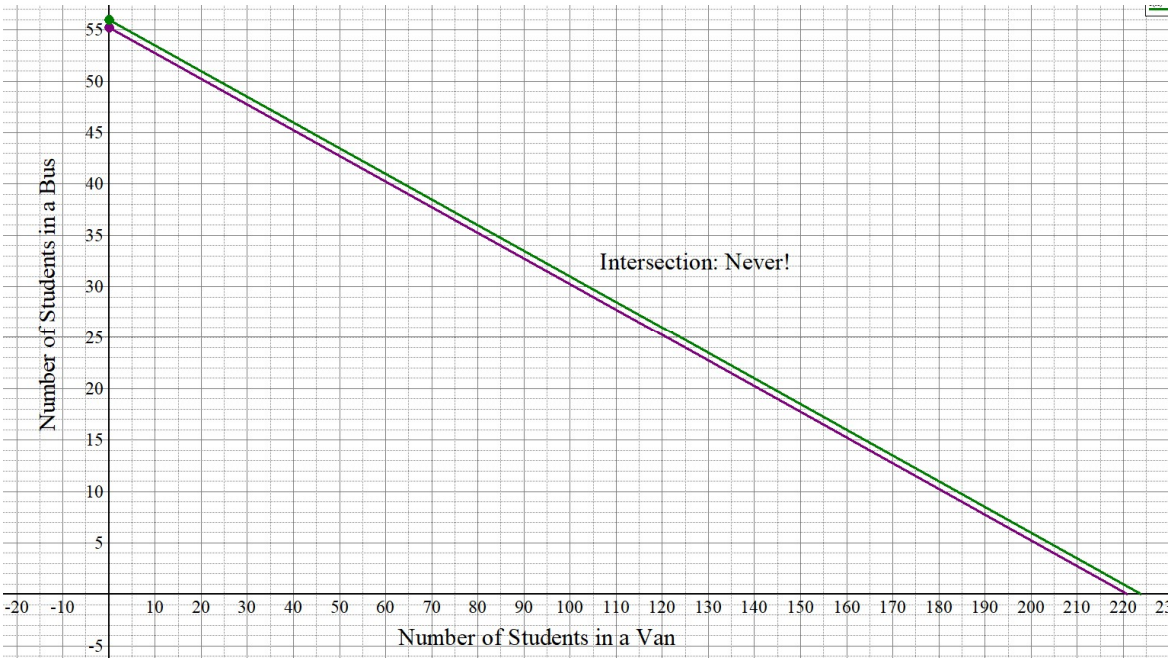
$8y = -2x + 442$ $4y = -x + 224$

$\frac{8y}{8} = -\frac{2x}{8} + \frac{442}{8}$ $\frac{4y}{4} = -\frac{x}{4} + \frac{224}{4}$

$y = -\frac{1}{4}x + 55\frac{1}{4}$ $y = -\frac{1}{4}x + 56$

Start at $55\frac{1}{4}$ students on a bus and decrease $\frac{1}{4}$ student per student on a van (or we can decrease one student on a bus per 4 students in a van).

Start at 56 students on a bus and decrease $\frac{1}{4}$ student per student on a van (or we can decrease one student on a bus per 4 students in a van).



Since the lines never cross, this is an impossible situation. There is no solution.

Module 4 Lesson 1 Notes GRAPHING SYSTEMS OF EQUATIONS

Boat Travel

A boat traveled 189 miles each way downstream and back. The trip downstream took 9 hours. The trip back took 21 hours. What is the speed of the boat in still water? What is the speed of the current?

This problem relies on using the formula for distance: distance = rate • time. (Rate is another word for speed)
Let's start by defining our variables. I chose to use speed of the boat on the x-axis, and the speed of the current on the y-axis
We will write two equations, one for the trip upstream and one for the trip downstream.
Downstream:
As the coal barge goes downstream its rate is comprised of two parts: the speed of the coal barge and the speed of the current. When going downstream the current increases the speed of the coal barge because the current is working with the barge. The rate of travel downstream is $x + y$.
This means the equation for the upstream trip is $189 = (x + y) \cdot 9$ or $189 = 9x + 9y$.

Upstream:
As the coal barge goes upstream its rate is comprised of two parts: the speed of the coal barge and the speed of the current. When going upstream the current actually slows the coal barge down because the current is working against the speed of the barge. The rate of travel upstream is $x - y$.
This means the equation for the upstream trip is $189 = (x - y) \cdot 21$ or $189 = 21x - 21y$.

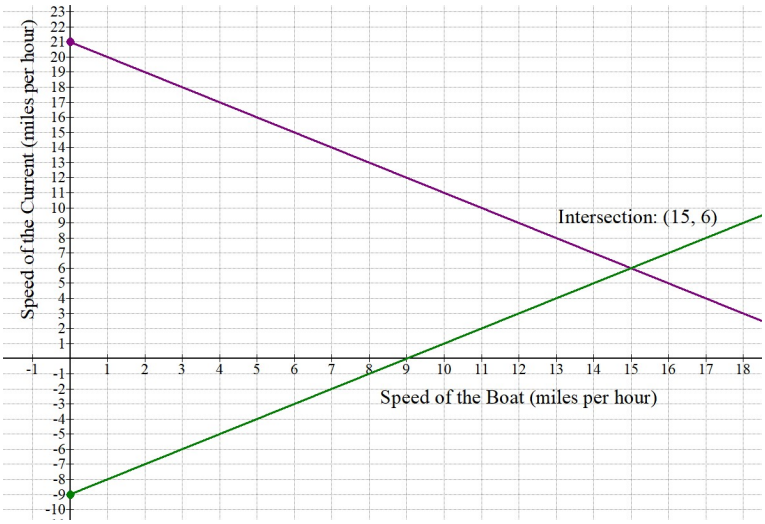
The next step is to solve both equations for y.

$9x + 9y = 189$
$-9x \quad -9x$
$9y = -9x + 189$
$\frac{9y}{9} = \frac{-9x}{9} + \frac{189}{9}$
$y = -x + 21$

$21x - 21y = 189$
$-21x \quad -21x$
$-21y = -21x + 189$
$\frac{-21y}{-21} = \frac{-21x}{-21} + \frac{189}{-21}$
$y = x - 9$

Start at 21 miles per hour for the current and decrease one mile per hour for each one mile per hour increase for the boat.

Start at -9 miles per hour for the current and increase one mile per hour for each one mile per hour increase for the boat.

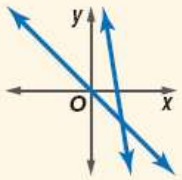
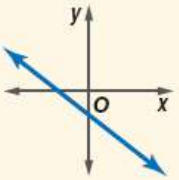
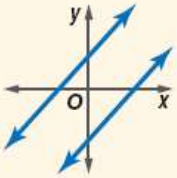


The speed of the boat is 15 miles per hour. The speed of the current is 6 miles per hour.

Module 4 Lesson 1 Notes GRAPHING SYSTEMS OF EQUATIONS

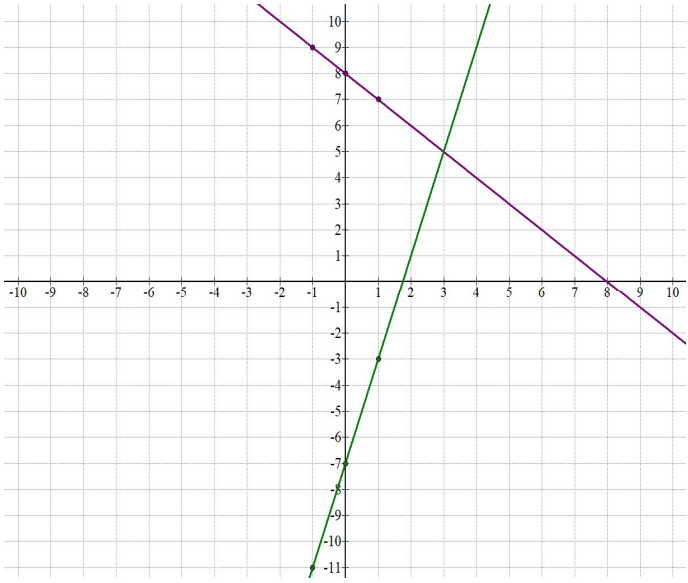
What is a system of equations?

--Two equations together are called a system of equations. A solution of a system is an ordered pair that satisfies both equations.

KEY CONCEPT — Graphing Systems of Equations			
Graph of a System			
Number of Solutions	exactly one solution	infinitely many	no solutions
Terminology	consistent and independent	consistent and dependent	inconsistent

Graph each system of equations. Then determine whether the system has *no* solution, *one* solution, or *infinitely many* solutions. If the system has one solution, name it.

1) $y = -x + 8$
$y = 4x - 7$



Remember that to graph, we need to start at the y-intercept.

$y = -\frac{1}{1}x + 8$ Start at 8 and go down 1 and to the right 1.

Start at 8 and go up 1 and to the left 1.

$y = \frac{4}{1}x - 7$ Start at -7 and go up 4 and to the right 1.

Start at -7 and go down 4 and to the left 1.

We can see that the lines cross at one point, so our system has one solution.

To name the solution, we name the point where the lines cross, in this case that is the point $(3, 5)$.

2) $x + 2y = 5$
$2x + 4y = 2$

Remember that to graph, we need to solve for y in both equations.

$x + 2y = 5$
$-x \quad -x$
$2y = -x + 5$
$\frac{}{2} \quad \frac{}{2}$
$y = -\frac{1}{2}x + 2\frac{1}{2}$

$2x + 4y = 2$
$-2x \quad -2x$
$4y = -2x + 2$
$\frac{}{4} \quad \frac{}{4}$
$y = -\frac{1}{2}x + \frac{1}{2}$

Module 4 Lesson 1 Notes GRAPHING SYSTEMS OF EQUATIONS

$y = -\frac{1}{2}x + 2\frac{1}{2}$ Start at 2 ½ and go down 1 and to the right 2.

Start at 2 ½ and go up 1 and to the left 2.

$y = -\frac{1}{2}x + \frac{1}{2}$ Start at ½ and go down 1 and to the right 2.

Start at ½ and go up 1 and to the left 2.

We can see that the lines are parallel and will not cross.

There is no solution to the system of equations.

3) $y = -2x - 3$
$2x + y = -3$

Remember that to graph, we need to solve for y in both equations.

$y = -2x - 3$
This equation is already solved for y.

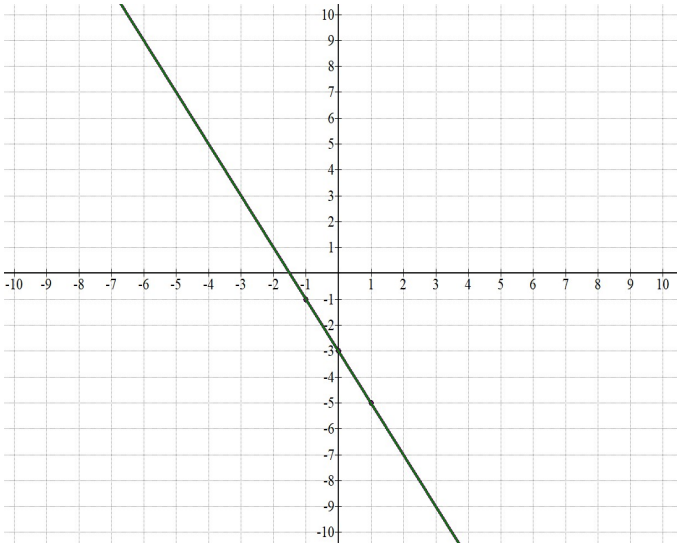
$2x + y = -3$
$-2x \quad -2x$
$y = -2x - 3$

$y = -\frac{2}{1}x - 3$ Start at -3 and go down 2 and to the right 1.

Start at -3 and go up 2 and to the left 1.

$y = -\frac{2}{1}x - 3$ Start at -3 and go down 2 and to the right 1.

Start at -3 and go up 2 and to the left 1.



We can see that the lines are a line on top of a line.

There are infinitely many solutions to the system of equations.

4) $y = -6$
$4x + y = 2$

Remember that to graph, we need to solve for y in both equations.

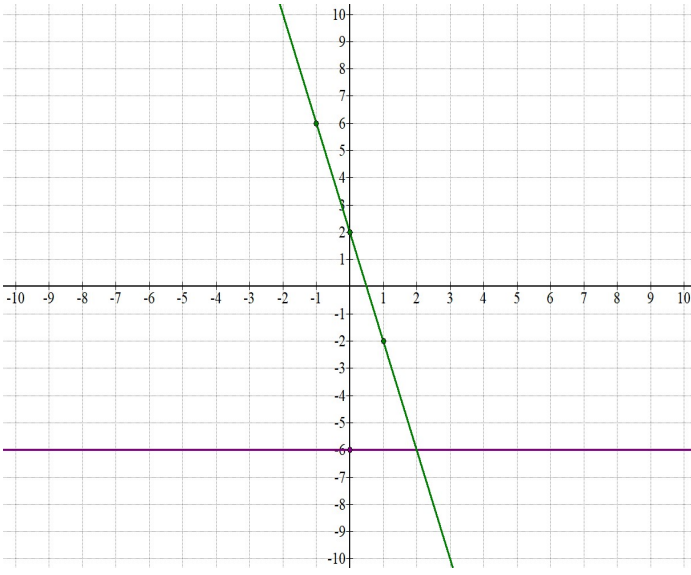
$y = -6$
This equation is already solved for y.

$4x + y = 2$
$-4x \quad -4x$
$y = -4x + 2$

$y = -6$ This is a horizontal line at -6 on the y-axis.

$y = -\frac{4}{1}x + 2$ Start at 2 and go down 4 and to the right 1.

Start at 2 and go up 4 and to the left 1.



We can see that the lines cross at one point, so our system has one solution.

To name the solution, we name the point where the lines cross, in this case that is the point $(2, -6)$.