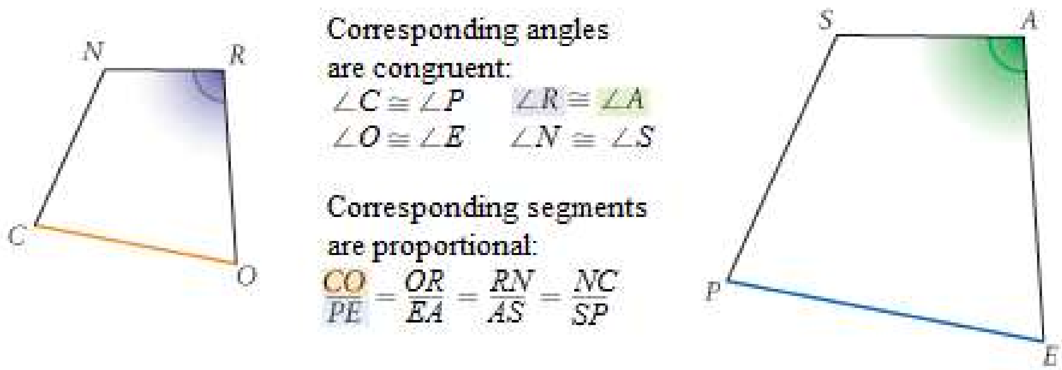
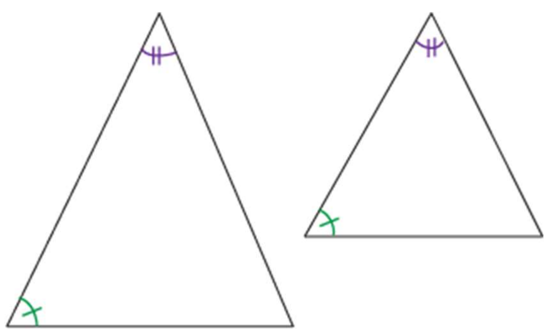
Lessons 11.1 & 11.2 – Similar Polygons and Similar Triangles

Similar (~) - Two polygons are similar if and only if the corresponding angles are congruent and the corresponding sides are proportional.

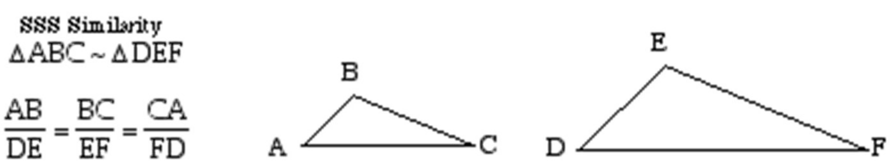


$CORN \sim PEAS$

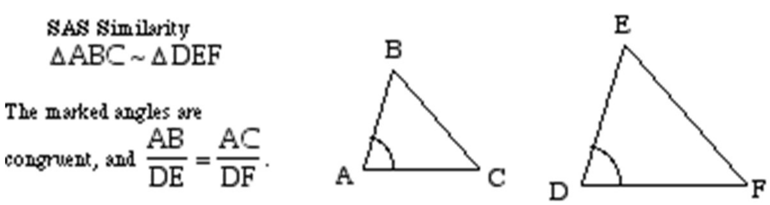
AA Similarity Conjecture - If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.



SSS Similarity Conjecture - If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.



SAS Similarity Conjecture - If two sides of one triangle are proportional to two sides of another triangle and the included angles are congruent, then the triangles are similar.



Example 1: Find the missing measurements. All measurements are in centimeters.

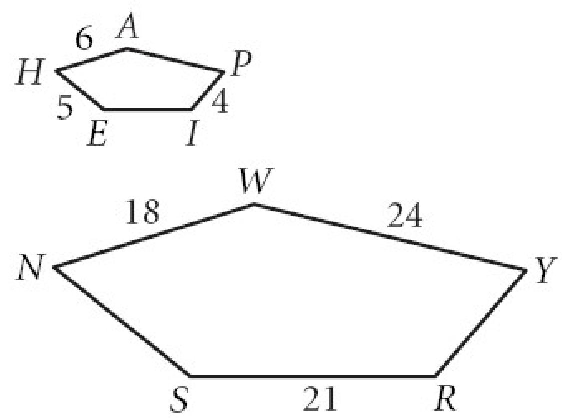
$HAPIE \sim NWYRS$

$AP =$________

$EI =$________

$SN =$________

$YR =$________



Since we are told that the two polygons are similar, we need to find corresponding sides on each figure that are known so that we can determine the constant of proportionality. You usually want to use the names of the figures to determine which sides are corresponding. HA is corresponding to NW and both are known.

$\frac{NW}{HA} = \frac{18}{6} = 3$, so 3 is our constant of proportionality.

This means that to find a side length in the smaller figure, we will divide the larger figure measure by 3. To find the side length in the larger figure, we will multiply the smaller figure measure by 3.

To find side AP, we will use the length of its corresponding side WY. Since WY is on the larger figure, we will divide its length by 3 to find the length of AP. So, $\frac{24}{3} = 8$.

$\boldsymbol{AP = 8}$ **cm**

To find side EI, we will use the length of its corresponding side SR. Since SR is on the larger figure, we will divide its length by 3 to find the length of EI. So, $\frac{21}{3} = 7$.

$\boldsymbol{EI = 7}$ **cm**

To find side SN, we will use the length of its corresponding side EH. Since EH is on the smaller figure, we will multiply its length by 3 to find the length of SN. So, $5 \cdot 3 = 15$.

$\boldsymbol{SN = 15}$ **cm**

To find side YR, we will use the length of its corresponding side PI. Since PI is on the smaller figure, we will multiply its length by 3 to find the length of YR. So, $4 \cdot 3 = 12$.

$\boldsymbol{YR = 12}$ **cm**

Example 2: Find the missing measurements. All measurements are in centimeters.

$QUAD \sim SIML$

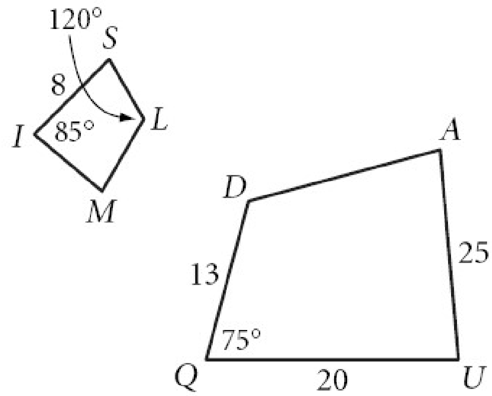
$SL =$________

$MI =$________

$m\angle D =$________

$m\angle U =$________

$m\angle A =$________



Since we are told that the two polygons are similar, we need to find corresponding sides on each figure that are known so that we can determine the constant of proportionality. You usually want to use the names of the figures to determine which sides are corresponding. QU is corresponding to SI and both are known.

$\frac{QU}{SI} = \frac{20}{8} = 2\frac{1}{2}$, so $2\frac{1}{2}$ is our constant of proportionality.

This means that to find a side length in the smaller figure, we will divide the larger figure measure by $2\frac{1}{2}$. To find the side length in the larger figure, we will multiply the smaller figure measure by $2\frac{1}{2}$.

To find side SL, we will use the length of its corresponding side QD. Since QD is on the larger figure, we will divide its length by $2\frac{1}{2}$ to find the length of SL. So, $\frac{13}{2\frac{1}{2}} = 5\frac{1}{4}$.

$\boldsymbol{SL = 5\frac{1}{4}}$ **cm**

To find side MI, we will use the length of its corresponding side AU. Since AU is on the larger figure, we will divide its length by $2\frac{1}{2}$ to find the length of MI. So, $\frac{25}{2\frac{1}{2}} = 10$.

$\boldsymbol{MI = 10}$ **cm**

To find $m\angle D$, we will use the measure of its corresponding angle $m\angle L$. Since corresponding angles in similar figures must be congruent, we know that $m\angle D = m\angle L$.

$\boldsymbol{m\angle D = 120°}$

To find $m\angle U$, we will use the measure of its corresponding angle $m\angle I$. Since corresponding angles in similar figures must be congruent, we know that $m\angle U = m\angle I$.

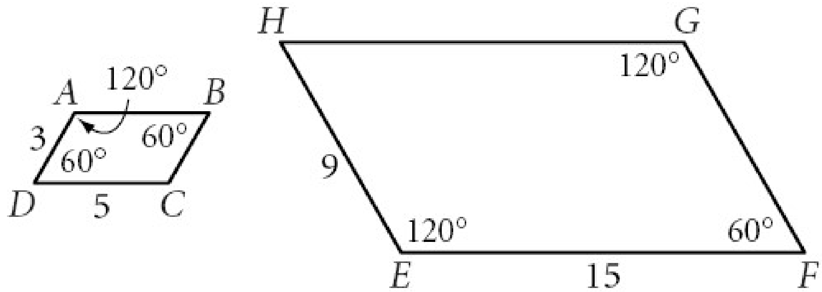
$\boldsymbol{m\angle U = 85°}$

To find $m\angle A$, we will us the fact that $QUAD$ is a quadrilateral and its angles must add to 360°.
$m\angle Q + m\angle U + m\angle A + m\angle D = 360° \quad \Rightarrow \quad 75° + 85° + m\angle A + 120° = 360° \quad \Rightarrow$
$280° + m\angle A = 360°$.

$\boldsymbol{m\angle A = 80°}$

Example 3: Determine whether or not the figures are similar. Explain why or why not.

$ABCD$ and $EFGH$



To determine whether two figures are similar, we need to verify two things:

1) Corresponding angles are congruent.
2) Corresponding sides are proportional.

Let's start by determining whether the first condition is true.

We can use the fact that $ABCD$ is a quadrilateral and its angles must add to $360°$ to find the missing angle measure.

$60° + 120° + 60° + m\angle C = 360°$

$240° + m\angle C = 360°$

$m\angle C = 120°$

We can use the fact that $EFGH$ is a quadrilateral and its angles must add to $360°$ to find the missing angle measure.

$120° + 60° + 120° + m\angle H = 360°$

$300° + m\angle H = 360°$

$m\angle H = 60°$

We can see that every angle in $ABCD$ has a congruent corresponding angle to an angle in $EFGH$.

Now, let's check if the corresponding sides are proportional.

We can see that sides EF and DC are corresponding. Sides EH and DA are also corresponding.

Sides GH and AB are also corresponding, but since they are equal in length to EF and DC we don't need to check them. Likewise for sides GF and CB.

So, we need to know if $\frac{EF}{DC} = \frac{EH}{DA}$.

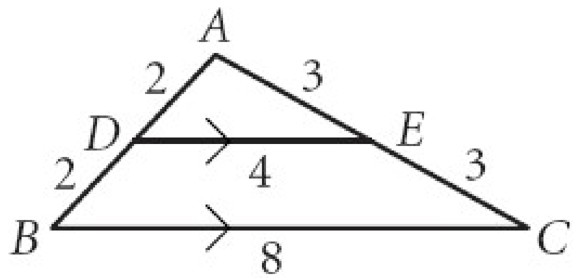
$\frac{EF}{DC} = \frac{15}{5} = 3$ $\qquad$ $\frac{EH}{DA} = \frac{9}{3} = 3$

Since both are equal, we can say that the sides are proportional.

$DCBA \sim EFGH$ because the corresponding angles are congruent and corresponding sides are proportional.

<u>Example 4: Determine whether or not the figures are similar. Explain why or why not.</u>

ΔABC and ΔADE



To determine whether two figures are similar, we need to verify two things:

1) Corresponding angles are congruent.
2) Corresponding sides are proportional.

Let's start by determining whether the first condition is true.

$\angle A \cong \angle A$ because they are the same angle.

$\angle ADE \cong \angle ABC$ because they are corresponding angles on parallel lines

$\angle AED \cong \angle ACB$ because they are corresponding angles on parallel lines

We can see that every angle in ΔABC has a congruent corresponding angle to an angle in ΔADE.

Now, let's check if the corresponding sides are proportional.

Sides DE and BC are corresponding. Sides AD and AB are also corresponding. The final set of corresponding sides is AE and AC.

**To find the length of side AB, we will need to add $AD + DB = 4$.

**To find the length of side AC we will need to add $AD + EC = 6$

We need to know if $\frac{BC}{DE} = \frac{AB}{AD} = \frac{AC}{AE}$.

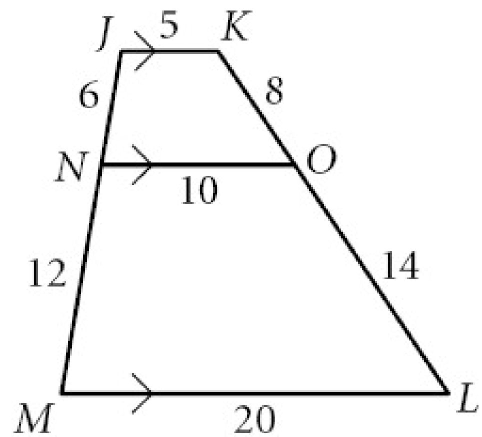
$\frac{BC}{DE} = \frac{8}{4} = 2$ $\frac{AB}{AD} = \frac{4}{2} = 2$ $\frac{AC}{AD} = \frac{6}{3} = 2$

Since all are equal, we can say that the sides are proportional.

$\Delta ABC \sim \Delta ADE$ because the corresponding angles are congruent and corresponding sides are proportional.

Example 5: Determine whether or not the figures are similar. Explain why or why not.

$JKON$ and $JKLM$



To determine whether two figures are similar, we need to verify two things:

1) Corresponding angles are congruent.
2) Corresponding sides are proportional.

Let's start by determining whether the first condition is true.

$\angle J \cong \angle J$ because they are the same angle.

$\angle K \cong \angle K$ because they are the same angle.

$\angle JNO \cong \angle JML$ because they are corresponding angles on parallel lines

$\angle KON \cong \angle KLM$ because they are corresponding angles on parallel lines

We can see that every angle in $JKON$ has a congruent corresponding angle to an angle in $JKLM$.

Now, let's check if the corresponding sides are proportional.

Sides ML and NO are corresponding. Sides JM and JN are also corresponding. Sides KL and KO are corresponding. The final set of corresponding sides is JK and JK.

**To find the length of side JM, we will need to add $JN + NM = 18$.

**To find the length of side KL we will need to add $KO + OL = 22$

We need to know if $\frac{ML}{NO} = \frac{JM}{JN} = \frac{KL}{KO} = \frac{JK}{JK}$.

$\frac{ML}{NO} = \frac{20}{10} = 2$ $\qquad$ $\frac{JM}{JN} = \frac{18}{6} = 3$ $\qquad$ $\frac{KL}{KO} = \frac{22}{8} = 2\frac{3}{4}$ $\qquad$ $\frac{JK}{JK} = \frac{5}{5} = 1$

Since all of the proportions are not equal, we must conclude that the figures are not similar.

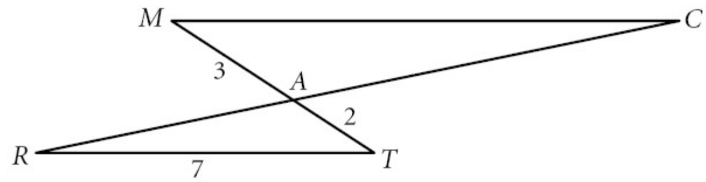
$JKON \nsim JKLM$ because the sides are not proportional.

**The line through ~ means "not".

Example 6: Find the missing measurements. All measurements are in centimeters.

$\Delta TAR \sim \Delta MAC$

$MC =$________



Since we are told that the two triangles are similar, we need to find corresponding sides on each figure that are known so that we can determine the constant of proportionality. You usually want to use the names of the figures to determine which sides are corresponding. TA is corresponding to MA and both are known.

$\frac{MA}{TA} = \frac{3}{2} = 1\frac{1}{2}$, so $1\frac{1}{2}$ is our constant of proportionality.

This means that to find a side length in the smaller figure, we will divide the larger figure measure by $1\frac{1}{2}$. To find the side length in the larger figure, we will multiply the smaller figure measure by $1\frac{1}{2}$.

To find side MC, we will use the length of its corresponding side RT. Since RT is on the smaller figure, we will multiply its length by $1\frac{1}{2}$ to find the length of MC. So, $7 \cdot 1\frac{1}{2} = 10\frac{1}{2}$.

$\boldsymbol{MC = 10\frac{1}{2}}$ **cm**

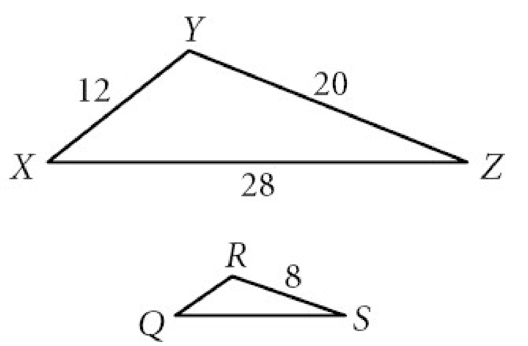
Example 7: Find the missing measurements. All measurements are in centimeters.

$\Delta XYZ \sim \Delta QRS$

$\angle Q \cong$________

$QR =$________

$QS =$________



Since we are told that the two triangles are similar, we need to find corresponding sides on each figure that are known so that we can determine the constant of proportionality. You usually want to use the names of the figures to determine which sides are corresponding. YZ is corresponding to RS and both are known.

$\frac{YZ}{RS} = \frac{20}{8} = 2\frac{1}{2}$, so $2\frac{1}{2}$ is our constant of proportionality.

This means that to find a side length in the smaller figure, we will divide the larger figure measure by $2\frac{1}{2}$. To find the side length in the larger figure, we will multiply the smaller figure measure by $2\frac{1}{2}$.

To find side QR, we will use the length of its corresponding side XY. Since XY is on the larger figure, we will divide its length by $2\frac{1}{2}$ to find the length of QR. So, $\frac{12}{2\frac{1}{2}} = 4\frac{4}{5}$.

$\boldsymbol{QR = 4\frac{4}{5}}$ **cm**

To find side QS, we will use the length of its corresponding sideXZ. Since XZ is on the larger figure, we will divide its length by $2\frac{1}{2}$ to find the length of QS. So,$\frac{28}{2\frac{1}{2}} = 11\frac{1}{5}$.

$\boldsymbol{QS = 11\frac{1}{5}}$ **cm**

The first question asks what $\angle Q$ is congruent to. It does not ask us to find a measure, so we will not be giving a numerical answer to that problem. We simply need to complete the statement with the congruent angle to $\angle Q$, which will be the corresponding angle.

$\boldsymbol{\angle Q \cong \angle X}$

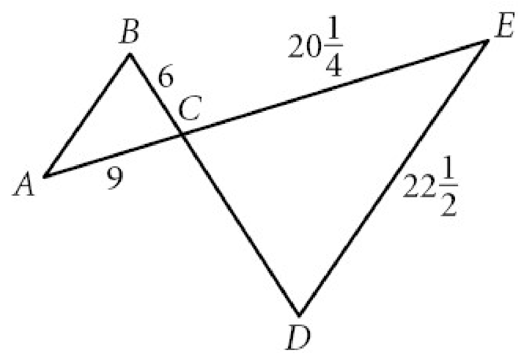
<u>Example 8: Find the missing measurements. All measurements are in centimeters.</u>

$\Delta ABC \sim \Delta EDC$

$\angle A \cong$________

$CD =$________

$AB =$________



Since we are told that the two triangles are similar, we need to find corresponding sides on each figure that are known so that we can determine the constant of proportionality. AC is corresponding to EC and both are known.

$\frac{EC}{AC} = \frac{20\frac{1}{4}}{9} = 2\frac{1}{4}$, so $2\frac{1}{4}$ is our constant of proportionality.

To find side CD, we will use the length of its corresponding side CB. Since CB is on the smaller figure, we will multiply its length by $2\frac{1}{4}$ to find the length of CD. So, $6 \cdot 2\frac{1}{4} = 13\frac{1}{2}$.

$\boldsymbol{CD = 13\frac{1}{2}}$ **cm**

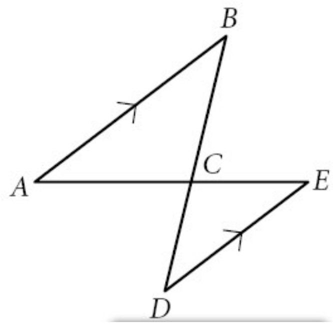
To find side AB, we will use the length of its corresponding side ED. Since ED is on the larger figure, we will divide its length by $2\frac{1}{4}$ to find the length of QS. So,$\frac{22\frac{1}{2}}{2\frac{1}{4}} = 10$.

$\boldsymbol{AB = 10}$ **cm**

The first question asks what $\angle A$ is congruent to which will be the corresponding angle.

$\boldsymbol{\angle A \cong \angle E}$

Example 9: Find similar triangles. Explain why they are similar.



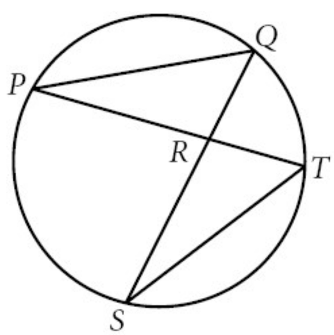
$\angle A \cong \angle E$ because they are alternate interior angles on parallel lines.

$\angle B \cong \angle D$ because they are alternate interior angles on parallel lines.

Make sure that when you name figures, the names are order specific.

$\Delta ABC \sim \Delta EDC$ by AA similarity

Example 10: Find similar triangles. Explain why they are similar.



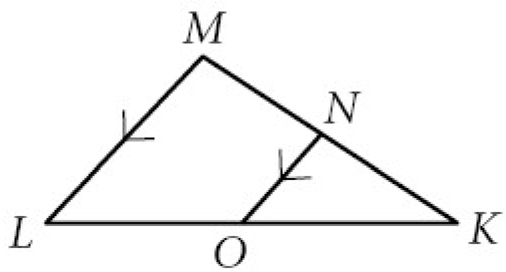
$\angle Q \cong \angle T$ because both are inscribed angles to the same arc.

$\angle P \cong \angle S$ because both are inscribed angles to the same arc.

Make sure that when you name figures, the names are order specific.

$\Delta PQR \sim \Delta STR$ by AA similarity

Example 11: Find similar triangles. Explain why they are similar.



$\angle LMK \cong \angle ONK$ because they are alternate interior angles on parallel lines.

$\angle MLK \cong \angle NOK$ because they are alternate interior angles on parallel lines.

Make sure that when you name figures, the names are order specific.

$\Delta KML \sim \Delta KNO$ by AA similarity