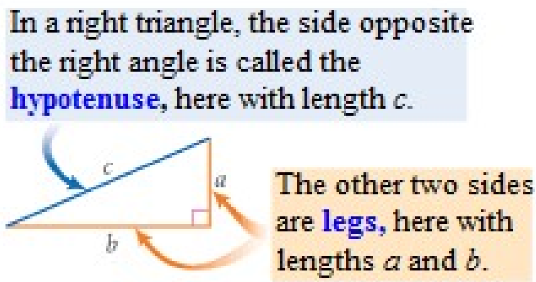
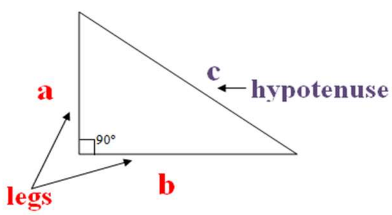
Lesson 9.1 – The Pythagorean Theorem



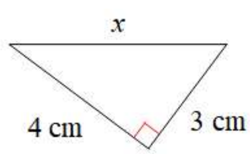
The Pythagorean Theorem:

In a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse.



$a^2 + b^2 = c^2$

Example 1: Finding the length of the hypotenuse given the lengths of the legs



**It doesn't matter which leg we call a and which leg we call b. The important thing is that the hypotenuse must be side c. So, $c = x$, but 4 cm and 3 cm can go in for either a or b. I know that x is the hypotenuse because it is across from the right angle.

$a^2 + b^2 = c^2$

$4^2 + 3^2 = x^2$ **If you switch a and b it just makes it $3^2 + 4^2 = x^2$, which will put you in the same spot since you can switch the order of addition.

$16 + 9 = x^2$

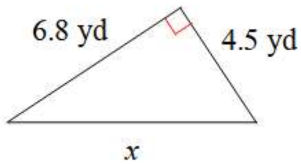
$25 = x^2$

$\sqrt{25} = \sqrt{x^2}$

$5 = x$

$\boldsymbol{x} = \mathbf{5\ cm}$

<u>Example 2: Finding the length of the hypotenuse given the lengths of the legs</u>



**I know that x is the hypotenuse because it is across from the right angle. That means 6.8 yd and 4.5 yd are the legs of the right triangle.

$a^2 + b^2 = c^2$

$6.8^2 + 4.5^2 = x^2$

$46.24 + 20.25 = x^2$

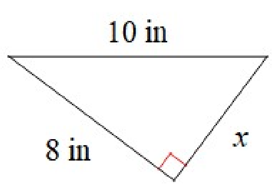
$66.49 = x^2$

$\sqrt{66.49} = \sqrt{x^2}$

$8.15 \approx x$

$\boldsymbol{x} \approx \mathbf{8.2\ yd}$

<u>Example 3: Finding a leg length given the length of one leg and the hypotenuse</u>



**I know that 10 in is the hypotenuse because it is across from the right angle. That means 8 in and x are the legs of the right triangle.

$a^2 + b^2 = c^2$

$8^2 + x^2 = 10^2$

$64 + x^2 = 100$

$-64 \qquad\quad -64$

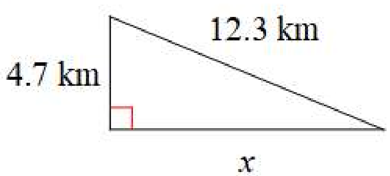
$x^2 = 36$

$\sqrt{x^2} = \sqrt{36}$

$x = 6$

$\boldsymbol{x = 6}$ **in**

<u>Example 4: Finding a leg length given the length of one leg and the hypotenuse</u>



**I know that 12.3 km is the hypotenuse because it is across from the right angle. That means 4.7 km and x are the legs of the right triangle.

$a^2 + b^2 = c^2$

$4.7^2 + x^2 = 12.3^2$

$22.09 + x^2 = 151.29$

$-22.09 \qquad -22.09$

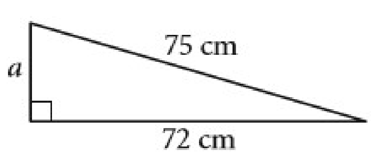
$x^2 = 129.2$

$\sqrt{x^2} = \sqrt{129.2}$

$x \approx 11.36667$

$\boldsymbol{x \approx 11.4}$ **km**

<u>Example 5: Finding a leg length given the length of one leg and the hypotenuse</u>



**I know that 75 cm is the hypotenuse because it is across from the right angle. That means a and 72 cm are the legs of the right triangle.

$a^2 + b^2 = c^2$

$a^2 + 72^2 = 75^2$

$a^2 + 5184 = 5625$

$\quad -5184 \quad -5184$

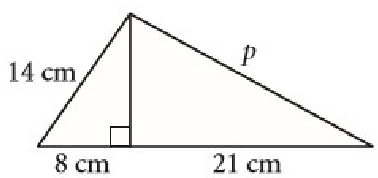
$a^2 = 441$

$\sqrt{a^2} = \sqrt{441}$

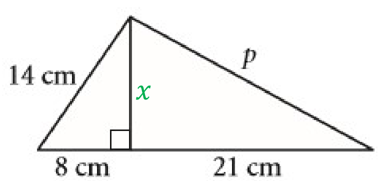
$a = 21$

$\boldsymbol{a = 21}$ **cm**

Example 6: Using the Pythagorean Theorem twice



**We do not know enough information to solve the right triangle that contains p. So, let's label the unknown, unlabeled side.



Let's start by solving for x.

I know that 14 cm is the hypotenuse in that right triangle, because it is across from the right angle. That means 8 and x are the legs of the right triangle.

$8^2 + x^2 = 14^2$

$64 + x^2 = 196$

$-64 \qquad -64$

$x^2 = 132$ **I'm not going to find the length of x, I'm going to leave this at x^2.

Now, let's switch our focus to the other triangle. I know that p is the hypotenuse because it is across from the right angle. That makes 21 and x the legs of the right triangle.

$x^2 + 21^2 = p^2$

$132 + 441 = p^2$

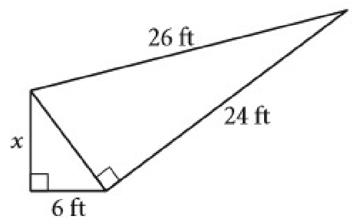
$573 = p^2$

$\sqrt{573} = \sqrt{p^2}$

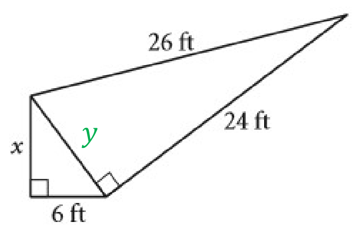
$23.9374 \approx p$

$\boldsymbol{p \approx 23.9}$ **cm**

Example 7: Using the Pythagorean Theorem twice



**We do not know enough information to solve the right triangle that contains x. So, let's label the unknown, unlabeled side.



Let's start with the triangle we can solve. 26 will be the hypotenuse. y and 24 will be the legs of the right triangle.

$y^2 + 24^2 = 26^2$

$y^2 + 576 = 676$

$\quad -576 \quad\quad -576$

$y^2 = 100$ **Because this isn't my final answer, I will leave this at y^2.

Now, let's switch our focus to the other triangle. y is the hypotenuse. 6 ft and x are the legs of the right triangle.

$x^2 + 6^2 = y^2$

$x^2 + 36 = 100$

$-36 \quad -36$

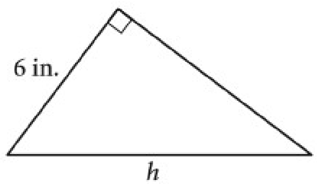
$x^2 = 64$

$\sqrt{x^2} = \sqrt{64}$

$x = 8$

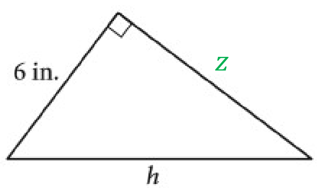
$\boldsymbol{x = 8}$ **ft**

Example 8: Area and the Pythagorean Theorem



Area = 39 in^2 $\qquad h \approx$__________

We know that h is the hypotenuse of this right triangle. However, we do not have enough information to solve the right triangle using the Pythagorean Theorem yet. We can use the area to solve for the missing leg.



Because 6 in and z meet at a right angle, they can be the base and height of the triangle.

$A = \frac{1}{2}bh$

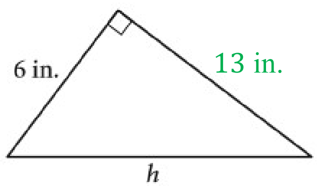
$39 = \frac{1}{2}(6)(z)$

$39 = 3z$

$\frac{39}{3} = \frac{3z}{3}$

$13 = z$

This means that the triangle looks like this…



Now we can use the Pythagorean Theorem. 6 in and 13 in are the legs of the right triangle and h is the hypotenuse.

$a^2 + b^2 = c^2$

$6^2 + 13^2 = h^2$

$36 + 169 = h^2$

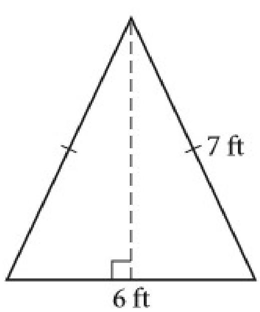
$175 = h^2$

$\sqrt{175} = \sqrt{h^2}$

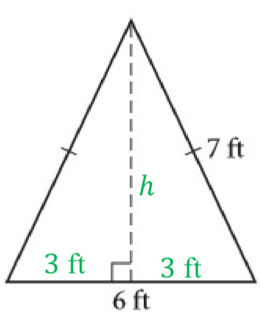
$13.2288 \approx h$

$\boldsymbol{h \approx 13.2}$ **in.**

<u>Example 9: Area and the Pythagorean Theorem</u>



Because this triangle is isosceles, we know that the altitude is also the bisector of the base.



If we take one-half of the triangle and use that to try to find the height, we know that 3 ft and h are the legs of the right triangle and 7 ft is the hypotenuse.

$h^2 + 3^2 = 7^2$

$h^2 + 9 = 49$

$\quad -9 \qquad -9$

$h^2 = 40$

$\sqrt{h^2} = \sqrt{40}$

$h \approx 6.32$

Now, we can use the height to find the area of the triangle.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(6)(6.32)$ **I'm using 6 ft as the base because I want the area of the entire triangle.

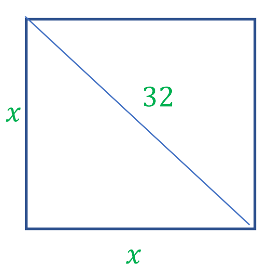
$A \approx 18.96$

$\boldsymbol{A \approx 18.96}$ **ft²**

Example 10: Squares and the Pythagorean Theorem

The diagonal of a square measures 32 meters. What is the area of the square?

Let's draw a picture:



So, to use the Pythagorean Theorem we know that 32 is the hypotenuse, and both legs are the same length (x).

$x^2 + x^2 = 32^2$

$x^2 + x^2 = 1024$

$2x^2 = 1024$

$\frac{2x^2}{2} = \frac{1024}{2}$

$x^2 = 512$

$x \approx 22.63$ **This means that each side of the square is about 22.63 meters.

Since this problem asked us for the area of the square, we should multiply base times height. Both the base and height of a square are the same. So, we multiply x by itself.

$x \cdot x = A$

$x^2 = A$

We already know x^2 from another step…

$\boldsymbol{A = 512}$ **m²**