

Lesson 1.9 Notes

For each set of sequences, find the first five terms. Then compare the growth of the arithmetic sequence and the geometric sequence. Which grows faster?

1. Arithmetic sequence: $f(1) = 2$, common difference, $d = 3$

Geometric sequence: $g(1) = 2$, common ratio, $r = 3$

Arithmetic	Geometric
$f(1) = 2$	$g(1) = 2$
$f(2) = 5$	$g(2) = 6$
$f(3) = 8$	$g(3) = 18$
$f(4) = 11$	$g(4) = 54$
$f(5) = 14$	$g(5) = 162$

Which value do you think will be more, $f(100)$ or $g(100)$?

$g(100)$ will be greater.

Why?

Multiplying by 3 increases faster than adding 3.

2. Arithmetic sequence: $f(1) = 2$, common difference, $d = 10$

Geometric sequence: $g(1) = 128$, common ratio, $r = \frac{1}{2}$

Arithmetic	Geometric
$f(1) = 2$	$g(1) = 128$
$f(2) = 12$	$g(2) = 64$
$f(3) = 22$	$g(3) = 32$
$f(4) = 32$	$g(4) = 16$
$f(5) = 42$	$g(5) = 8$

Which value do you think will be more, $f(100)$ or $g(100)$?

$f(100)$ will be greater.

Why?

f is an increasing sequence, while g is a decreasing sequence.

3. Arithmetic sequence: $f(1) = 20$, $d = 10$

Geometric sequence: $g(1) = 2$, $r = 2$

Arithmetic	Geometric
$f(1) = 20$	$g(1) = 2$
$f(2) = 30$	$g(2) = 4$
$f(3) = 40$	$g(3) = 8$
$f(4) = 50$	$g(4) = 16$
$f(5) = 60$	$g(5) = 32$

Which value do you think will be more, $f(100)$ or $g(100)$?

$g(100)$ will be greater.

Why?

Multiplying by two will increase much faster as the x -values increase than adding by 10.

4. Arithmetic sequence: $f(1) = 50$, common difference, $d = -10$

Geometric sequence: $g(1) = 1$, common ratio, $r = 2$

Arithmetic	Geometric
$f(1) = 50$	$g(1) = 1$
$f(2) = 40$	$g(2) = 2$
$f(3) = 30$	$g(3) = 4$
$f(4) = 20$	$g(4) = 8$
$f(5) = 10$	$g(5) = 16$

Which value do you think will be more, $f(100)$ or $g(100)$?
 $g(100)$ will be greater.

Why?

g is an increasing sequence, while f is a decreasing sequence.

5. Arithmetic sequence: $f(1) = 64$, common difference, $d = -2$

Geometric sequence: $g(1) = 64$, common ratio, $r = \frac{1}{2}$

Arithmetic	Geometric
$f(1) = 64$	$g(1) = 64$
$f(2) = 62$	$g(2) = 32$
$f(3) = 60$	$g(3) = 16$
$f(4) = 58$	$g(4) = 8$
$f(5) = 56$	$g(5) = 4$

Which value do you think will be more, $f(100)$ or $g(100)$?
 $g(100)$ will be greater.

Why?

The geometric sequence will never be negative.

6. Considering arithmetic and geometric sequences, would there ever be a time that a geometric sequence does not out grow an arithmetic sequence in the long run as the number of terms of the sequences becomes really large? Explain.

If a geometric sequence is multiplying by a number less than 1, then it is actually a decreasing sequence and will not outgrow an arithmetic sequence.

Other important notes:

Given the following information, determine the explicit equation for each sequence.

1) $f(1) = 6$, $f(n) = 2f(n - 1)$

The common ratio is 2. $f(0) = \frac{6}{2} = 3$.

Explicit Equation: $f(x) = 2^x \cdot 3$

2) $f(n) = 5f(n - 1)$, $f(1) = \frac{2}{3}$

The common ratio is 5. $f(0) = \frac{2}{3} \div 5 =$

$$\frac{2}{3} \cdot \frac{1}{5} = \frac{2}{15}.$$

Explicit Equation: $f(x) = 5^x \cdot \frac{2}{15}$

3) $f(1) = 6$, $f(n) = 2 + f(n - 1)$

The common difference is 2. $f(0) = 6 - 2 = 4$.

Explicit Equation: $f(x) = 2x + 4$

4) $f(n) = -5 + f(n - 1)$, $f(1) = \frac{2}{3}$

The common difference is -5. $f(0) = \frac{2}{3} + 5 = \frac{2}{3} + \frac{15}{3} = \frac{17}{3} = 5\frac{2}{3}$.

Explicit Equation: $f(x) = -5x + 5\frac{2}{3}$