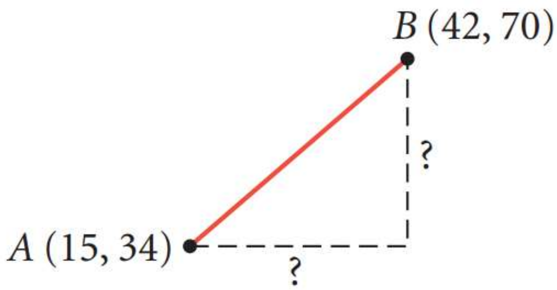
Lesson 9.5 – Distance in Coordinate Geometry

The Distance Formula - The distance between points $A(x_1,\ y_1)$ and $B(x_2,\ y_2)$ is given by $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



$$AB = \sqrt{(42-15)^2 + (70-34)^2}$$
$$AB = \sqrt{(27)^2 + (36)^2}$$
$$AB = \sqrt{729+1296}$$
$$AB = \sqrt{2025}$$
$$AB = 45$$

Example 1: Find the distance between the pair of points.

$(-5,-5),(1,3)$

$(-5,-5),(1,3)$

$x_1, y_1 \quad x_2, y_2$

$\sqrt{(1 - -5)^2 + (3 - -5)^2}$

$\sqrt{(1+5)^2 + (3+5)^2}$

$\sqrt{(6)^2 + (8)^2}$

$\sqrt{36 + 64}$

$\sqrt{100}$

10 units

Example 2: Find the distance between the pair of points.

$(-11,-5),(5,7)$

$(-11,-5),(5,7)$

$x_1, y_1 \quad x_2, y_2$

$\sqrt{(5--11)^2+(7--5)^2}$

$\sqrt{(5+11)^2+(7+5)^2}$

$\sqrt{(16)^2+(12)^2}$

$\sqrt{256+144}$

$\sqrt{400}$

20 units

<u>Example 3: Find the distance between the pair of points.</u>

$(8,-2),(-7,6)$

$(8,-2),(-7,6)$

$x_1, y_1 \quad x_2, y_2$

$\sqrt{(-7-8)^2+(6--2)^2}$

$\sqrt{(-7-8)^2+(6+2)^2}$

$\sqrt{(-15)^2+(8)^2}$

$\sqrt{225+64}$

$\sqrt{289}$

17 units

<u>Example 4: Use the distance and slope formulas to identify the type of quadrilateral.</u>

$A(-2,1), B(3,-2), C(8,1), D(3,4)$

$A(-2,1), B(3,-2)$

$x_1, y_1 \quad x_2, y_2$

$AB=\sqrt{(3--2)^2+(-2-1)^2}$

$B(3,-2), C(8,1)$

$x_1, y_1 \quad x_2, y_2$

$BC=\sqrt{(8-3)^2+(1--2)^2}$

$AB = \sqrt{(3+2)^2 + (-2-1)^2}$

$AB = \sqrt{(5)^2 + (-3)^2}$

$AB = \sqrt{25+9}$

$AB = \sqrt{34}$

$BC = \sqrt{(8-3)^2 + (1+2)^2}$

$BC = \sqrt{(5)^2 + (3)^2}$

$BC = \sqrt{25+9}$

$BC = \sqrt{34}$

$C(8,1), D(3,4)$
$x_1, y_1 \quad x_2, y_2$

$CD = \sqrt{(3-8)^2 + (4-1)^2}$

$CD = \sqrt{(-5)^2 + (3)^2}$

$CD = \sqrt{25+9}$

$CD = \sqrt{34}$

$D(3,4), A(-2,1)$
$x_1, y_1 \quad x_2, y_2$

$DA = \sqrt{(-2-3)^2 + (1-4)^2}$

$DA = \sqrt{(-5)^2 + (-3)^2}$

$DA = \sqrt{25+9}$

$DA = \sqrt{34}$

Since all four sides are congruent in length, we know this quadrilateral is either a rhombus or a square. If the quadrilateral is a square, the sides must meet at a right angle. So, we will use the slope formula to determine if sides AB and BC meet at a right angle.

$A(-2,1), B(3,-2)$
$x_1, y_1 \quad x_2, y_2$

$m = \frac{-2-1}{3--2} = -\frac{3}{5}$

$B(3,-2), C(8,1)$
$x_1, y_1 \quad x_2, y_2$

$m = \frac{1--2}{8-3} = \frac{3}{5}$

Since the slopes are not negative reciprocals of each other, the sides do not meet at right angles and the quadrilateral is not a square.

Rhombus

<u>Example 5: Use the distance and slope formulas to identify the type of quadrilateral.</u>

$T(-3,-3), U(4,4), V(0,6), W(-5,1)$

$T(-3,-3), U(4,4)$
$x_1, y_1 \quad x_2, y_2$

$TU = \sqrt{(4--3)^2 + (4--3)^2}$

$TU = \sqrt{(4+3)^2 + (4+3)^2}$

$U(4,4), V(0,6)$
$x_1, y_1 \quad x_2, y_2$

$UV = \sqrt{(0-4)^2 + (6-4)^2}$

$TU = \sqrt{(7)^2 + (7)^2}$

$TU = \sqrt{49 + 49}$

$TU = \sqrt{98}$

$UV = \sqrt{(-4)^2 + (2)^2}$

$UV = \sqrt{16 + 4}$

$UV = \sqrt{20}$

$V(0, 6), W(-5, 1)$
$x_1, y_1 \quad x_2, y_2$

$VW = \sqrt{(-5 - 0)^2 + (1 - 6)^2}$

$VW = \sqrt{(-5)^2 + (-5)^2}$

$VW = \sqrt{25 + 25}$

$VW = \sqrt{50}$

$W(-5, 1), T(-3, -3)$
$x_1, y_1 \quad x_2, y_2$

$WT = \sqrt{(-3 - -5)^2 + (-3 - 1)^2}$

$WT = \sqrt{(2)^2 + (-4)^2}$

$WT = \sqrt{4 + 16}$

$WT = \sqrt{20}$

Since one set of opposite sides is congruent, but the other is not, this quadrilateral can only be a trapezoid.

Isosceles Trapezoid

Example 6: Determine whether ΔABC is scalene, isosceles, or equilateral. Find the perimeter of the triangle.

$A(4, 14), B(10, 6), C(16, 14)$

$A(4, 14), B(10, 6)$
$x_1, y_1 \quad x_2, y_2$

$AB = \sqrt{(10 - 4)^2 + (6 - 14)^2}$

$AB = \sqrt{(6)^2 + (-8)^2}$

$AB = \sqrt{36 + 64}$

$AB = \sqrt{100}$

$AB = 10$

$B(10, 6), C(16, 14)$
$x_1, y_1 \quad x_2, y_2$

$BC = \sqrt{(16 - 10)^2 + (14 - 6)^2}$

$BC = \sqrt{(6)^2 + (8)^2}$

$BC = \sqrt{36 + 64}$

$BC = \sqrt{100}$

$BC = 10$

$C(16, 14), A(4, 14)$
$x_1, y_1 \quad x_2, y_2$

$CA = \sqrt{(4 - 16)^2 + (14 - 14)^2}$

$CA = \sqrt{(-12)^2 + (0)^2}$

$CA = \sqrt{144 + 0}$

$CA = \sqrt{144}$

$CA = 12$

Since two sides of the triangle are congruent, it is isosceles.

Isosceles Triangle

The perimeter can be found by adding the lengths of the three sides $(10 + 10 + 12 = 32)$

$\boldsymbol{P = 32}$ **units**