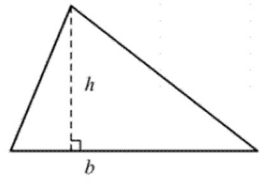
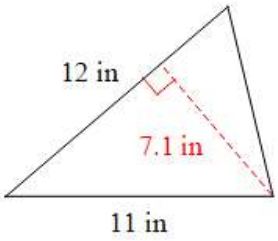
Lesson 8.2 – Areas of Triangles, Trapezoids, and Kites

***Remember to pay attention to units. Units of area should be squared (power of 2). Units of length like base, height, and perimeter should have a power of 1.

Triangle Area Conjecture - The area of a triangle is given by the formula $A = \frac{1}{2}bh$, where A is the area, b is the length of the base, and h is the height of the triangle. The base and height must be perpendicular.



Example 1: Finding area of a triangle given base and height



$A =$ __?___

***Remember that base and height of a triangle must be perpendicular (meet at a right angle). This means that the 11 in is just extra information that we don't need to use.

$A = \frac{1}{2}bh$

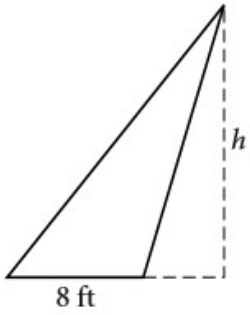
$b = 12, h = 7.1$

$A = \frac{1}{2}(12)(7.1)$ **To calculate this, you can turn the $\frac{1}{2}$ into a 0.5 and multiply (i.e. $0.5 \cdot 12 \cdot 7.1$) or you can multiply 12 and 7.1 and then divide by 2 (i.e. $\left(\frac{12 \cdot 7.1}{2}\right)$). Both methods will give you the same answer.

$A = 42.6$

The area is **42.6 in^2**.

Example 2: Finding height of a triangle given area and base



$A = 64$ ft^2 $h =$ ___?___

$A = \frac{1}{2}bh$

$A = 64, b = 8$

$64 = \frac{1}{2}(8)(h)$

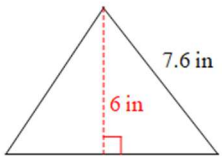
$64 = (4)(h)$ **We can multiply the $\frac{1}{2} \cdot 8$ as $0.5 \cdot 8$ or think of this as half of 8.

$\frac{64}{4} = \frac{(4)(h)}{4}$

$16 = h$

The height is **16 ft**.

Example 3: Finding base of a triangle given area and height



Area = 26.1 in^2 $b =$ ___?___

$A = \frac{1}{2}bh$

$A = 26.1, h = 6$

$26.1 = \frac{1}{2}(b)(6)$

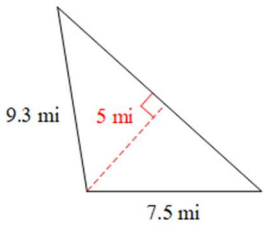
$26.1 = (3)(b)$ **We can reorder the multiplication in this problem so that our numbers are together and our variable is last $\left(\frac{1}{2}(6)(b)\right)$. This allows us to calculate as we did in Example 2.

$$\frac{26.1}{3} = \frac{(3)(b)}{3}$$

$$8.7 = b$$

The base is **8.7 in**.

Example 4: Finding perimeter of a triangle



Area = 29.5 mi²

$P =$?

We will need to use the area to find the remaining side which will help us find perimeter.

$$A = \frac{1}{2}bh$$

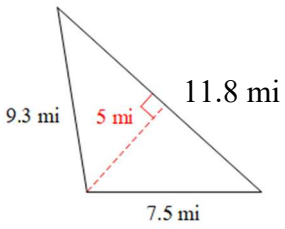
$$A = 29.5, h = 5$$

$$29.5 = \frac{1}{2}(b)(5)$$

$$29.5 = (2.5)(b)$$

$$\frac{29.5}{2.5} = \frac{(2.5)(b)}{2.5}$$

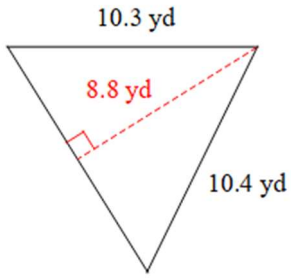
$$11.8 = b$$



$P = 9.3 + 7.5 + 11.8 = 28.6$

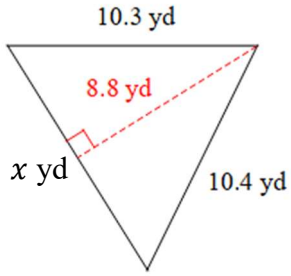
The perimeter is **28.6 mi**.

Example 5: Finding area of a triangle given perimeter, height, and two sides



$P = 31.7$ yd $\quad A =$?

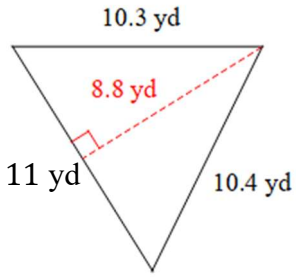
We can use the perimeter to find the length of the third side that will be the base of the triangle.



$$31.7 = 10.3 + 10.4 + x$$

$$31.7 = 20.7 + x$$

$$11 = x$$



$$A = \frac{1}{2}bh$$

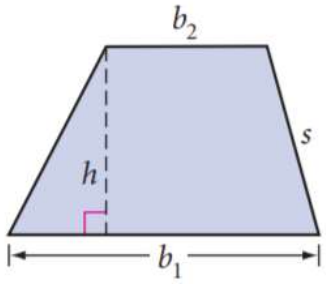
$$b = 11, h = 8.8$$

$$A = \frac{1}{2}(11)(8.8)$$

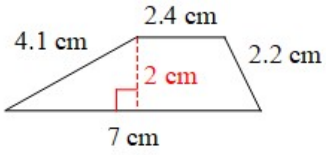
$$A = 48.4$$

The area is **48.4 yd²**.

Trapezoid Area Conjecture - The area of a trapezoid is given by the formula $A = \frac{1}{2}(b_1 + b_2)h$, where A is the area, b_1 and b_2 are the lengths of the bases, and h is the height of the trapezoid. The height must be perpendicular to both bases of the trapezoid.



Example 6: Finding area of a trapezoid



$A =$?

The bases of a trapezoid are always the parallel sides. So, the bases will be the 2.4 cm and 7 cm sides. The other two sides are just extra, an unnecessary, information.

$A = \frac{1}{2}(b_1 + b_2)h$

$b_1 = 7, b_2 = 2.4, h = 2$ **It doesn't matter which base you call b_1 and which base you call b_2

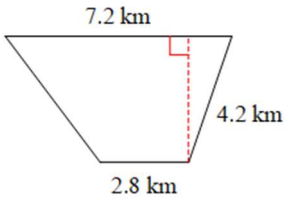
$A = \frac{1}{2}(7 + 2.4)(2)$

$A = \frac{1}{2}(9.4)(2)$ **To calculate this, you can turn the $\frac{1}{2}$ into a 0.5 and multiply (i.e. $0.5 \cdot 9.4 \cdot 2$) or you can multiply 9.4 and 2 and then divide by 2 (i.e. $\left(\frac{9.4 \cdot 2}{2}\right)$). Both methods will give you the same answer.

$A = 9.4$

The area is **9.4 cm^2**.

Example 7: Finding height of a trapezoid



Area = 20 km² $h =$?

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$A = 20, b_1 = 2.8, b_2 = 7.2$$

$$20 = \frac{1}{2}(2.8 + 7.2)h$$

$$20 = \frac{1}{2}(10)h$$

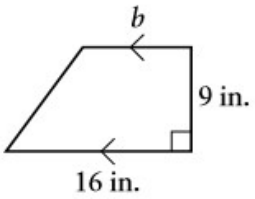
$$20 = 5h$$

$$\frac{20}{5} = \frac{5h}{5}$$

$$4 = h$$

The height of the trapezoid is **4 km**.

Example 8: Finding a missing base of a trapezoid



$A = 126$ in² $b =$?

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$A = 126, b_1 = 16, h = 9$$

$$126 = \frac{1}{2}(16 + b)(9)$$

$$126 = \tfrac{1}{2}(9)(16 + b)$$ **We can switch the order of the multiplication

$126 = 4.5(16 + b)$ **Calculate $\frac{1}{2} \cdot 9$.

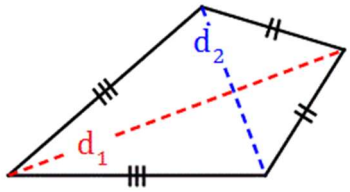
$\frac{126}{4.5} = \frac{4.5(16 + b)}{4.5}$

$28 = 16 + b$

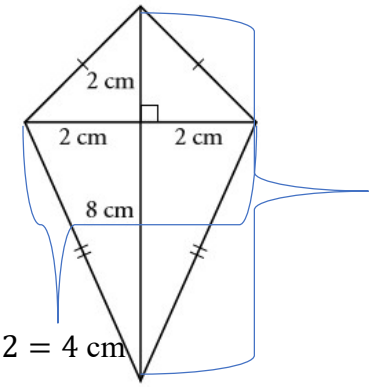
$12 = b$

The base is **12 in**.

Kite Area Conjecture - The area of a kite is given by the formula $A = \frac{1}{2} d_1 d_2$, where A is the area, d_1 and d_2 are the lengths of the diagonals.



Example 9: Finding area of a kite



$d_1 = 8 + 2 = 10$ cm

$d_2 = 2 + 2 = 4$ cm

$A =$ __?___

$A = \frac{1}{2} d_1 d_2$

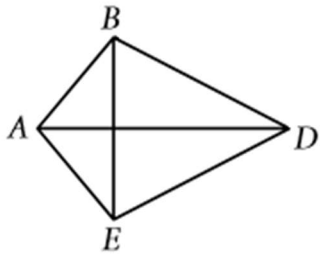
$d_1 = 10, d_2 = 4$

$A = \frac{1}{2}(10)(4)$

$A = 20$

The area is **20 cm²**.

Example 10: Finding a missing diagonal of a kite



$ABDE$ is a kite. $A = 120$ in^2 $AD = 60$ in $BE =$___?___

$A = \frac{1}{2} d_1 d_2$

$A = 120, d_1 = 60$

$120 = \frac{1}{2}(60)(d_2)$

$120 = 30(d_2)$

$\frac{120}{30} = \frac{30d_2}{30}$

$40 = d_2$

$\boldsymbol{BE = 40}$ **in.**