Lesson 2.1 Notes

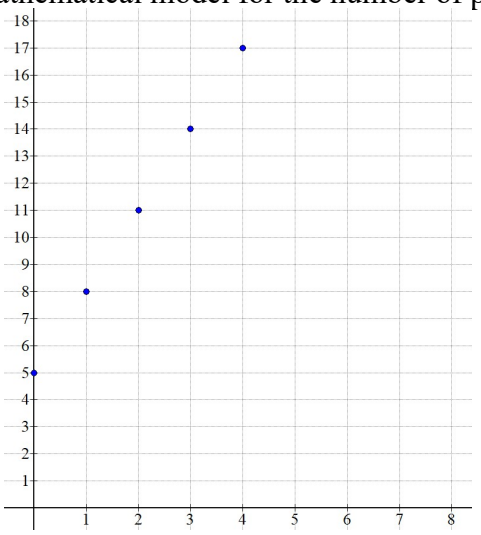
1. My little sister, Savannah, is three years old. She has a piggy bank that she wants to fill. She started with five pennies and each day when I come home from school, she is excited when I give her three pennies that are left over from my lunch money.
   a. What values should represent the domain (x-values)?
      Time is typically domain. So, number of days, in this case.
   b. What values should represent the range (y-values)?
      The range would be the number of pennies in Savannah's piggy bank.
   c. Would it make sense to talk about giving my sister pennies on day -2? Why?
      It would not make sense to talk about day -2. I cannot go back in time and give my sister pennies.
   d. Would it make sense for me to talk about giving my sister pennies on day 13.5? Why?
      It would not make sense to talk about giving my sister pennies on day 13.5. I give her pennies once per day. So, the fact that a part of a day has elapsed, does not impact the amount of pennies she has.
   e. Is this function discrete or continuous?
      The function is discrete because the number of pennies increases in chunks. The number of pennies does not continuously change. She increases the number of pennies by 3 each time and that change happens all at once.
   f. Identify the domain of the function.
      Domain is the possible values for input in the function. Since I don't bring my sister pennies on day 1.5 or on day -3, it does not make sense for the domain to include negative or partial values. The domain is positive integers. **Integers are numbers that do not have fractional or decimal parts. They are whole numbers.
   g. Use a table, a graph, and an equation to create a mathematical model for the number of pennies in the piggy bank on day n.

n	$f(n)$
0	5
1	8
2	11
3	14

Equation:

$$f(n) = 3n + 5$$

**We chose to write the explicit form but we could have written recursive.



**We do not connect the points on our graph because the function is discrete.

2. Our family has a small pool for relaxing in the summer that holds 1500 gallons of water. I decided to fill the pool for the summer. When I had 5 gallons of water in the pool, I decided that I didn't want to stand outside and watch the pool fill, so I had to figure out how long it would take so that I could leave, but come back to turn off the water at the right time. I checked the flow on the hose and found that it was filling the pool at a rate of 2 gallons every minute.
   a. What values should represent the domain (x-values)?
      Time is typically domain. So, number of minutes, in this case.
   b. What values should represent the range (y-values)?
      The range would be the number of gallons of water in the pool.

c. Would it make sense to talk about filling the pool at -10 minutes? Why?
It would not make sense to talk about -10 minutes. I can't go back in time and start filling the pool. It has to start at time zero.

d. Would it make sense to talk about filling the pool at 20.5 minutes? Why?
It would make sense to talk about 20.5 minutes. Because the water is always flowing in, the fact that a partial minute has elapsed impacts the amount of water in the pool.

e. Is this function discrete or continuous?
The function is continuous because the amount of water in the pool is continuously changing. Water is always flowing into the pool changing the amount of water in the pool.

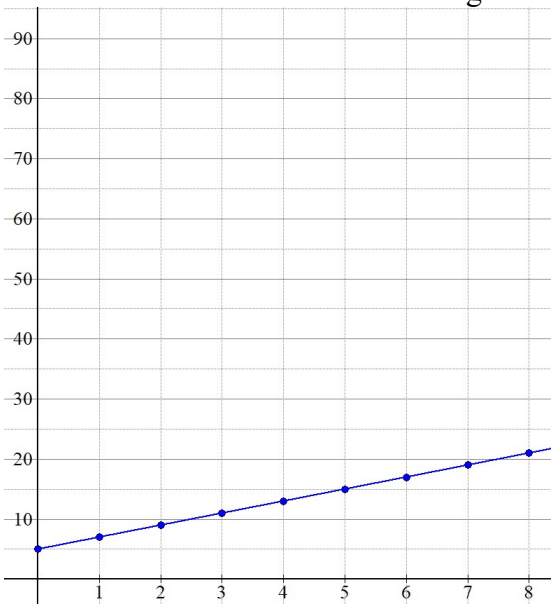
f. Identify the domain of the function.
Domain is the possible values for input in the function. The pool is not being filled at -30 minutes, so negative input values do not make sense. However, it does make sense for the pool to be filling with water at 2.5 minutes or 4.65 minutes. So, we could have partial values. It would take 747.5 minutes for the pool to fill up, so we know that the domain should be less than that.
The domain of the function is positive real numbers less than or equal to 747.5.
**Real numbers are all numbers (whole, fractional, and decimal).
**To figure out how long it takes to fill the pool, solve for the time it takes to get to 1500 gallons: $1500 = 2t + 5 \implies 1495 = 2t \implies 747.5 = t$

g. Use a table, a graph, and an equation to create a mathematical model for the number of gallons of water in the pool at t minutes.

t	$f(t)$
0	5
1	7
2	9
3	11

Equation:

$$f(t) = 2t + 5$$



**We connect the points on the graph with a line because the function is continuous.

3. I'm more sophisticated than my little sister so I save my money in a bank account that pays me 3% interest on the money in the account at the end of each month. (If I take my money out before the end of the month, I don't earn any interest for the month.) I started the account with $50 that I got for my birthday. Use a table, a graph, and an equation to create a mathematical model of the amount of money I will have in the account after m months.
   a. What values should represent the domain (x-values)?
   Time is typically domain. So, number of months, in this case.
   b. What values should represent the range (y-values)?
   The range would be the amount of money in my bank account.
   c. Would it make sense to talk about how much money is in my account after -1 month? Why?
   It would not make sense to talk about -1 month. I cannot have money in the account prior to opening the account on day zero.

d. Would it make sense to talk about how much money is in my account after 2.5 months? Why?
Because I don't earn interest until a full month has elapsed, the fact that a part of a month has elapsed doesn't affect the amount of money in the account. Therefore, it does not make sense to talk about 2.5 months.

e. Is this function discrete or continuous?
The function is discrete because the amount of money in the account changes all at once. It does not continuously change.

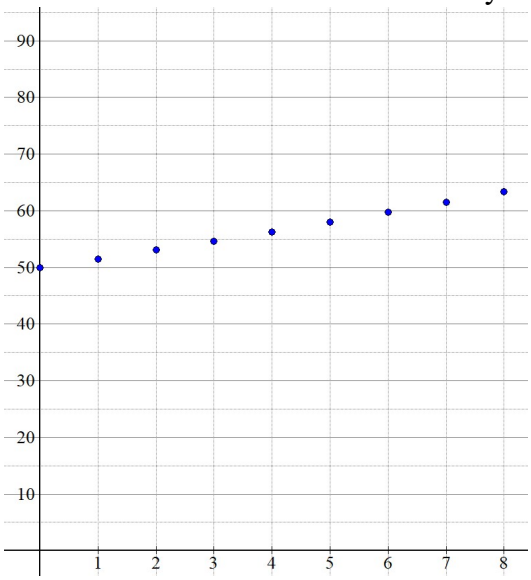
f. Identify the domain of the function.
The domain of a function is input values. My money does not earn interest at a negative value since it can't earn interest before I put it into my account. My money only earns interest at the end of each month (once per month). I do not earn interest part way through a month, so partial values for input do not make sense.
The domain is positive integers.

g. Use a table, a graph, and an equation to create a mathematical model of the amount of money I will have in the account after m months.

m	$f(m)$
0	$50
1	$51.50
2	$53.05
3	$54.64

Equation:

$$f(m) = 1.03^x \cdot 50$$



**We do not connect the points on our graph because the function is discrete.

4. At the end of the summer, I decide to drain the 1500-gallon swimming pool. I noticed that it drains faster when there is more water in the pool. That was interesting to me, so I decided to measure the rate at which it drains. I found that 3% was draining out of the pool every minute.

a. What values should represent the domain (x-values)?
Time is typically domain. So, number of minutes, in this case.

b. What values should represent the range (y-values)?
The range would be the number of gallons of water remaining in the pool.

c. Would it make sense to talk about draining water from the pool at -3 minutes? Why?
It would not make sense to talk about -3 minutes. I can't go back in time and start draining the pool. It has to start at time zero.

d. Would it make sense to talk about draining water from the pool at 7.5 minutes? Why?
It would make sense to talk about 7.5 minutes. Because the water is always flowing out of the pool, the fact that a partial minute has elapsed impacts the amount of water in the pool.

e. Is this function discrete or continuous?
The function is continuous because the amount of water in the pool is continuously changing. Water is always flowing out of the pool changing the amount of water in the pool.

f. Identify the domain of the function.
Domain is the possible values for input in the function. The pool is not being emptied at -30 minutes, so negative input values do not make sense. However, it does make sense for the pool to be emptying at 2.5 minutes or 4.65 minutes. So, we could have partial values. We don't have

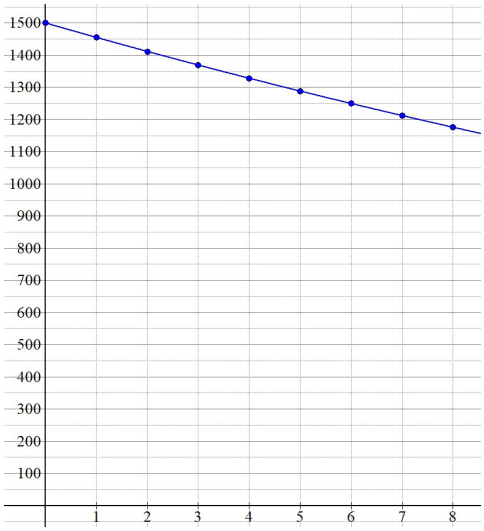
the tools to solve for when the pool would be empty, so we are just going to say that the domain can go forever.
The domain of the function is positive real numbers.

g. Use a table, a graph, and an equation to create a mathematical model for the number of gallons of water in the pool at t minutes.

t	$f(t)$
0	1500
1	1455
2	1411.35
3	1369.0095

Equation:

$$f(t) = 0.97^t \cdot 1500$$



**We connect the points on the graph with a line because the function is continuous.

5. Compare problems 1 and 3. What similarities do you see? What differences do you notice?
   Similarities:
   Both functions are increasing and discrete.
   Differences:
   The problem in #1 is arithmetic. The problem in #3 is geometric.

6. Compare problems 1 and 2. What similarities do you see? What differences do you notice?
   Similarities:
   Both functions are increasing and have a constant difference.
   Differences:
   The problem in #1 is discrete. The problem in #2 is continuous.

7. Compare problems 3 and 4. What similarities do you see? What differences do you notice?
   Similarities:
   Both functions have a constant ratio.
   Differences:
   The problem in #3 is discrete and increasing. The problem in #4 is continuous and decreasing.