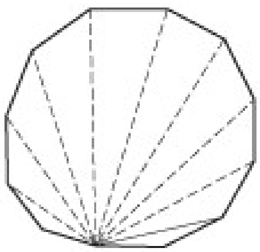
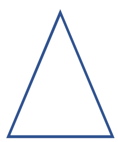
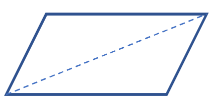
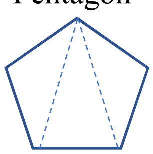
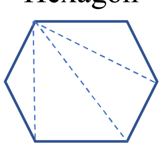
<u>Lessons 5.1 & 5.2 – Polygon Sum Conjecture and Exterior Angles of a Polygon</u>

Polygon Sum Conjecture - The sum of the measures of the n interior angles of an n-gon is $180°(n-2)$.



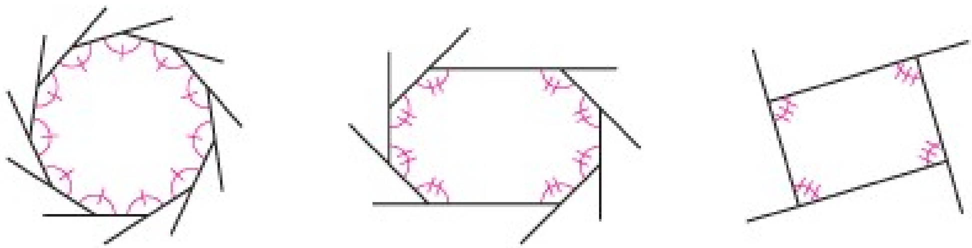
Why $180°(n-2)$?

Let's look at a pattern of figures.

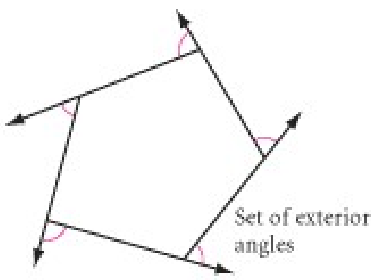
Figure	# of sides	# of triangles the figure can be split into	Sum of interior angles
Triangle 	3	1	$1\,\Delta \cdot 180° = 180°$
Quadrilateral 	4	2	$2\,\Delta \cdot 180° = 360°$
Pentagon 	5	3	$3\,\Delta \cdot 180° = 540°$
Hexagon 	6	4	$4\,\Delta \cdot 180° = 720°$

Notice that each figure makes a number of triangles equal to the number of sides minus two (hence $n-2$). Then we can multiply the number of triangles by the sum of the angles of a triangle, which is why we have the formula $180°(n-2)$.

Equiangular Polygon Conjecture - You can find the measure of each interior angle of an equiangular n-gon by using $\frac{180°(n-2)}{n}$.



Exterior Angle Sum Conjecture - For any polygon the sum of the measures of a set of exterior angles is 360°.



<u>Example 1: Complete.</u>

The sum of the measures of the interior angles of a regular polygon is $2340°$. How many sides does the polygon have?

The sum of the measures of the interior angles of a polygon can be found using the formula 180°(n–2). So, we set that equal to $2340°$ and solve for n.

$180°(n-2) = 2340°$

$\overline{180°} \qquad\qquad \overline{180°}$

$n - 2 = 13$

$\quad +2 \qquad +2$

$n = 15$

The polygon has 15 sides.

Example 2: Complete.

The sum of the measures of the interior angles of a regular polygon is $3960°$. How many sides does the polygon have?

The sum of the measures of the interior angles of a polygon can be found using the formula $180°(n–2)$. So, we set that equal to $3960°$ and solve for n.

$$\frac{180°(n-2)}{180°} = \frac{3960°}{180°}$$

$n - 2 = 22$

$+2 \quad + 2$

$n = 24$

The polygon has 24 sides.

Example 3: Complete

One exterior angle of a regular polygon measure $10°$. What is the measure of each interior angle? How many sides does the polygon have?

Each interior angle has measure $180° - 10° = 170°$ since an exterior and interior angle are a linear pair of angles.

Each interior angle is $\mathbf{170°}$.

All exterior angles of a polygon always add to $360°$. So, we can divide to find the number of sides.

$$\frac{360°}{10°} = 36$$

The polygon has 36 sides.

Example 4: Complete

One exterior angle of a regular polygon measure $30°$. How many sides does the polygon have?

All exterior angles of a polygon always add to $360°$. So, we can divide to find the number of sides.

$\frac{360°}{30°} = 12$

The polygon has 12 sides.

Example 5: Complete

If the sum of the measures of the interior angles of a polygon equals the sum of the measures of its exterior angles, how many sides does it have?

We know that the exterior angles of a polygon always add to $360°$ regardless of how many sides the polygon has. If the sum of the measures of the interior angles must be equal to the sum of the measures of the exterior angles, then the interior angles must add to $360°$ as well.

The sum of the measures of the interior angles of a polygon can be found using the formula 180°(n–2). So, we set that equal to $360°$ and solve for n.

$180°(n-2) = 360°$

$\overline{180°} \qquad \overline{180°}$

$n - 2 = 2$

$+2 \quad +2$

$n = 4$

The polygon has 4 sides. It is a quadrilateral.

Example 6: Complete

If the sum of the measures of the interior angles of a polygon s twice the sum of the measures of its exterior angles, how many sides does it have?

We know that the exterior angles of a polygon always add to $360°$ regardless of how many sides the polygon has. If the sum of the measures of the interior angles is equal to twice the sum of the measures of the exterior angles, then the interior angles must add to $2 \cdot 360° = 720°$.

The sum of the measures of the interior angles of a polygon can be found using the formula 180°(n–2). So, we set that equal to $720°$ and solve for n.

$180°(n-2) = 720°$

$\overline{180°} \qquad \overline{180°}$

$n - 2 = 4$

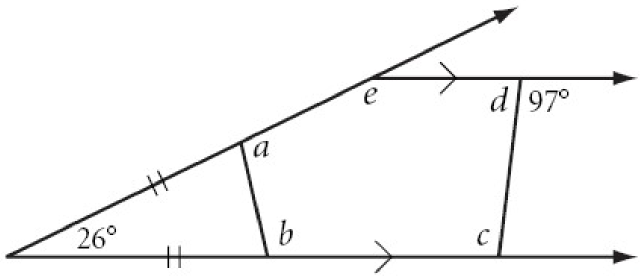
$\quad +2 \quad +2$

$n = 6$

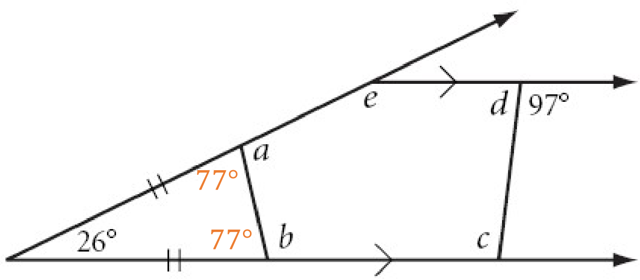
The polygon has 6 sides. It is a hexagon.

Example 7: Find each lettered angle measure

$a =$________, $b =$________, $c =$ ________, $d =$________, $e =$________



The triangle on the left is isosceles and can be solved. Since they gave us the measure of the vertex angle, we can solve for the measure of each of the base angles $\left(\frac{180-26}{2} = 77\right)$.



a is a linear pair with one of the base angles, so $180 - 77 = 103$.

$\boldsymbol{a = 103°}$

b is also a linear pair with one of the base angles, so $180 - 77 = 103$.

$\boldsymbol{b = 103°}$

c is an alternate interior angle on parallel lines to the 97° angle, so it must be congruent to that angle.

$\boldsymbol{c = 97°}$

d is a linear pair with the 97° angle, so $180 - 97 = 83$.

$\boldsymbol{d = 83°}$

We now know 4 of the five angles from the pentagon with interior angles $a, b, c, d,$ and e. We know that a pentagon should add to $180(5-2) = 180(3) = 540°$.

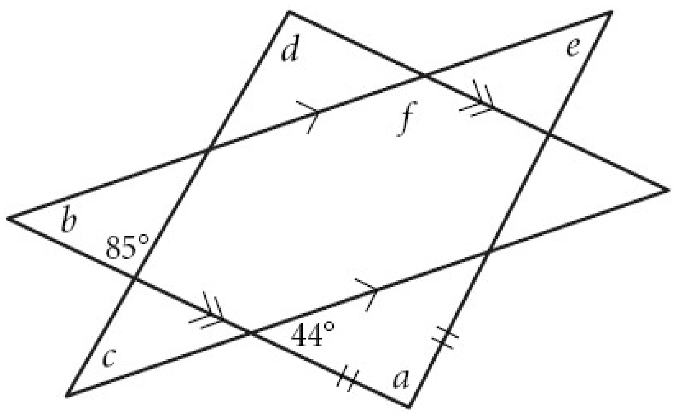
$103 + 103 + 97 + 83 = 386$

$540 - 386 = 154$

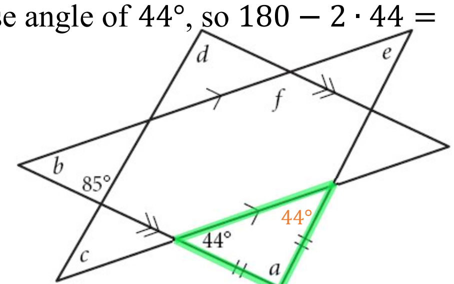
$\boldsymbol{e = 154°}$

Example 8: Find each lettered angle measure

$a =$________, $b =$________, $c =$ ________, $d =$________, $e =$________, $f =$________

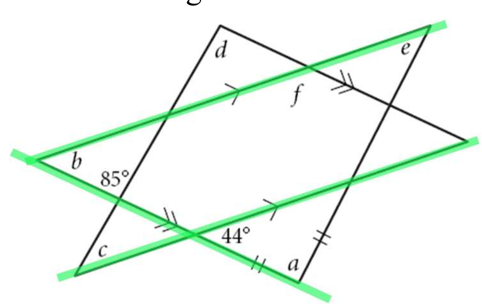


a is the vertex angle of an isosceles triangle that has a base angle of $44°$, so $180 - 2 \cdot 44 = 180 - 88 = 92$.



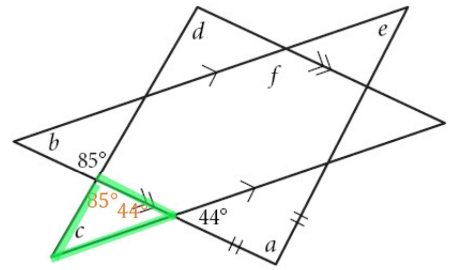
$\boldsymbol{a = 92°}$

b is a corresponding angle on parallel lines with the $44°$ angle.



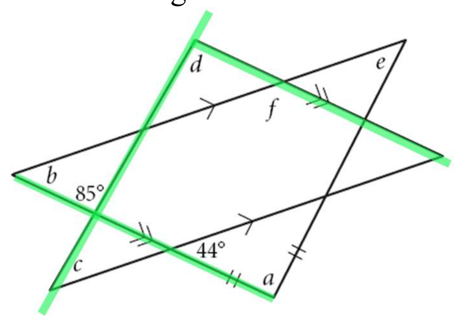
$\boldsymbol{b = 44°}$

c is the third angle in a triangle with vertical angles to $85°$ and $44°$, so $180 - 85 - 44 = 51$



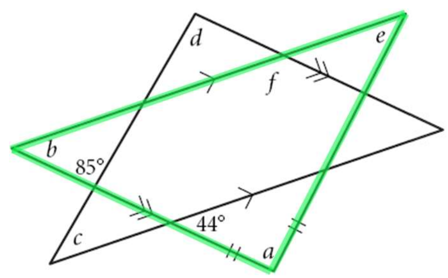
$\boldsymbol{c = 51°}$

d is an alternate interior angle on parallel lines with the $85°$ angle.



$\boldsymbol{d = 85°}$

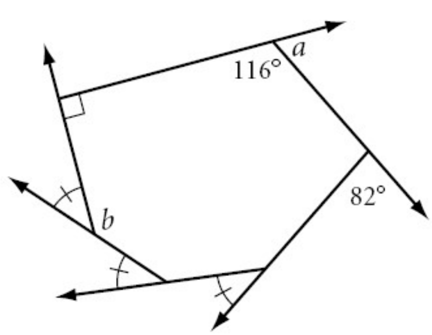
$a, b,$ and e form a triangle, so $180 - 92 - 44 = 44$.



$\boldsymbol{e = 44°}$

<u>Example 9: Find each lettered angle measure</u>

$a =$________, $b =$________



a is a linear pair with the $116°$ angle. $180 - 116 = 64$.

$\boldsymbol{a = 64°}$

We know that all of the exterior angles of a polygon should add to $360°$. So, the linear pair to the right angle is $90°$. We know 3 of the exterior angles of this hexagon. The remaining three are all the same measure. So, $360 = 90 + 64 + 82 + 3x$.

$360 = 90 + 64 + 82 + 3x$

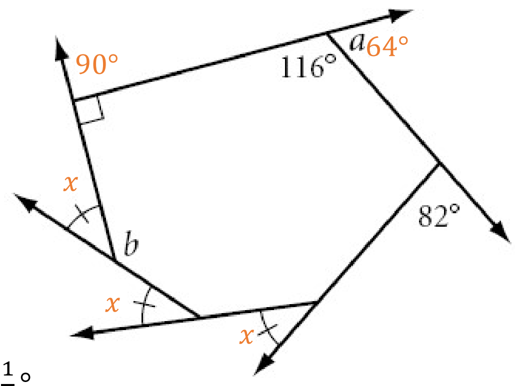
$360 = 236 + 3x$

$-236 \quad -236$

$124 = 3x$

$\overline{3} \qquad \overline{3}$

$41\frac{1}{3} = x$



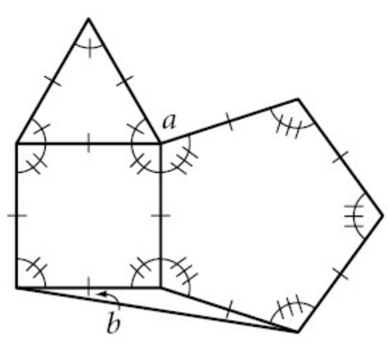
b is a linear pair with the exterior angle that is $41\frac{1}{3}°$.

$180 - 41\frac{1}{3} = 138\frac{2}{3}$

$\boldsymbol{b = 138\frac{2}{3}°}$

<u>Example 10: Find each lettered angle measure</u>

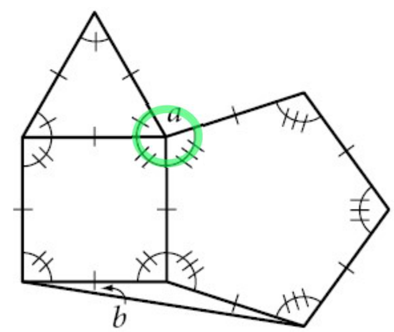
$a =$________, $b =$________



Let's start by finding the angle measures of each of the equiangular polygons.

Triangle	Quadrilateral	Pentagon
$180(3-2) = 180$	$180(4-2) = 360$	$180(5-2) = 540$
$\frac{180}{3} = 60$	$\frac{360}{4} = 90$	$\frac{540}{5} = 108$

Each angle in the triangle is 60°, each angle in the quadrilateral is 90°, and each angle in the pentagon is 108°.

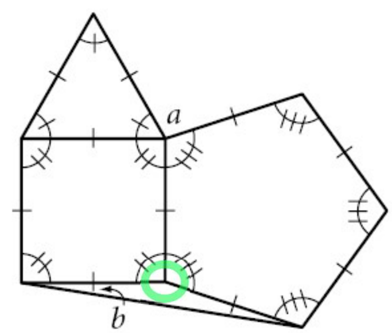


All of the angles with the same vertex as a should add to 360°.

$108 + 90 + 60 = 258$

$360 - 258 = 102$

$\boldsymbol{a = 102}°$



All of the angles with the same vertex as b should add to 360°.

$108 + 90 = 198$

$360 - 198 = 162$

The 162° angle is the vertex angle of an isosceles triangle. To find the base angles, we subtract the vertex angle from 180° and divide the remainder by 2.

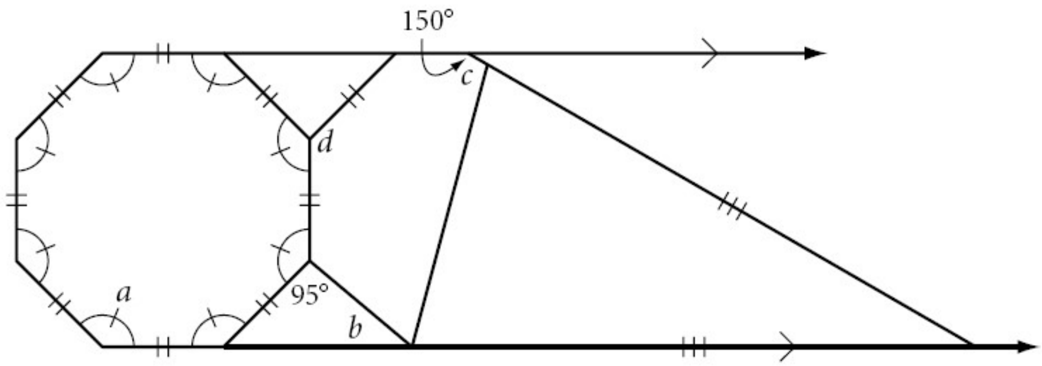
$180 - 162 = 18$

$\frac{18}{2} = 9$

$\boldsymbol{b = 9}°$

Example 11: Find each lettered angle measure

$a =$________, $b =$________, $c =$ ________, $d =$________

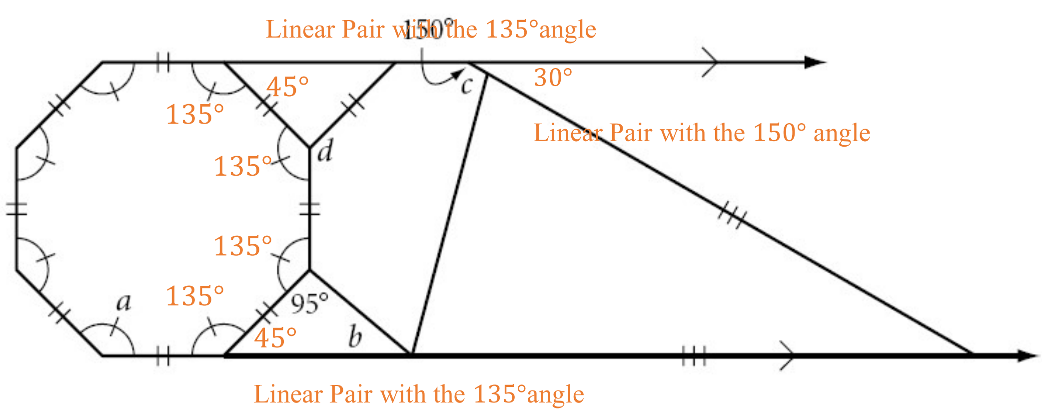


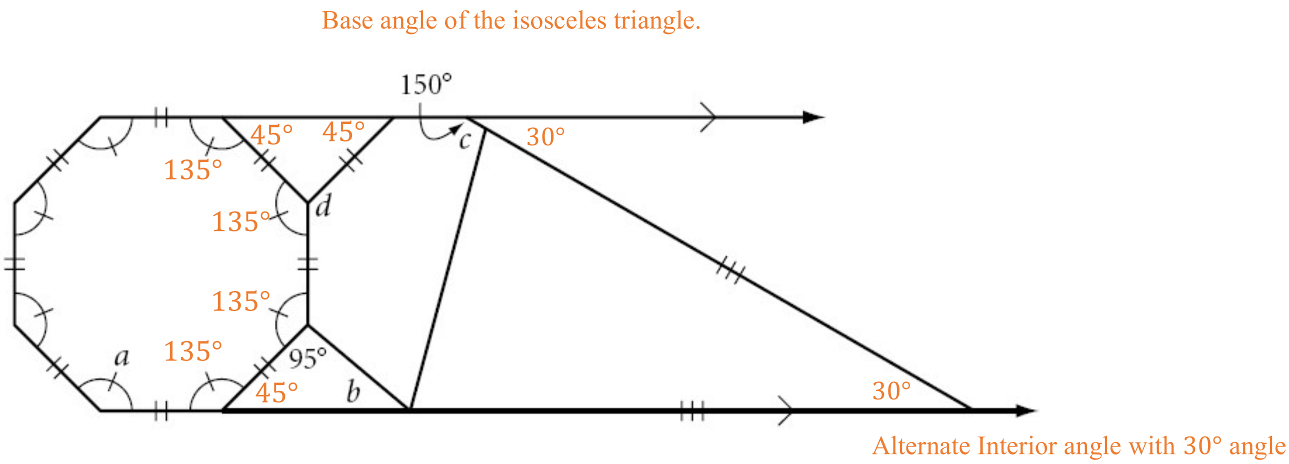
a is an angle of an equiangular octagon.

$180(8-2) = 1080$

$\frac{1080}{8} = 135$

$\boldsymbol{a = 135°}$



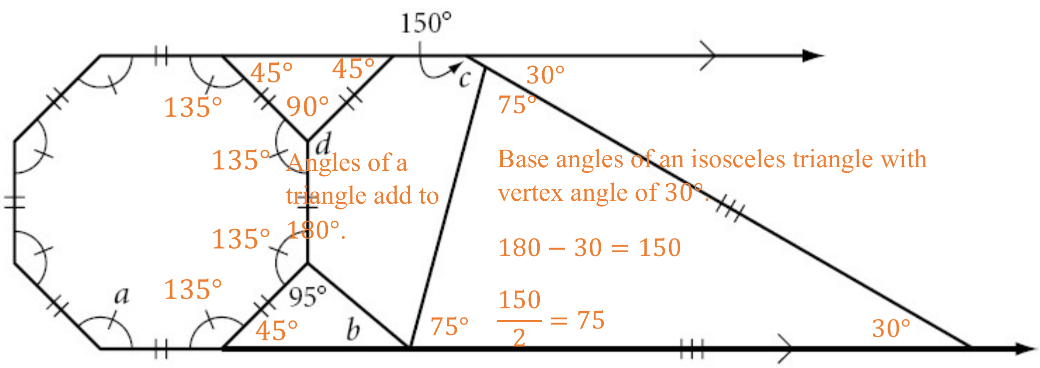


b is the third angle in a triangle with the 95° angle and the linear pair to one of the angles in the octagon.

$45 + 95 = 140$

$180 - 140 = 40$

$\boldsymbol{b = 40°}$



c is a linear pair with the 75° base angle of the isosceles triangle.

$180 - 75 = 105$

$\boldsymbol{c = 105°}$

d makes a 360° circle with the 135° and 90° angles.

$135 + 90 = 225$

$360 - 225 = 135$

$\boldsymbol{d = 135°}$