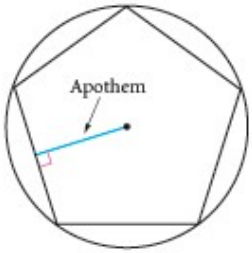
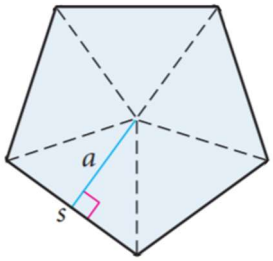
Lesson 8.4 – Areas of Regular Polygons

***Remember that a regular polygon is one that is equiangular (all angles are congruent) and equilateral (all sides are congruent)

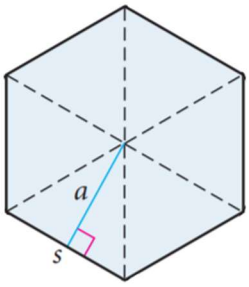
An **apothem** of a regular polygon is a perpendicular segment from the center of the polygon's circumscribed circle to a side of the polygon.



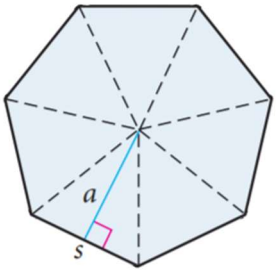
Regular Polygon Area Conjecture – The area of a regular polygon is given by the formula $A = \frac{1}{2}asn$ or $A = \frac{1}{2}aP$, where A is the area, P is the perimeter, a is the apothem, s is the length of each side, and n is the number of sides.



Regular pentagon



Regular hexagon



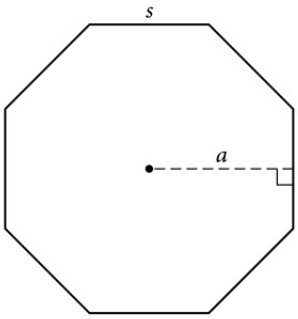
Regular heptagon

**Notice that there are two formulas for area of a regular polygon. Which one you use will depend on what information you are given, or what information you are asked for.

**As you go through this lesson, keep in mind there is a difference between A and a.

**You will see $\approx$ and $=$ as you go through these notes. $\approx$ means "approximately".

Example 1: Finding area of a regular polygon given apothem, side length, and number of sides



$s = 12$ cm $\quad a \approx 14.5$ cm $\quad A \approx$ ___?___

Because we are given side length, we will use $A = \frac{1}{2}asn$. We are explicitly given the side length and apothem length. To find the number of sides, we must count. The polygon is an octagon, so the number of sides is 8.

$a \approx 14.5, s = 12, n = 8$

$A \approx \frac{1}{2}(14.5)(12)(8)$ **To calculate this, you can turn the $\frac{1}{2}$ into a 0.5 and multiply (i.e. $0.5 \cdot 14.5 \cdot 12 \cdot 8$) or you can multiply the three non-fractional numbers and then divide by 2 (i.e. $\left(\frac{14.5 \cdot 12 \cdot 8}{2}\right)$). Both methods will give you the same answer.

$A \approx 696$

The area is about **696 cm²**.

Example 2: Finding area of a regular polygon given apothem and perimeter

Regular hexagon: $a \approx 15$ in and $P = 45$ in, $A \approx$ ___?___

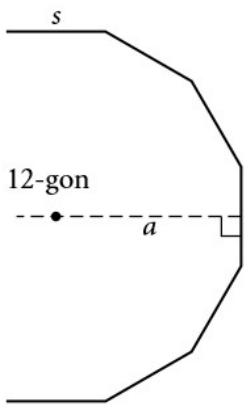
Because we are given perimeter, we will use $A = \frac{1}{2}aP$. This means that it doesn't actually matter that the figure is a hexagon or has 6 sides.

$A \approx \frac{1}{2}(15)(45)$

$A \approx 337.5$

The area is about **337.5 in²**.

Example 3: Finding apothem length given area, side length, and number of sides



$s = 4.2$ cm $\quad A \approx 197$ cm^2 $\quad a \approx$ ___?___

Since we are given side length, we will use $A = \frac{1}{2}asn$.

$A \approx 197, s = 4.2, n = 12$

$197 \approx \frac{1}{2}(a)(4.2)(12)$

$197 \approx \frac{1}{2}(4.2)(12)(a)$ **We can reorder the multiplication however we want, so I shifted all the numbers to the front of the multiplication problem and the variable to the end so that we can calculate.

$197 \approx 25.2(a)$ $**\frac{1}{2}(4.2)(12) = 25.2$

$\frac{197}{25.2} \approx \frac{25.2a}{25.2}$

$7.82 \approx a$

The apothem length is about **7.82 cm**.

Example 4: Finding side length given area, apothem length, and number of sides

Regular pentagon: $a \approx 1.2$ m, $A \approx 9$ m^2. Find s.

Since we are asked to find side length, we will use $A = \frac{1}{2}asn$. We know that the figure is a pentagon and has 5 sides.

$A \approx 9, a \approx 1.2, n = 5$

$9 = \frac{1}{2}(1.2)(s)(5)$

$9 = \frac{1}{2}(1.2)(5)(s)$

$9 = 3s$

$\frac{9}{3} = \frac{3s}{3}$

$3 = s$

The side length is **3 m**.

Example 5: Finding number of sides given area, apothem length, and side length

In a regular n-gon, $s = 4.8$ cm, $a \approx 7.4$ cm, and $A \approx 177.6$ cm^2. Find n.

Since we are given side length, we will use $A = \frac{1}{2}asn$.

$A \approx 177.6, a \approx 7.4, s = 4.8$

$177.6 = \frac{1}{2}(7.4)(4.8)(n)$

$177.6 = 17.76n$

$\frac{177.6}{17.76} = \frac{17.76n}{n}$

$10 = n$

The polygon has **10 sides**. **We are finding number of sides on this one, not a length.

Example 6: Finding apothem length given area and perimeter

Regular n-gon: $P = 45$ ft, $A \approx 126$ ft^2, $a \approx$ ___?___

Because we are given perimeter, we will use $A = \frac{1}{2}aP$. This means that it doesn't actually matter that how many sides the figure has.

$A \approx 126, P = 45$

$$126 = \frac{1}{2}(a)(45)$$

$$126 = \frac{1}{2}(45)(a)$$

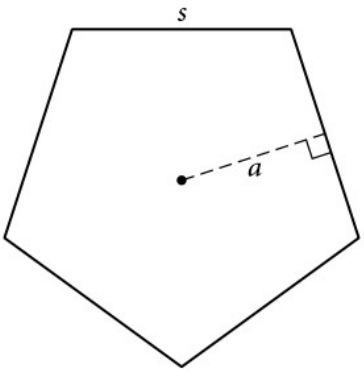
$$126 = 22.5a$$

$$\frac{126}{22.5} = \frac{22.5a}{a}$$

$$5.6 \approx a$$

The apothem length is approximately **5.6 ft**.

Example 7: Finding perimeter given area and apothem



$a = 6$ cm $A \approx 130.8$ cm^2 $P \approx$ ___?___

Because we are asked to find perimeter, we will use $A = \frac{1}{2}aP$. This means that it doesn't actually matter that the figure is a pentagon or has 5 sides.

$$A \approx 130.8, a = 6$$

$$130.8 = \frac{1}{2}(6)(P)$$

$$130.8 = 3P$$

$$\frac{130.8}{3} = \frac{3P}{3}$$

$$43.6 \approx P$$

The perimeter of the pentagon is approximately 43.6 cm.