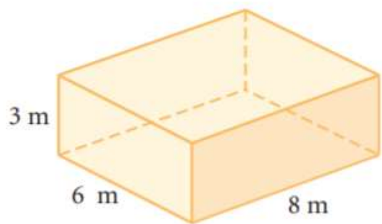
Lesson 8.7 – Surface Area

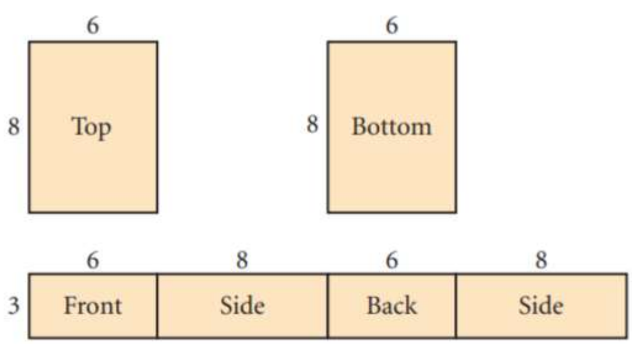
The **surface area** of a solid is the sum of the area of the faces or surfaces that enclose the solid.

Rectangular Prism Surface Area:

Find the surface area of the rectangular prism.



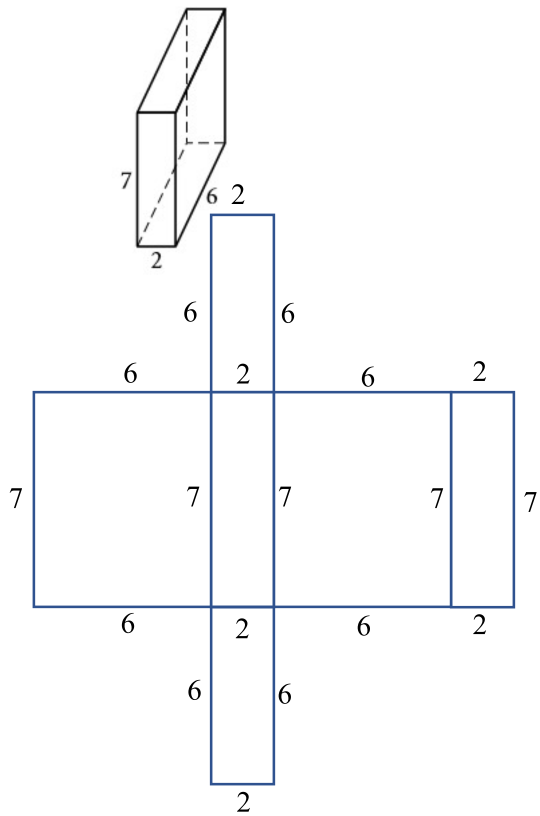
Draw and label the two congruent bases, and the four lateral faces unfolded into one rectangle. Then find the areas of all the rectangular faces.



$$
\begin{aligned}
\text{surface area} &= 2(\text{base area}) + (\text{lateral surface area}) \\
&= 2(6 \cdot 8) + 3(6 + 8 + 6 + 8) \\
&= 2(48) + 3(28) \\
&= 96 + 84 \\
&= 180
\end{aligned}
$$

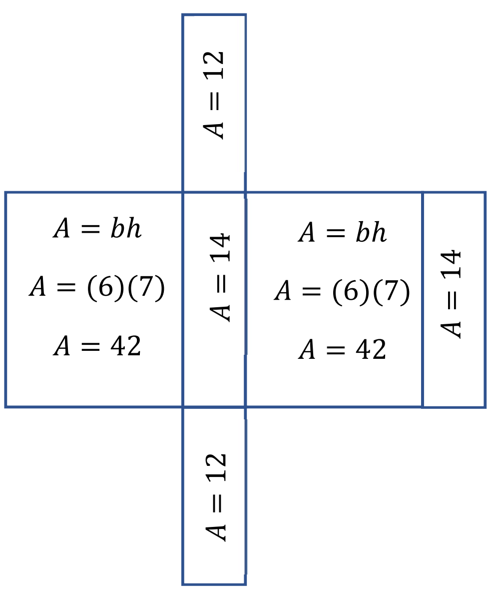
The surface area of the prism is 180 m^2.

Example 1: Surface Area of a Rectangular Prism



This is a net of the rectangular prism. I, obviously, went a little overboard on labeling sides, but I wanted to be sure that you recognize what all the measurements are for this net.

Now let's find the area of each of the faces of the rectangular prism.



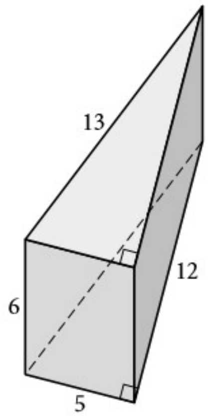
To find the surface area, we add the area of the 6 faces.

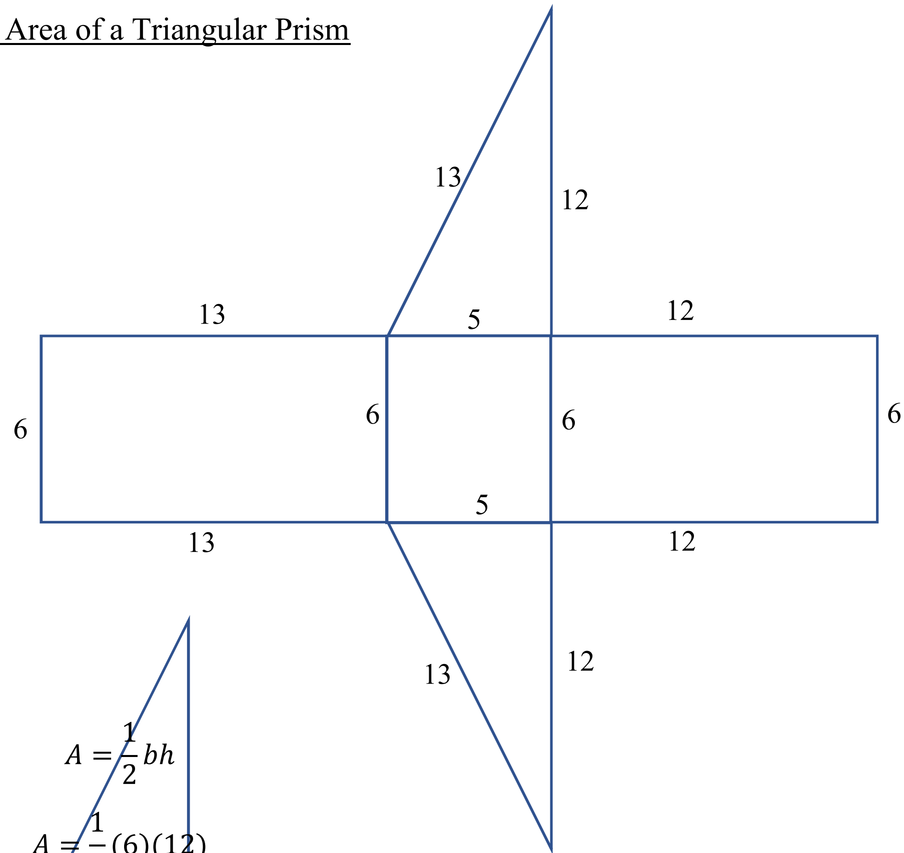
$$SA = 2(42) + 2(14) + 2(12)$$

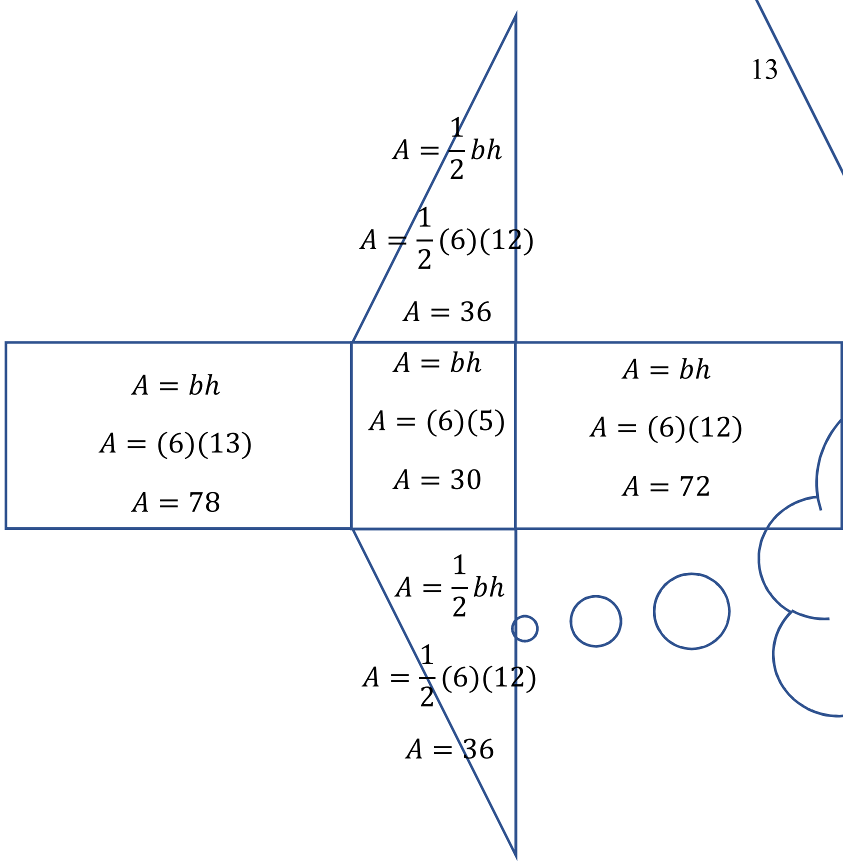
$$SA = 84 + 28 + 24$$

$$\boldsymbol{SA = 136 \text{ units}^2}$$

Example 2: Surface Area of a Triangular Prism







$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(6)(12)$$

$$A = 36$$

$$A = bh$$

$$A = (6)(13)$$

$$A = 78$$

$$A = bh$$

$$A = (6)(5)$$

$$A = 30$$

$$A = bh$$

$$A = (6)(12)$$

$$A = 72$$

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(6)(12)$$

$$A = 36$$

Remember that the 6 and 12 are the sides that meet at a right angle and therefore they must be the base and height of the triangle.

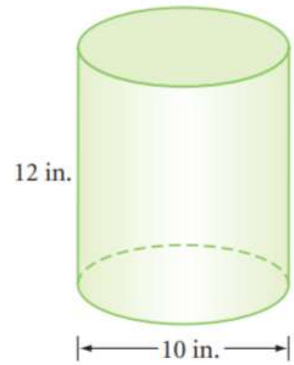
$$SA = 2(36) + 78 + 30 + 72$$

$$SA = 72 + 180$$

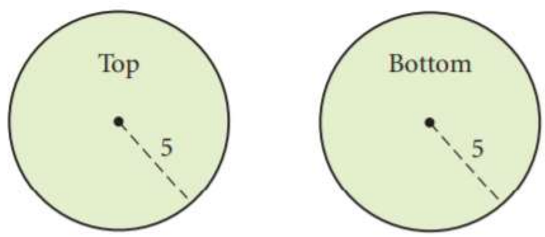
$$\mathbf{SA = 252\ units^2}$$

Cylinder Surface Area:

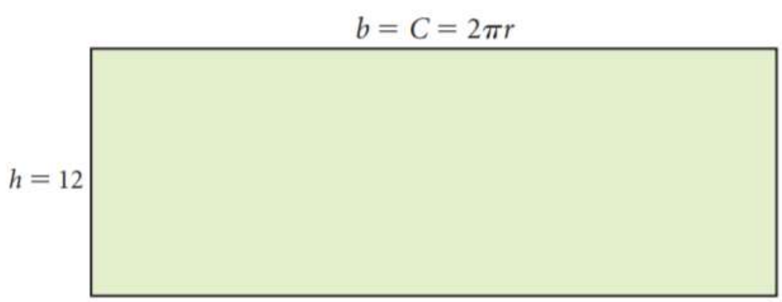
Find the surface area of the cylinder.



Imagine cutting apart the cylinder. The two bases are circular regions, so you need to find the areas of two circles. Think of the lateral surface as a wrapper. Slice it and lay it flat to get a rectangular region. You'll need the area of this rectangle. The height of the rectangle is the height of the cylinder. The base of the rectangle is the circumference of the circular base.



Bases

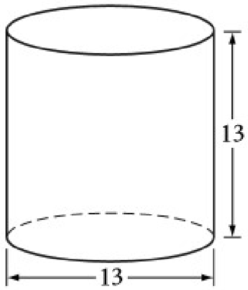


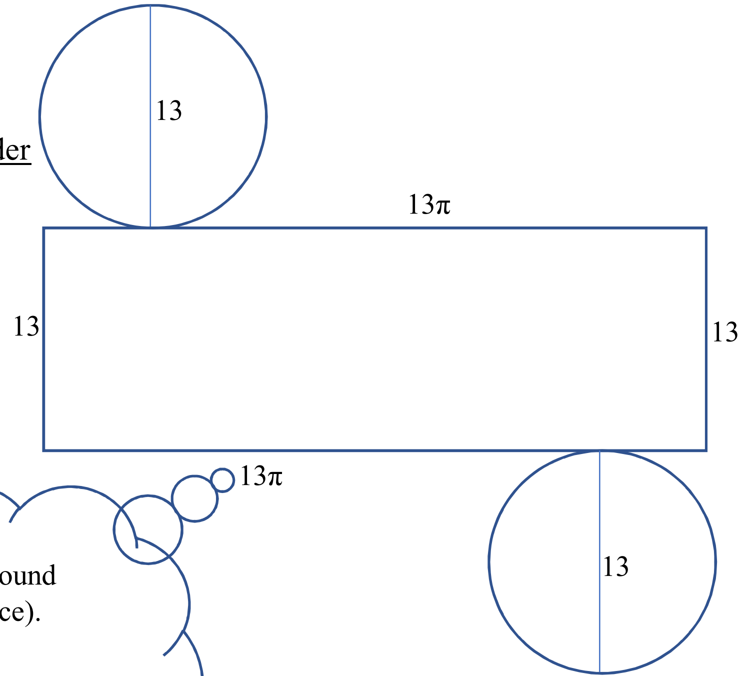
Lateral surface

$$\begin{aligned} \text{surface area} &= 2(\pi r^2) + (2\pi r)h \\ &= 2(\pi \cdot 5^2) + (2 \cdot \pi \cdot 5) \cdot 12 \\ &\approx 534 \end{aligned}$$

The surface area of the cylinder is about 534 in^2.

Example 3: Surface Area of a Cylinder





This length will wrap around the circle (circumference).

$$C = \pi d$$

$$C = \pi(13)$$

$$C = 13\pi$$

The 13 is a diameter. The radius of the circle is half of that, or 6.5.

$$A = \pi r^2$$

$$A = \pi(6.5)^2$$

$$A = 42.25\pi$$

$$A = bh$$

$$A = (13\pi)(13)$$

$$A = 169\pi$$

$$A = \pi r^2$$

$$A = \pi(6.5)^2$$

$$A = 42.25\pi$$

$SA = 2(42.25\pi) + 169\pi$

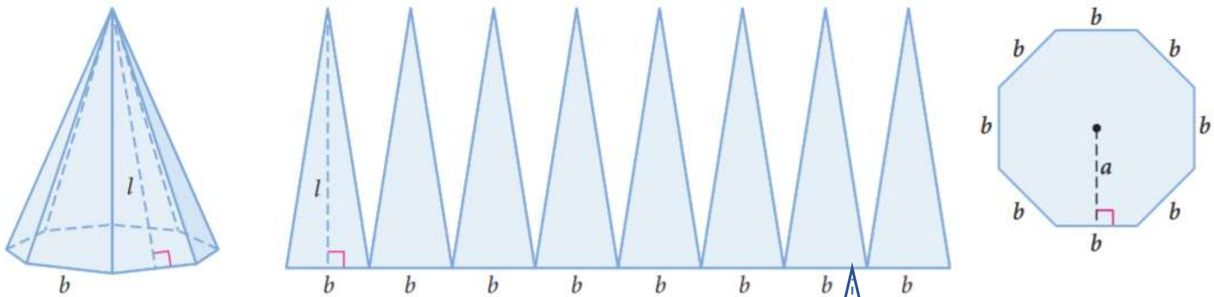
$SA = 84.5\pi + 169\pi$

$SA = 253.5\pi$ units^2

You can choose to multiply π out and get an approximation. If you do, you should calculate that $SA \approx 796.39$ units^2.

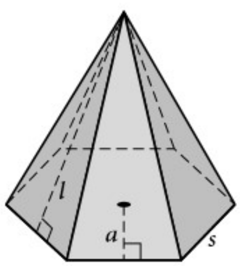
Surface Area of a Regular Pyramid:

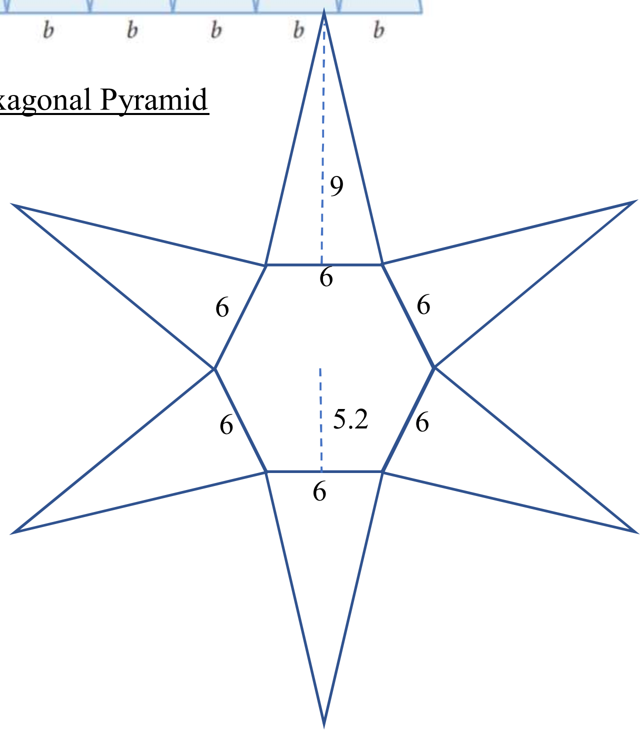
You can cut and unfold the surface of a regular pyramid into these shapes.

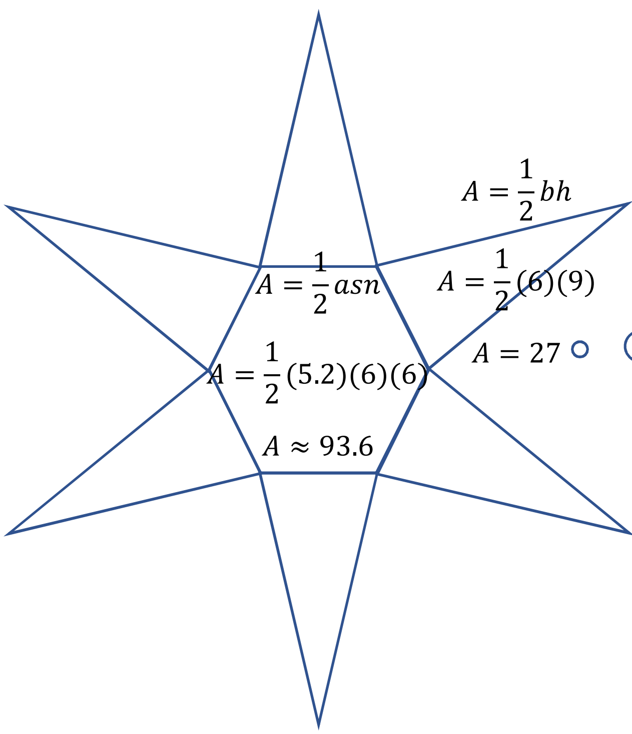


Example 4: Surface Area of a Regular Hexagonal Pyramid

Base is a regular hexagon.
$s = 6$, $a \approx 5.2$, and $l = 9$.







$A = \frac{1}{2}asn$

$A = \frac{1}{2}(5.2)(6)(6)$

$A \approx 93.6$

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(6)(9)$

$A = 27$

All six triangular sides will have the exact same area.

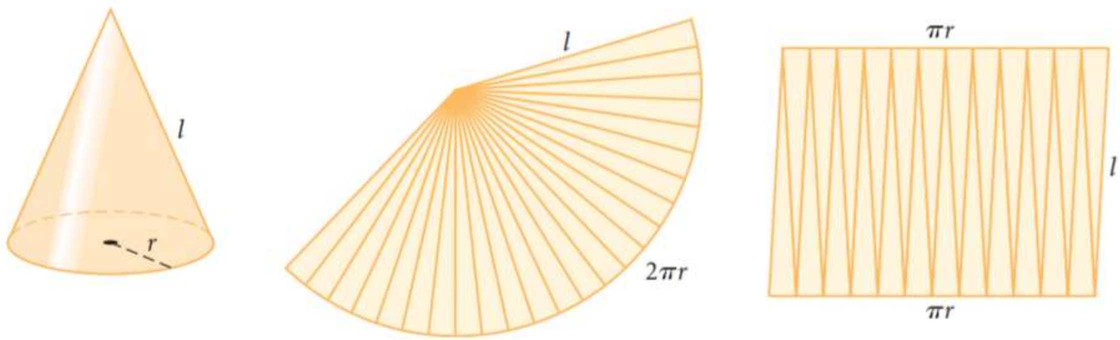
$SA \approx 6(27) + 93.6$

$SA \approx 162 + 93.6$

$\boldsymbol{SA \approx 255.6}$ **units**^{2}

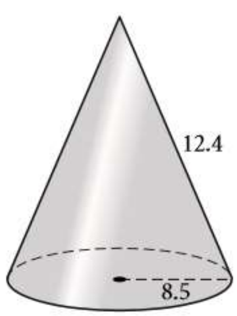
Surface Area of a Cone:

As the number of faces of a pyramid increases, it begins to look like a cone. You can think of the lateral surface as many small triangles or as a sector of a circle.



***Cones are one of those things that is tough to visualize how to get all of the pieces, so I'm going to give you a formula for surface area of a cone. $SA = \pi r l + \pi r^2$

<u>Example 5: Surface Area of a Cone</u>



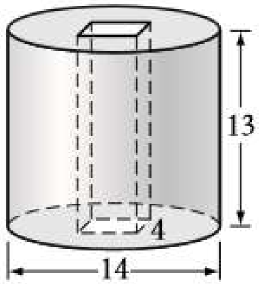
$SA = \pi r l + \pi r^2$

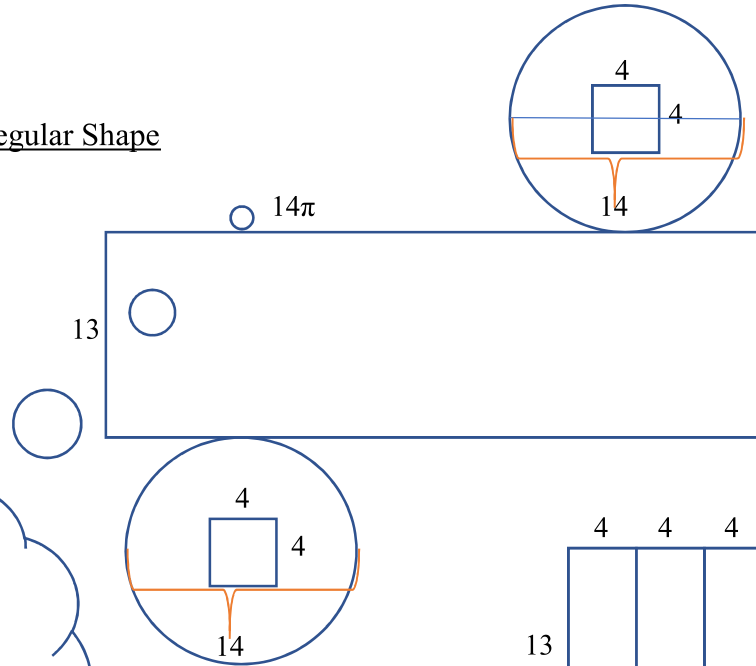
$SA = \pi(8.5)(12.4) + \pi(8.5)^2$

$SA = \pi(105.4) + \pi(72.25)$

$\boldsymbol{SA = 177.65\pi \approx 558.10}$ **units²**

Example 6: Surface Area of an Irregular Shape



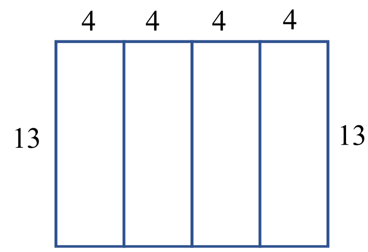


This length will wrap around the circle (circumference).

$C = \pi d$

$C = \pi(14)$

$C = 14\pi$



$A = \pi r^2 - bh$

$A = \pi(14)^2 - (4)(4)$

$A = 196\pi - 16$

The area of this figure is the area of the circle minus the area of the square inside.

$A = bh$

$A = (14\pi)(13)$

$A = 182\pi$

$A = \pi r^2 - bh$

$A = \pi(14)^2 - (4)(4)$

$A = 196\pi - 16$

$A = bh$

$A = (4)(13)$

$A = 52$

All four of these rectangles have the same area.

$SA = 182\pi + 2(196\pi - 16) + 4(52)$

$SA = 182\pi + 392\pi - 32 + 208$

$SA = (574\pi + 176)$ units^2

Or, as an approximation…

$SA \approx 1803.27 + 176 \approx 1979.3$ units^2