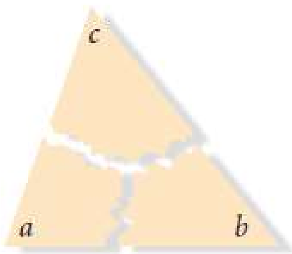
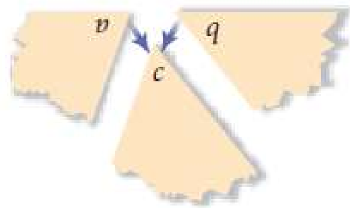
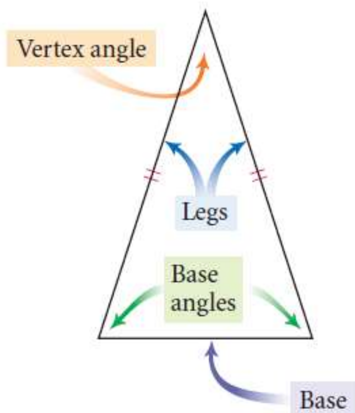
Lessons 4.1 & 4.2 – Triangle Sum Conjecture and Properties of Isosceles Triangles

Triangle Sum Conjecture - The sum of the measures of the angles in every triangle is 180°.

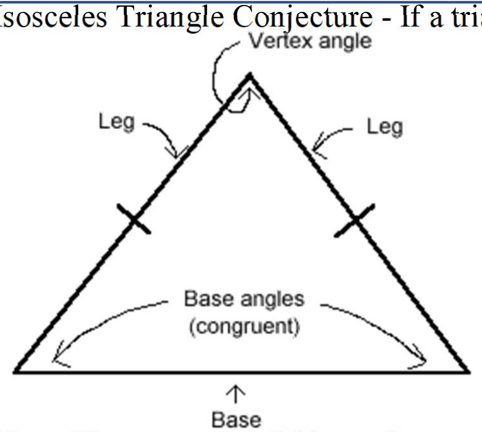


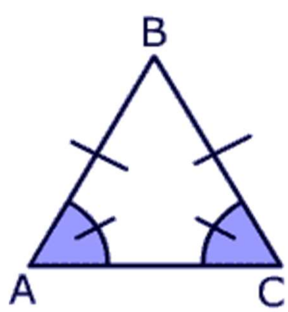


Isosceles Triangle Review:



Isosceles Triangle Conjecture - If a triangle is isosceles, then its base angles are congruent.

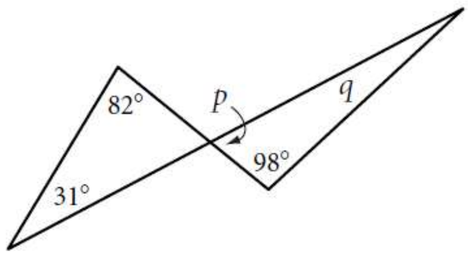




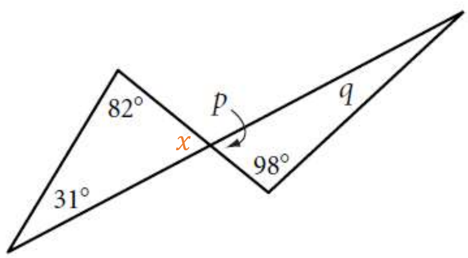
Note: The converse of this conjecture is also *true*. *If a triangle* has two congruent angles, then it is an isosceles triangle.

Example 1: Determine the angle measures.

$p =$ _______, $q =$ _______



Let's label the unlabeled angle to make references easier.



The sum of the measures of the angles in every triangle is 180°, so $31° + 82° + x = 180°$. This means $x = 67°$.

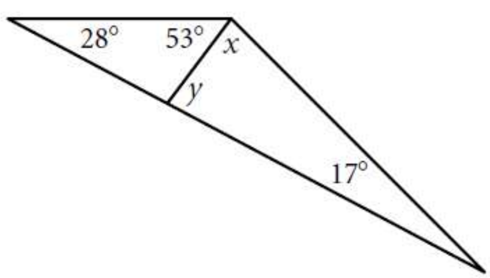
$\boldsymbol{p = 67°}$ p is a vertical angle to x and must be congruent to x.

$\boldsymbol{q = 15°}$ $p, q,$ and the 98° angle form a triangle and must add to 180°.

So, $67° + 98° + q = 180°$.

Example 2: Determine the angle measures.

$x =$ _______, $y =$ _______



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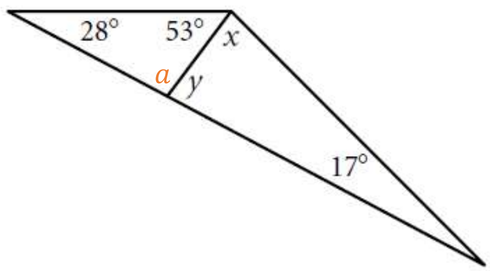


The sum of the measures of the angles in every triangle is 180°, so $28° + 53° + a = 180°$. This means $a = 99°$.

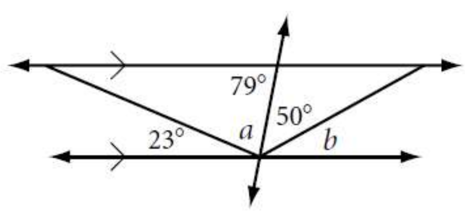
$\boldsymbol{y = 81°}$ y is a linear pair with a and must add to 180° with a.

$\boldsymbol{x = 82°}$ $x, y,$ and the 17° angle form a triangle and must add to 180°.

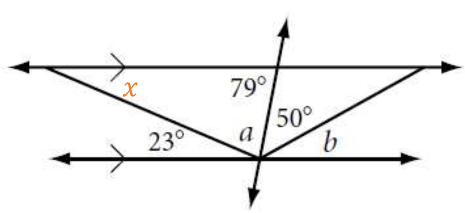
So, $81° + 17° + x = 180°$

Example 3: Determine the angle measures.

$a =$ ______, $b =$ ______



Let's label an unlabeled angle to make references easier.



x and the 23° angle are alternate interior angles on parallel lines and must be congruent, so $x = 23°$.

$\boldsymbol{a = 78°}$ $x, a,$ and the 79° angle form a triangle and must add to 180°.

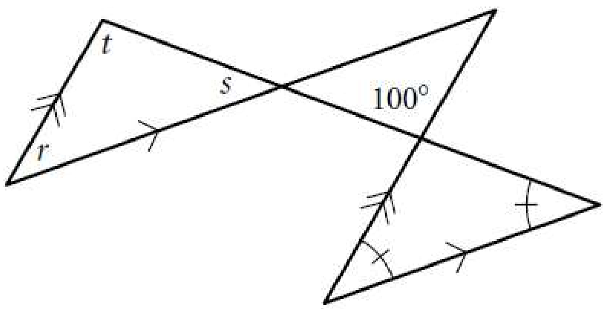
So, $23° + 79° + a = 180°$.

$\boldsymbol{b = 29°}$ $b, a,$ the 23° angle, and the 50° angle are supplementary angles and must add to 180°. So, $23° + 50° + 78° + b = 180°$.

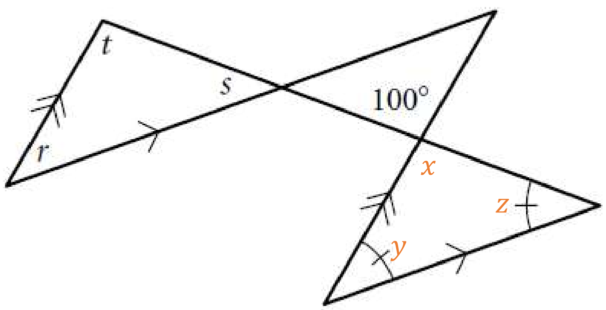
Example 4: Determine the angle measures.

$r =$ ________, $s =$ ________,

$t =$ ________



Let's label some angles to make references easier.



x is a vertical angle to the $100°$ angle, so $x = 100°$.

The sum of the measures of the angles in every triangle is 180°, so $100° + y + z = 180°$. Since y and z are marked congruent, we can say $100° + 2y = 180°$. This means that $2y = 80°$ and $y = 40°$. Since y and z are marked congruent, $z = 40°$.

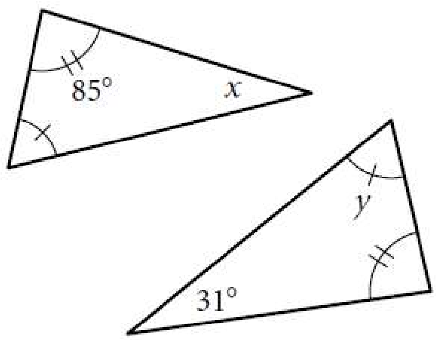
$\boldsymbol{t = 100°}$ t is an alternate interior angle on parallel lines to the $100°$ angle and must be congruent to $100°$.

$\boldsymbol{s = 40°}$ s is a corresponding angle on parallel lines (the ones marked with one arrow) to z and must be congruent to z.

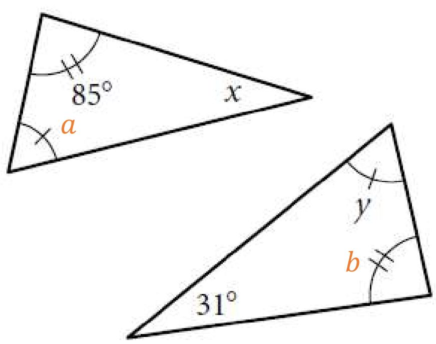
$\boldsymbol{r = 40°}$ $r, s,$ and t form a triangle and must add to 180°. So, $100° + 40° + q = 180°$.

Example 5: Determine the angle measures.

$x =$ _______, $y =$ _______



Let's label some angles to make references easier.



b is marked congruent to the $85°$ angle, so $b = 85°$.

$\boldsymbol{y = 64}°$ $\quad$ $b, y,$ and the $31°$ angle form a triangle and must add to $180°$.

So, $31° + 85° + y = 180°$.

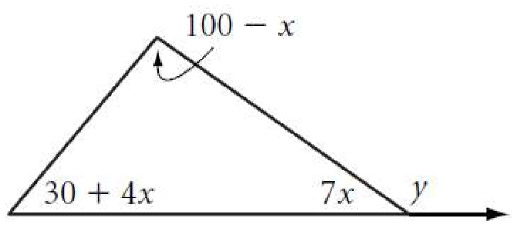
a is marked congruent to y, so $a = 64°$.

$\boldsymbol{x = 31}°$ $\quad$ $a, x,$ and the $85°$ angle form a triangle and must add to $180°$.

So, $64° + 85° + x = 180°$.

Example 6: Determine the angle measures.

$y =$ ________



We need to find the value of x before we can find the value of y.

The three angles of a triangle must add to 180°, so $(100 - x) + (30 + 4x) + (7x) = 180°$.

$100 - x + 30 + 4x + 7x = 180$	Remove parentheses
$100 + 30 + 10x = 180$	Combine terms with x
$130 + 10x = 180$	Combine constant terms
$10x = 50$	Subtract 130 from both sides
$x = 5$	Divide by 10 on both sides

$7x$ and y are a linear pair of angles and must add to 180°, so $7x + y = 180°$.

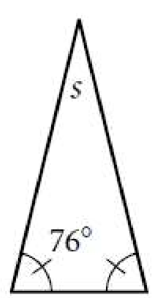
We know that $x = 5$, so we can substitute that in: $7(5) + y = 180°$.

$35 + y = 180$	Multiply
$y = 145$	Subtract 35 from each side

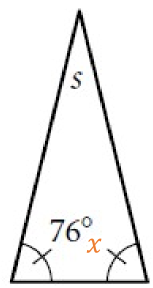
$\boldsymbol{y = 145°}$

Example 7: Determine the angle measures.

$s =$ ________



Let’s label the unlabeled angle to make references easier.



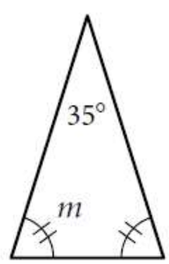
x is marked congruent to the $76°$ angle, so $x = 76°$.

$\boldsymbol{s = 28°}$ $s, x,$ and the $76°$ angle form a triangle and must add to $180°$.

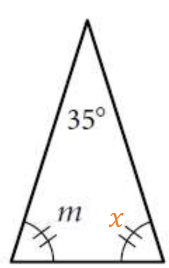
So, $76° + 76° + s = 180°$.

Example 8: Determine the angle measures.

$m =$ ________



Let’s label the unlabeled angle to make references easier.



$m, x,$ and the $35°$ angle form a triangle and must add to $180°$. So, $35° + m + x = 180°$.

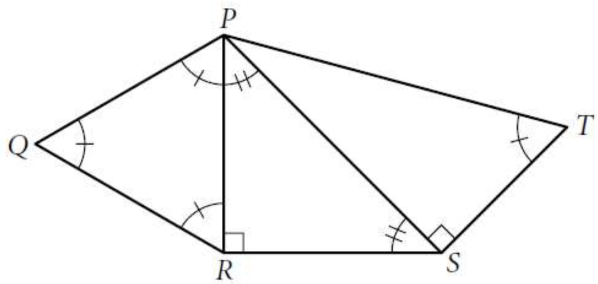
x is marked congruent to m, so $35° + 2m = 180°$.

$2m = 145°$.

$\boldsymbol{m = 72\frac{1}{2}°}$

Example 9: Determine the angle measures.

Find the measure of $\angle QPT$.



In triangle QPR, $\angle PQR, \angle QRP$ and $\angle RPQ$ are marked congruent, so we can split $180°$ three ways to find the measure of each angle in that triangle. $m\angle PQR = m\angle QRP = m\angle RPQ = 60°$.

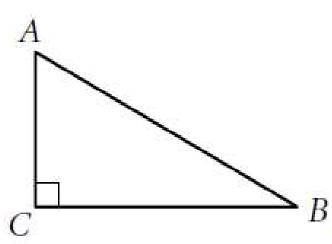
In triangle $PRS, m\angle PRS = 90°$, and $\angle RSP$ and $\angle SPR$ are marked congruent. Let's use x as the variable to find those two angles since they both have the same measure. So, $90° + 2x = 180° \;\Rightarrow\; 2x = 90° \;\Rightarrow\; x = 45°$. This means that $m\angle RSP = m\angle SPR = 45°$.

In triangle PST, $m\angle PST = 90°$. $\angle STP$ is marked congruent to $\angle PQR, \angle QRP$ and $\angle RPQ$ which means that $m\angle STP = 60°$. To find $m\angle TPS$, we can subtract $60°$ and $90°$ from $180°$. So, $m\angle TPS = 30°$

$\boldsymbol{m\angle QPT = 135°}$ $\quad m\angle QPT = m\angle QPR + m\angle RPS + m\angle SPT$.

So, $m\angle QPT = 60° + 45° + 30°$.

Example 10: Use the diagram to explain why $\angle A$ and $\angle B$ are complementary.

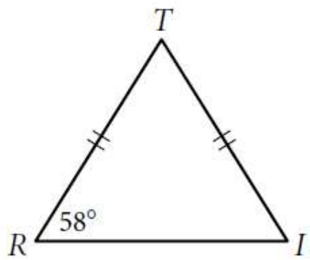


$m\angle A + m\angle B + m\angle C = 180°$	Angles of a triangle add to 180°
$m\angle A + m\angle B + 90° = 180°$	Substitute the angle measure of $\angle C$
$m\angle A + m\angle B = 90°$	Subtract 90° from both sides

The definition of complementary is that two angles add to $90°$. So, we just showed that $m\angle A + m\angle B = 90°$. Therefore $\angle A$ and $\angle B$ are complementary.

Example 11: Determine the angle measures.

$m\angle T =$ ________



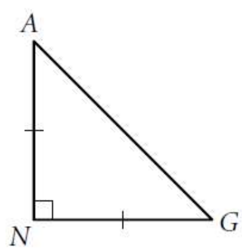
Triangle RTI is isosceles with $\overline{RT} \cong \overline{TI}$. The angles opposite the congruent sides are the base angles and must be congruent. So, $\angle R$ and $\angle I$ are the base angles and $\angle R \cong \angle I$. This means that $m\angle R = m\angle I = 58°$.

All angles of a triangle must add to 180°. So, $m\angle R + m\angle T + m\angle I = 180°$.

$58° + m\angle T + 58° = 180°$	Substitute known values
$116° + m\angle T = 180°$	Combine constant values
$\boldsymbol{m\angle T = 64°}$	Subtract 116 from both sides

Example 12: Determine the angle measures.

$m\angle G =$ _______



Triangle ANG is isosceles with $\overline{NA} \cong \overline{NG}$. The angles opposite the congruent sides are the base angles and must be congruent. So, $\angle A$ and $\angle G$ are the base angles and $\angle A \cong \angle G$.

All angles of a triangle must add to 180°. So, $m\angle A + m\angle N + m\angle G = 180°$.

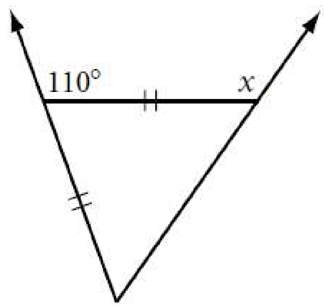
$m\angle A + 90° + m\angle G = 180°$	Substitute known values
$90° + 2m\angle G = 180°$	Since $\angle A \cong \angle G$, we can say that there are basically two of one or the other angle.

$2m\angle G = 90°$ Subtract 90 from each side

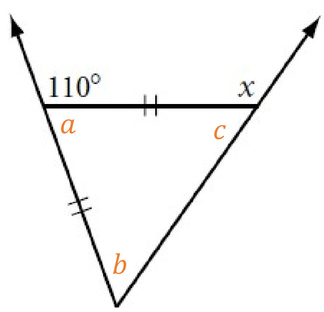
$\boldsymbol{m\angle G = 45°}$ Divide by 2 on both sides

<u>Example 13: Determine the angle measures.</u>

$x =$ ______



Let's label some angles to make references easier.



a is a linear pair with the $110°$ angle, so $a + 110° = 180°$, and $a = 70°$.

The triangle is isosceles with b and c as the base angles, so $b = c$.

$a + b + c = 180°$ angles of a triangle

$70° + b + c = 180°$ substitute known values

$70° + 2c = 180°$ Since $b = c$, we can say that $b + c = c + c = 2c$

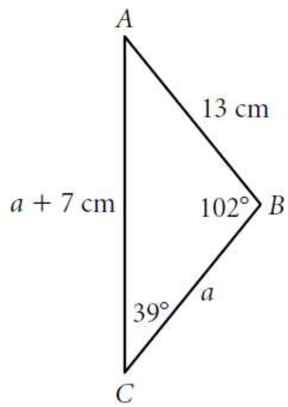
$2c = 110°$ subtract 70 from both sides

$c = 55°$ divide by two on both sides

$\boldsymbol{x = 125°}$ x is a linear pair with c, so $x + 55° = 180°$.

Example 14: Find the measures.

$m\angle A =$ _______, perimeter of $\triangle ABC =$ _______



All angles of a triangle must add to 180°. So, $m\angle A + m\angle B + m\angle C = 180°$.

$m\angle A + 102° + 39° = 180°$ Substitute known values

$141° + m\angle A = 180°$ Combine constant values

$\boldsymbol{m\angle A = 39°}$ Subtract 141 from both sides

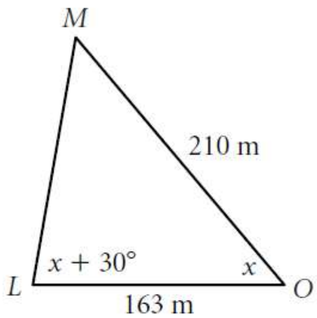
Since, $m\angle A = m\angle C = 39°$, $\angle A$ and $\angle C$ are the base angles of an isosceles triangle with $\overline{AB} \cong \overline{BC}$. This means that $m\overline{AB} = m\overline{BC} = 13$ cm. So, $a = 13$ cm.

To find $m\overline{AC}$, we can substitute 13 for a. $m\overline{AC} = a + 7 = 13 + 7 = 20$ cm.

Perimeter of $\boldsymbol{\Delta ABC = 46}$ cm $m\overline{AB} + m\overline{BC} + m\overline{AC} = 13 + 13 + 20$

Example 15: Find the measures.

The perimeter of $\triangle LMO$ is 536 m. $LM =$ ______, $m\angle M =$ ______



$LM + MO + LO = 536$	The perimeter of ΔLMO is 536 m
$LM + 210 + 163 = 536$	Substitute known values
$LM + 373 = 536$	Combine constant values
$\boldsymbol{LM = 163}$ **m**	Subtract 373 from both sides

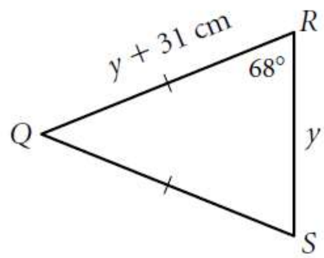
Triangle LMO is isosceles with $\overline{LM} \cong \overline{LO}$. The angles opposite the congruent sides are the base angles and must be congruent. So, $\angle M$ and $\angle O$ are the base angles, $\angle M \cong \angle O$, and $m\angle M = m\angle O = x$.

All angles of a triangle must add to 180°. So, $m\angle L + m\angle M + m\angle O = 180°$.

$(x + 30°) + (x) + (x) = 180°$	Substitute for angles
$3x + 30° = 180°$	Combine like terms
$3x = 150°$	Subtract 30 from both sides
$x = 50°$	Divide by 3 on both sides
$\boldsymbol{m\angle M = 50°}$	$m\angle M = m\angle O = x$ and $x = 50°$

Example 16: Find the measures.

The perimeter of $\triangle QRS$ is 344 cm. $m\angle Q =$ _______, $QR =$ _______



Triangle QRS is isosceles with $\overline{QR} \cong \overline{QS}$. The angles opposite the congruent sides are the base angles and must be congruent. So, $\angle R$ and $\angle S$ are the base angles, $\angle R \cong \angle S$, and $m\angle R = m\angle S = 68°$.

All angles of a triangle must add to 180°. So, $m\angle Q + m\angle R + m\angle S = 180°$.

$m\angle Q + 68° + 68° = 180°$	Substitute known values
$136° + m\angle Q = 180°$	Combine constant values
$\boldsymbol{m\angle Q = 44°}$	Subtract 136 from both sides

$\overline{QR} \cong \overline{QS}, QR = QS = y + 31$ cm

$QR + RS + QS = 344$	The perimeter of ΔQRS is 344 cm
$(y + 31) + (y) + (y + 31) = 344$	Substitute for side values
$3y + 62 = 344$	Combine like terms
$3y = 282$	Subtract 62 from both sides
$y = 94$	Divide by 3 on both sides
$\boldsymbol{QR = 125}$ **cm**	$QR = y + 31 = 94 + 31 = 125$