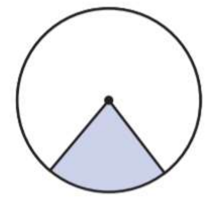
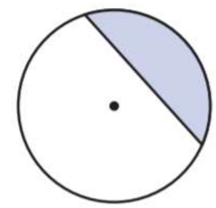
Lesson 8.6 – Areas of Slices of Circles

A sector of a circle is the region between two radii and an arc of the circle.



Sector of a circle

A segment of a circle in the region between a chord and an arc of a circle.

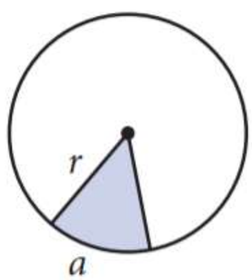


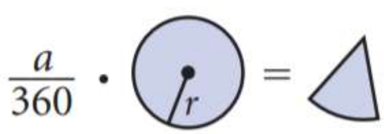
Segment of a circle

An annulus is the region between two concentric circles.

Annulus

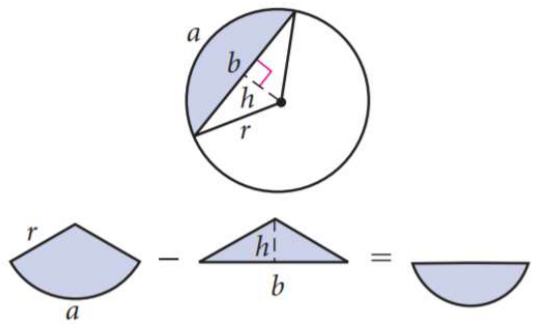
Sector Area Conjecture – The area of a sector is given by the formula $A_{sector} = \frac{a}{360}\pi r^2$ where A is the area, a is the arc measure, and r is the radius of the circle.





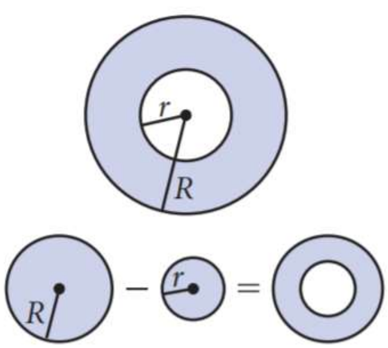
$$\frac{a}{360} \cdot \pi r^2 = A_{\text{sector}}$$

Segment Area Conjecture – The area of a segment is given by the formula $A_{segment} = \frac{a}{360}\pi r^2 - \frac{1}{2}bh$ where A is the area, a is the arc measure, r is the radius of the circle, b is the length of the chord, and h is the height of the triangle.



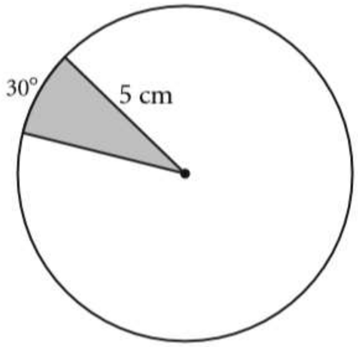
$$\frac{a}{360}\pi r^2 - \frac{1}{2}bh = A_{\text{segment}}$$

Annulus Area Conjecture – The area of an annulus is given by the formula $A_{annulus} = \pi R^2 - \pi r^2$ where A is the area, R is the radius of the large circle, and r is the radius of the small circle.



$$\pi R^2 - \pi r^2 = A_{\text{annulus}}$$

Example 1: Area of a sector

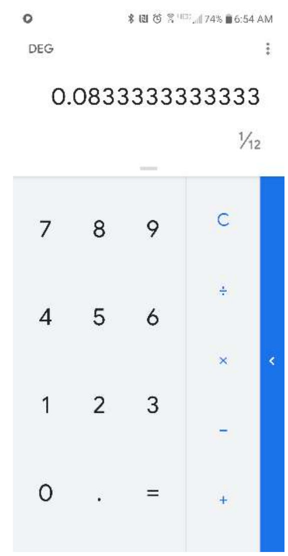


$$A_{sector} = \frac{a}{360}\pi r^2$$

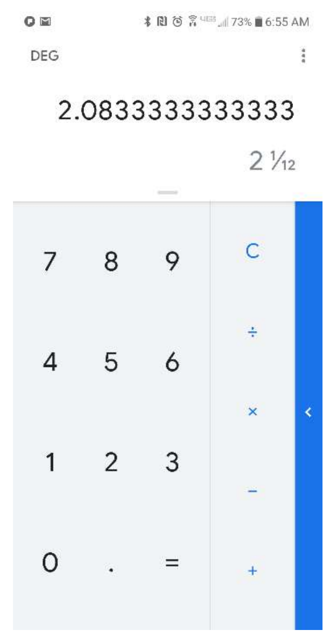
$a = 30, r = 5$

$$A = \frac{30}{360}\pi(5)^2$$

$A = \frac{1}{12}\pi(25)$ **$\frac{30 \div 30}{360 \div 30} = \frac{1}{12}$. If your calculator works like my phone $30 \div 360\ldots$



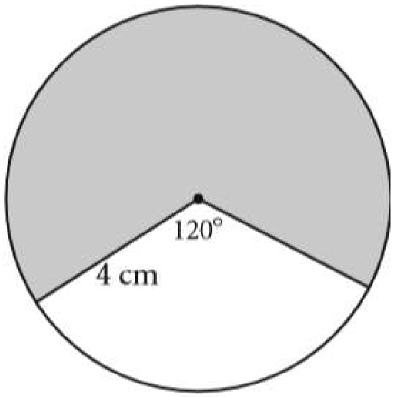
$A = \frac{1}{12}(25)\pi$ **$\frac{1}{12} \cdot \frac{25}{1} = \frac{25}{12}$. If you keep the $\frac{1}{12}$ in your calculator and multiply by 25…



$A = \frac{25}{12}\pi$ or $A = 2\frac{1}{12}\pi$

$\boldsymbol{A = \frac{25}{12}\pi}$ **cm²** or $\boldsymbol{A = 2\frac{1}{12}\pi}$ **cm²**

Example 2: Area of a sector



Remember that we are finding the area of the shaded region. So, the central angle of the shaded region is $360° - 120° = 240°$. The measures of the arc is equal to the measure of the central angle that forms it. So, $a = 240°$.

$$A_{sector} = \frac{a}{360}\pi r^2$$

$a = 240, r = 4$

$$A = \frac{240}{360}\pi(4)^2$$

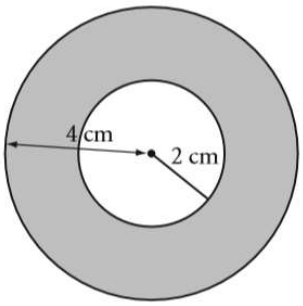
$A = \frac{2}{3}\pi(16)$ **$\frac{240 \div 120}{360 \div 120} = \frac{2}{3}$.

$A = \frac{2}{3}(16)\pi$ **$\frac{2}{3} \cdot \frac{16}{1} = \frac{32}{3}$.

$A = \frac{32}{3}\pi$ or $A = 10\frac{2}{3}\pi$

$A = \frac{32}{3}\pi$ cm² or **$A = 10\frac{2}{3}\pi$ cm²**

Example 3: Area of an annulus



$A_{annulus} = \pi R^2 - \pi r^2$

$R = 4, r = 2$

$A = \pi(4)^2 - \pi(2)^2$

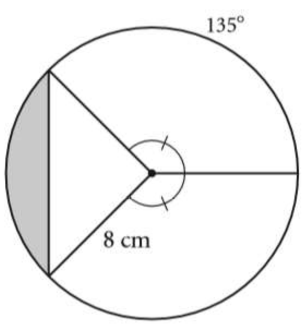
$A = \pi(16) - \pi(4)$

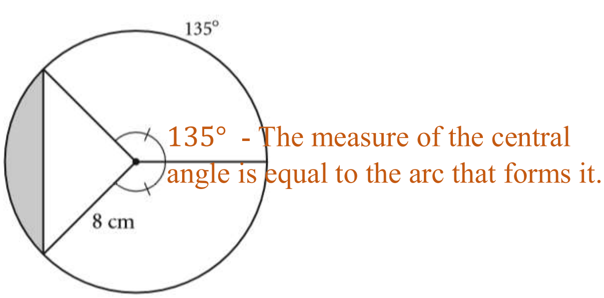
$A = 16\pi - 4\pi$ **Since both of these terms have π, we can combine them.

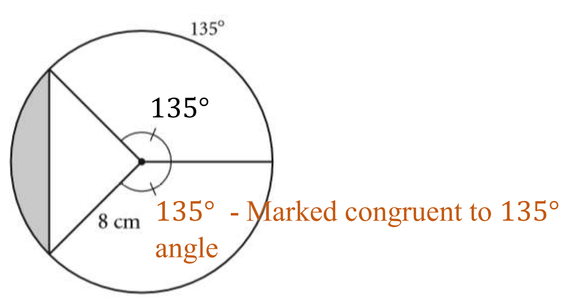
$A = 12\pi$

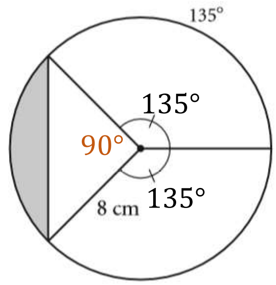
$\boldsymbol{A = 12\pi}$ **cm²**

Example 4: Area of a segment



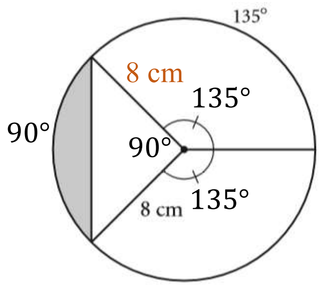






To find the missing central angle: $360° - 135° - 135° = 90°$

So, the arc of the segment is 90°.



The other radius of the circle must also be 8 cm.

Since the two radii of the circle meet at right angles, they can be the base and height of the triangle.

$$A_{segment} = \frac{a}{360}\pi r^2 - \frac{1}{2}bh$$

$a = 90, r = 8, b = 8, h = 8$

$$A = \frac{90}{360}\pi(8)^2 - \frac{1}{2}(8)(8)$$

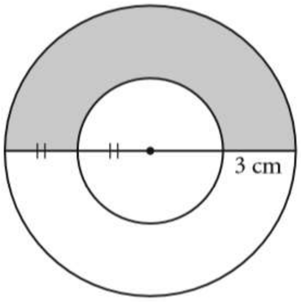
$A = \frac{1}{4}\pi(64) - \frac{1}{2}(64)$ **$\frac{90 \div 90}{360 \div 90} = \frac{1}{4}$.

$$A = \frac{1}{4}(64)\pi - 32$$

$A = 16\pi - 32$ **Since these terms do not both have π, they are not like terms and we must keep them separate in our answer.

$\boldsymbol{A = (16\pi - 32)\ \textbf{cm}^2}$

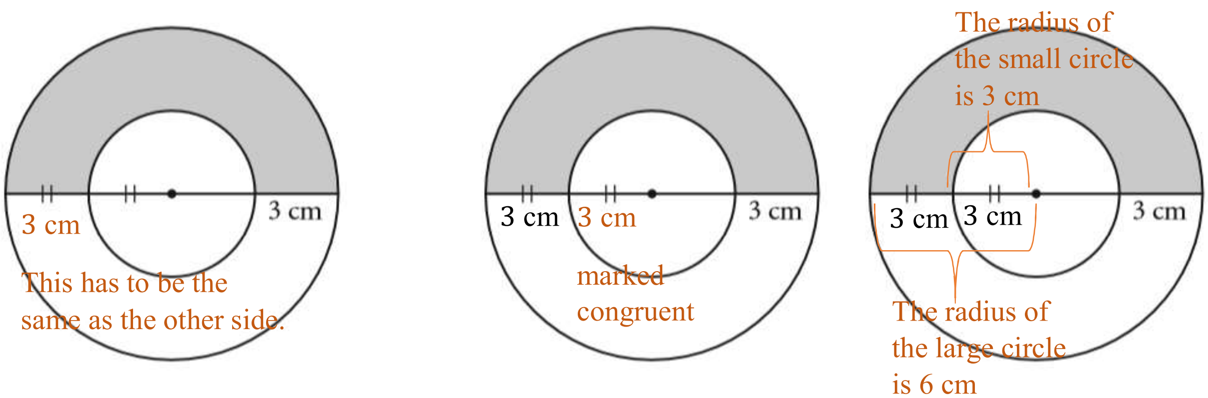
Example 5: Area of a sector of an annulus



The formula for an annulus applies, but we don't want the area of the entire annulus. We want a portion of the annulus. So, we will modify the annulus area conjecture ($A_{annulus} = \pi R^2 - \pi r^2$).

We want half of the annulus, or 180° of the 360° of the annulus $\left(\frac{180°}{360°} = \frac{1}{2}\right)$.

So, we will use the formula $A = \frac{180}{360}(\pi R^2 - \pi r^2)$ or $A = \frac{1}{2}(\pi R^2 - \pi r^2)$.



$$A = \frac{180}{360}(\pi R^2 - \pi r^2) = \frac{1}{2}(\pi R^2 - \pi r^2)$$

$$R = 6, r = 3$$

$$A = \frac{1}{2}(\pi(6)^2 - \pi(3)^2)$$

$$A = \frac{1}{2}\left(\pi(36) - \pi(9)\right)$$

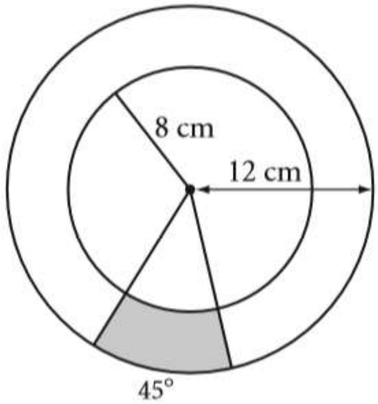
$$A = \frac{1}{2}(36\pi - 9\pi)$$

$$A = \frac{1}{2}(27\pi)$$

$$A = 13.5\pi$$

$$\boldsymbol{A = 13.5\pi\ \mathrm{cm}^2}$$

Example 6: Area of a sector of an annulus



We will modify the annulus area conjecture ($A_{annulus} = \pi R^2 - \pi r^2$) very similarly to Example 5. This time we only want 45° of the 360°.

$A = \dfrac{45}{360}(\pi R^2 - \pi r^2)$

$R = 12, r = 8$

$A = \frac{1}{8}(\pi(12)^2 - \pi(8)^2)$ **$\frac{45 \div 45}{360 \div 45} = \frac{1}{8}$

$A = \dfrac{1}{8}\left(\pi(144) - \pi(64)\right)$

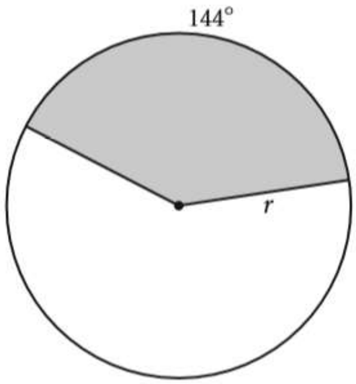
$A = \dfrac{1}{8}(144\pi - 64\pi)$

$A = \frac{1}{8}(80\pi)$ **$\frac{1}{8} \cdot \frac{80}{1} = \frac{80}{8} = 10$

$A = 10\pi$

$\boldsymbol{A = 10\pi}$ **cm²**

Example 7: Finding a radius given area of a sector



Shaded area is 40π cm^2. Find r.

$A_{sector} = \frac{a}{360}\pi r^2$

$A = 40\pi, a = 144$

$40\pi = \frac{144}{360}\pi r^2$

$\frac{40\pi}{\pi} = \frac{\frac{144}{360}\pi r^2}{\pi}$

$40 = \frac{144}{360}r^2$

$40 = \frac{2}{5}r^2$ **$\frac{144 \div 72}{360 \div 72} = \frac{2}{5}$

$\frac{5}{2} \cdot 40 = \frac{5}{2} \cdot \frac{2}{5}r^2$ **Multiplying both sides by the reciprocal (flip) of the fraction will cancel the fraction out.

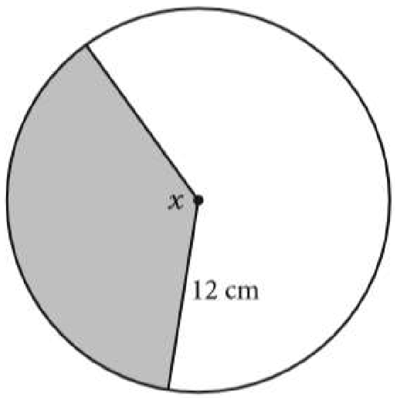
$16 = r^2$

$\sqrt{16} = \sqrt{r^2}$

$4 = r$

$\boldsymbol{r = 4}$ **cm**

Example 8: Finding the measure of a central angle given the area of a sector



Shaded area is 54π cm^2. Find x.

$$A_{sector} = \frac{a}{360}\pi r^2$$

$$A = 54\pi, r = 12$$

$$54\pi = \frac{x}{360}\pi(12)^2$$

$$\frac{54\pi}{\pi} = \frac{\frac{x}{360}\pi(12)^2}{\pi}$$

$$54 = \frac{x}{360}(12)^2$$

$$54 = \frac{x}{360} \cdot 144$$

$$360 \cdot 54 = 360 \cdot \frac{x}{360} \cdot 144$$

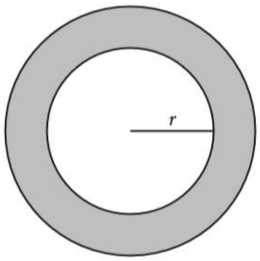
$$19440 = x \cdot 144$$

$$\frac{19440}{144} = \frac{x \cdot 144}{144}$$

$$135 = x$$

$$\boldsymbol{x = 135^\circ}$$

Example 9: Finding a missing radius given area of an annulus



Shaded area is 51π cm^2. The diameter of the larger circle is 20 cm. Find x.

The diameter of the larger circle is 20 cm. $r = \frac{20}{2} = 10$. So, the radius is 10 cm.

$A_{annulus} = \pi R^2 - \pi r^2$

$A = 51\pi, R = 10$

$51\pi = \pi(10)^2 - \pi r^2$

$51\pi = 100\pi - \pi r^2$

$\frac{51\pi}{\pi} = \frac{100\pi - \pi r^2}{\pi}$

$51 = 100 - r^2$ **Since there was a π in each term they can all cancel out

$-49 = -r^2$ **Both sides of the equation are negative, so we can turn them both positive.

$49 = r^2$

$\sqrt{49} = \sqrt{r^2}$

$7 = r$

$\boldsymbol{r}$ = 7 cm