Lesson 3.2 Notes

Alonzo, Maria, and Sierra were floating in inner tubes down a river, enjoying their day. Alonzo noticed that sometimes the water level was higher in some places than in other. Maria noticed there were times they seemed to be moving faster than at other times. Sierra laughed and said "Math is everywhere!" To learn more about the river, Alonzo and Maria collected data throughout the trip.

1) Use the data collected by Alonzo to interpret the key features of this relationship.

Time (in minutes)	0	10	20	30	40	50	60	70	80	90	100	110	120
Depth (in feet)	4	6	8	10	6	5	4	5	7	12	9	6.5	5

Increasing: $(0, 30) \cup (60, 90)$ Minimum: 4   Domain: $[0, 120]$ x-intercept: None
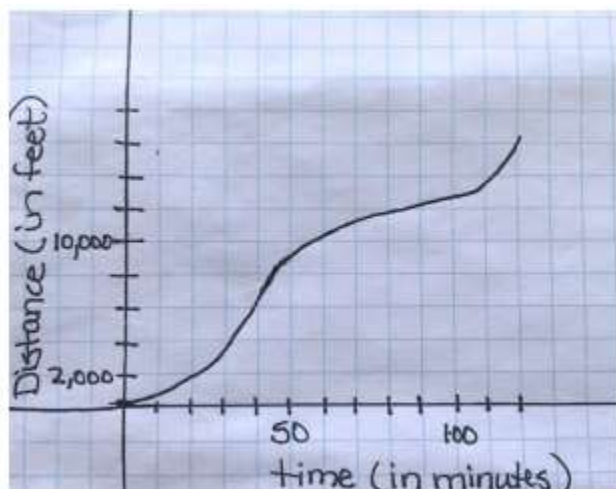
Decreasing: $(30, 60) \cup (90, 120)$ Maximum: 12   Range: $[4, 12]$ y-intercept: 4

Maria created a graph by collecting data on a GPS unit that told her the distance she had traveled over a period of time.

2) Using the graph created by Maria, describe the key features (increasing, decreasing, domain, range, maximum, minimum, intercepts) of this relationship.

Increasing: $(0, 120)$

Minimum: 0

Maximum: 16,000

Domain: $[0, 120]$

Range: $[0, 16000]$

x-intercept: 0

y-intercept: 0

3) Sierra looked at the data collected by her two friends and made several of her own observations. Explain why you either agree or disagree with each observation made.
   a. The depth of the water increases and decreases throughout the 120 minutes of floating down the river.
      Agree. The depth has intervals of both increasing and decreasing.
   b. The distance traveled is always increasing.
      Agree. Distance traveled cannot decrease.
   c. The distance traveled is a function of time.
      Agree. The time is the input value. At any point in time there is one, and only one, amount of distance traveled.

d. The distance traveled is greatest during the last ten minutes of the trip than during any other ten minute interval of time.
    Disagree. If we are referring to the steepest part of the graph, I believe that occurs between 40 and 50 minutes. If we are referring to when the maximum distance is reached, that happens at the end of the trip or at 120 minutes.

e. The domain of the distance/time graph is all real numbers.
    Disagree. The domain of the distance/time graph is $[0, 120]$.

f. The y-intercept of the depth of the water over time function is (0, 0)
    Disagree. The y-intercept occurs when the x-value is 0, so it is $(0, 4)$.

g. The distance traveled increases and decreases over time.
    Disagree. The distance traveled is always as increasing function.

h. The depth of the water is never 11 feet.
    Disagree. Somewhere between 80 and 90 minutes the depth of the water has to be 11 feet to get from 7 feet to 12 feet. The same is true as the depth decreases from 90 to 100 minutes.

i. The range of the distance/time graph is from [0, 15000].
    Disagree. The range of the distance/time graph is $[0, 16000]$.

j. The domain of the depth of water with respect to time is from [0, 120].
    Agree. The time starts at 0 minutes and ends at 120 minutes.

k. The range of the depth of water over time is from [4, 5].
    Disagree. The range is $[4, 12]$.

l. The distance/time graph has no maximum value.
    Disagree. The maximum value occurs at the end of the domain of the graph at about 16,000.

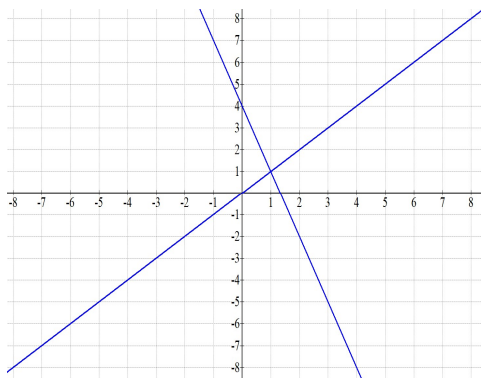m. The depth of water reached a maximum at 30 minutes.
    Disagree. The depth of water reached a maximum at 90 minutes.

Other important notes:

Graph each set of linear equations on the same set of axes. Name the coordinates of the point where the two lines intersect.
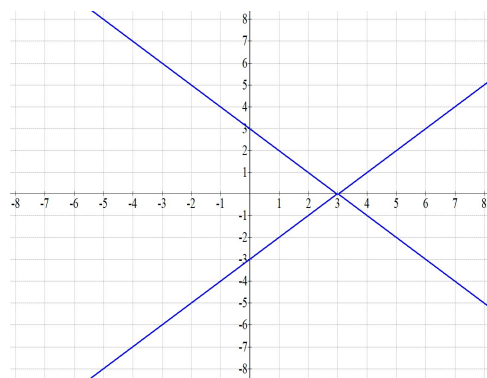
1. $\begin{cases} f(x) = x \\ g(x) = -3x + 4 \end{cases}$
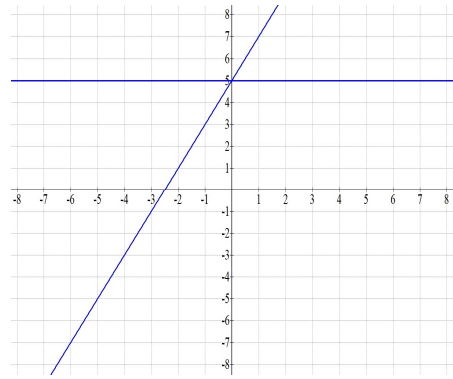   Point of Intersection: $(1, 1)$

2. $\begin{cases} f(x) = -x + 3 \\ g(x) = x - 3 \end{cases}$
    Point of Intersection: $(3, 0)$

3. $\begin{cases} f(x) = 2x + 5 \\ g(x) = 5 \end{cases}$

   Point of Intersection: $(0, 5)$

For each graph state a) the interval(s) where it is increasing, decreasing, or constant, b) the minimum or maximum value (if it exists), c) identify the domain and range, and d) determine whether the function is continuous, discrete or discontinuous. Use interval notation.

1.

   a. Increasing: $(-\infty, -5) \cup (5, \infty)$

   Decreasing: $(-5, 5)$

   Constant: None

   b. Minimum: $-\infty$

   Maximum: ∞

   c. Domain: $(-\infty, \infty)$

   Range: $(-\infty, \infty)$
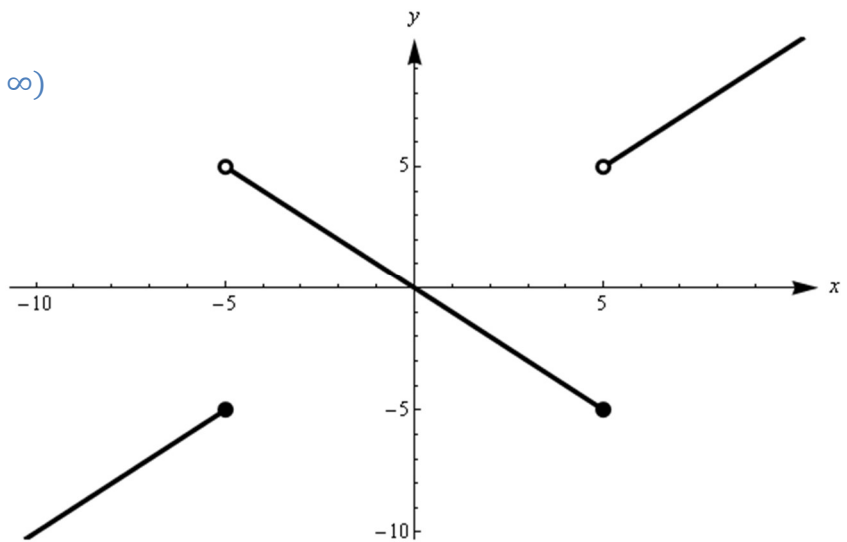
   d. x-intercept(s): $(0, 0)$

   y-intercept: $(0, 0)$

   e. Circle one: Continuous   Discrete   ⬭Discontinuous⬭

2.

$f(x) = \begin{cases} x^2, & 0 \le x \le 1 \\ -x + 2, & 1 < x \le 2 \\ x^2 - 3x + 2, & 2 < x \le 3 \end{cases}$

   a. Increasing: $(0, 1) \cup (2, 3)$
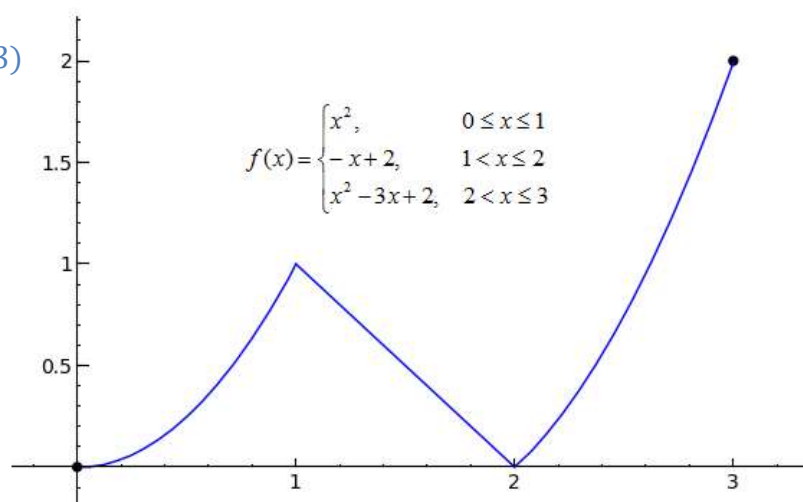
   Decreasing: $(1, 2)$

   Constant: None

   b. Minimum: 0

   Maximum: 2

   c. Domain: $[0, 3]$

Range: $[0, 2]$

d. x-intercept(s): $(0, 0)$ and $(2, 0)$

y-intercept: $(0, 0)$

e. Circle one: ~~Continuous~~ Discrete Discontinuous

3.

a. Increasing: $(2, 3)$

Decreasing: $(0, 1) \cup (3, 4)$
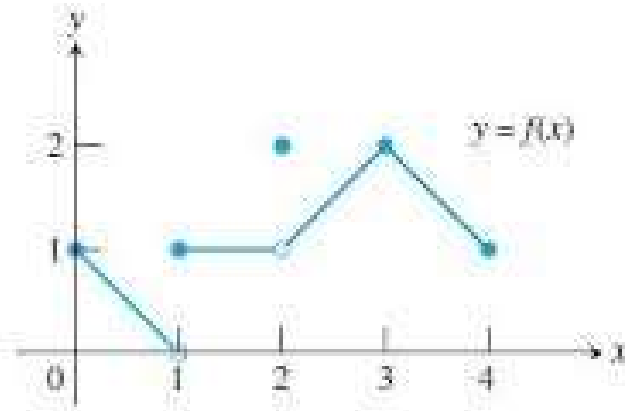
Constant: $(1, 2)$

b. Minimum: Approaching 0

Maximum: 2

c. Domain: $[0, 4]$

Range: $(0, 2]$

d. x-intercept(s): None

y-intercept: $(0, 1)$

e. Circle one: Continuous Discrete ~~Discontinuous~~

$y = f(x)$

4.

a. Increasing: $(-3, 0)$
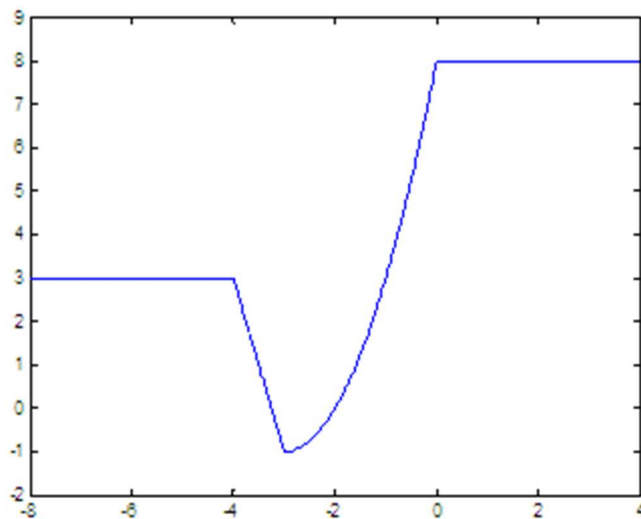
Decreasing: $(-4, -3)$

Constant: $(-\infty, -4) \cup (0, \infty)$

b. Minimum: -1

Maximum: 8

c. Domain: $(-\infty, \infty)$

Range: $[-1, 8]$

d. x-intercept(s): $(-3.5, 0)$ and $(-2, 0)$

y-intercept: $(0, 3)$

e. Circle one: ~~Continuous~~ Discrete Discontinuous

5.

   a. Increasing: None

       Decreasing: $(-\infty, \infty)$

       Constant: None

   b. Minimum: 5

       Maximum: ∞

   c. Domain: $(-\infty, \infty)$

       Range: $(5, \infty)$

   d. x-intercept(s): None
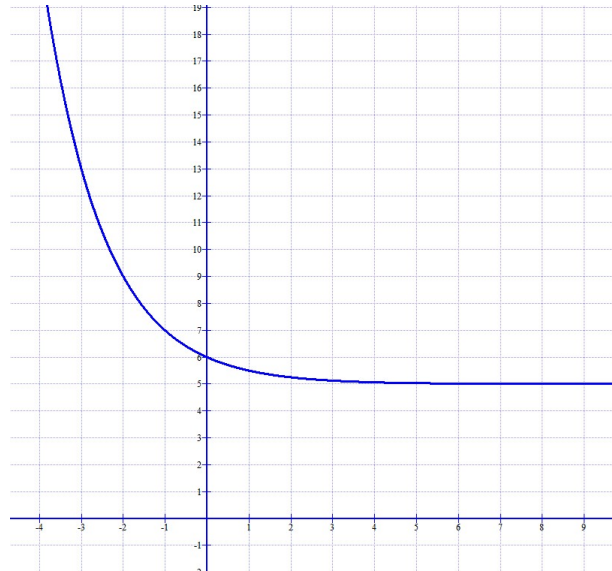
       y-intercept: $(0, 6)$

   e. Circle one: **Continuous**   Discrete   Discontinuous

Write equations for the given tables in both recursive and explicit form.

1.

n	$f(n)$
1	3
2	11
3	19

Recursive:
$f(0) = -5$
$f(x) = f(x - 1) + 8$
Explicit:
$f(x) = 8x - 5$

2.

n	$f(n)$
0	5
5	15
7	19

n	$f(n)$
0	5
1	
2	
3	
4	
5	15
6	
7	19

Let's try addition first and see whether we get a correct answer.

$$\frac{15 - 5}{5 - 0} = \frac{0}{5} = 2$$

If we add 2 each time, we get 5, 7, 9, 11, 13, 15, 19 which is what we need, so we are adding 2.

Recursive:
$f(0) = 5$
$f(x) = f(x - 1) + 2$
Explicit:
$f(x) = 2x + 5$

3.

n	$f(n)$
1	3
4	81
5	243

n	$f(n)$
1	3
2	
3	
4	81
5	243

Since we have two terms that are together (the 4ᵗʰ and 5ᵗʰ terms) and it appears to be increasing quickly, let's try multiplication.

$$\frac{243}{81} = 3$$

If we multiply by 3 our terms are 3, 9, 27, 81, 243 which is correct.

Recursive:
$f(0) = 1$
$f(x) = f(x - 1) \cdot 3$
Explicit:
$f(x) = 3^x$

4.

n	$f(n)$
0	-2
2	-8
3	-16

n	$f(n)$
0	-2
1	
2	-8
3	-16

Since we have two terms that are together (the 2ⁿᵈ and 3ʳᵈ terms) and it appears to be increasing quickly, let's try multiplication.

$$\frac{-16}{-8} = 2$$

If we multiply by 2 our terms are -2, -4, -8, -16 which is correct.

Recursive:
$f(0) = -2$
$f(x) = f(x - 1) \cdot 2$
Explicit:
$f(x) = 2^x \cdot -2$