

Definition of a Monomial: A monomial is a number, a variable, or a product of a number and one or more variables

**Monomials that are real numbers are called constants.

Expression	Monomial?	Reason
-5	yes	-5 is a real number and an example of a constant.
$p + q$	no	The expression involves the addition, not the product, of two variables.
x	yes	Single variables are monomials.

Check your progress:

Expression	Monomial?	Reason
$-x + 5$	NO	The expression involves the addition, not the product, of two variables.
$23abcd^2$	YES	The expression involves the product of a number and multiple variables.
$\frac{xyz^3}{2}$	YES	The expression involves the product of a number (dividing by two is just multiplying by $\frac{1}{2}$) and multiple variables.
$\frac{ab}{c}$	NO	The expression involves the quotient of variables.

Exponent and Base:

$$\begin{array}{c}
 \text{exponent} \downarrow \\
 2^5 = \overbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}^{5 \text{ factors}} \text{ or } 32 \\
 \text{base} \uparrow
 \end{array}$$

PROPERTIES OF EXPONENTS:

Name	Words	Symbols	Example	Justification
Product of Powers	To multiply two powers that have the same base, add their exponents.	$a^m \cdot a^n = a^{m+n}$	$a^4 \cdot a^{12} = a^{4+12} = a^{16}$	$2^3 \cdot 2^5 = \overbrace{2 \cdot 2 \cdot 2}^{3 \text{ factors}} \cdot \overbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}^{5 \text{ factors}} \text{ or } 2^8$ $3 + 5 \text{ or } 8 \text{ factors}$ $3^2 \cdot 3^4 = \overbrace{3 \cdot 3}^{2 \text{ factors}} \cdot \overbrace{3 \cdot 3 \cdot 3 \cdot 3}^{4 \text{ factors}} \text{ or } 3^6$ $2 + 4 \text{ or } 6 \text{ factors}$
Power of a Power	To find the power of a power, multiply the exponents.	$(a^m)^n = a^{m \cdot n}$	$(k^5)^9 = k^{5 \cdot 9} = k^{45}$	$(4^2)^5 = \overbrace{(4^2)(4^2)(4^2)(4^2)(4^2)}^{5 \text{ factors}}$ $= 4^{2+2+2+2+2} \leftarrow$ $= 4^{10}$

				$(z^8)^3 = \overbrace{(z^8)(z^8)(z^8)}^{3 \text{ factors}}$ $\longrightarrow = z^{8+8+8}$ $= z^{24}$
Power of a Product	To find the power of a product, find the power of each factor and multiply.	$(ab)^m = a^m b^m$	$(-2xy)^3$ $= (-2)^3 x^3 y^3$ $= -8x^3 y^3$	$(xy)^4 = (xy)(xy)(xy)(xy)$ $= (x \cdot x \cdot x \cdot x)(y \cdot y \cdot y \cdot y)$ $= x^4 y^4$ $(6ab)^3 = (6ab)(6ab)(6ab)$ $= (6 \cdot 6 \cdot 6)(a \cdot a \cdot a)(b \cdot b \cdot b)$ $= 6^3 a^3 b^3$ or $216a^3 b^3$

Example: Product of Powers

Simplify each expression.

a. $(5x^7)(x^6)$

$$(5x^7)(x^6) = (5)(1)(x^7)(x^6) \quad \text{Group the coefficients and the variables.}$$

$$= (5 \cdot 1)(x^{7+6}) \quad \text{Product of Powers}$$

$$= 5x^{13} \quad \text{Simplify.}$$

b. $(4ab^6)(-7a^2b^3)$

$$(4ab^6)(-7a^2b^3) = (4)(-7)(a \cdot a^2)(b^6 \cdot b^3) \quad \text{Group the coefficients and the variables.}$$

$$= -28(a^{1+2})(b^{6+3}) \quad \text{Product of Powers}$$

$$= -28a^3 b^9 \quad \text{Simplify.}$$

Check your progress:

- $(3y^4)(7y^5)$
 $(3 \cdot 7)(y^4 \cdot y^5) = (21)(y^{4+5}) = 21y^9$
- $(-4rs^2t^3)(-6r^5s^2t^3)$
 $(-4 \cdot -6)(r \cdot r^5)(s^2 \cdot s^2)(t^3 \cdot t^3) = (24)(r^{1+5})(s^{2+2})(t^{3+3}) = 24r^6 s^4 t^6$

Example: Power of a Power

Simplify $[(3^2)^3]^2$.

$$[(3^2)^3]^2 = (3^2 \cdot 3)^2 \quad \text{Power of a Power}$$

$$= (3^6)^2 \quad \text{Simplify.}$$

$$= 3^{6 \cdot 2} \quad \text{Power of a Power}$$

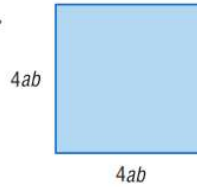
$$= 3^{12} \text{ or } 531,441 \quad \text{Simplify.}$$

Check your progress:

- $[(2^2)^2]^4$
 $[2^{2 \cdot 2}]^4 = [2^4]^4 = 2^{4 \cdot 4} = 2^{16}$

Example: Power of a Product**GEOMETRY** Express the area of the square as a monomial.

$$\begin{aligned}
 \text{Area} &= s^2 && \text{Formula for the area of a square} \\
 &= (4ab)^2 && \text{Replace } s \text{ with } 4ab. \\
 &= 4^2 a^2 b^2 && \text{Power of a Product} \\
 &= 16a^2 b^2 && \text{Simplify.}
 \end{aligned}$$

The area of the square is $16a^2b^2$ square units.Check your progress:

- 1) Express the area of a square with sides of length
- $2xy^2$
- as a monomial.

$$A = s^2$$

$$A = (2xy^2)^2$$

$$A = 2^2 x^2 (y^2)^2 = 4x^2 y^{2 \cdot 2} = 4x^2 y^4$$

To simplify an expression involving monomials, write an equivalent expression in which:

- each base appears exactly once,
- there are no powers of powers, and
- all fractions are in simplest form.

Example: Simplifying ExpressionsSimplify $(3xy^4)^2 [(-2y)^2]^3$.

$$\begin{aligned}
 (3xy^4)^2 [(-2y)^2]^3 &= (3xy^4)^2 (-2y)^6 && \text{Power of a Power} \\
 &= (3)^2 x^2 (y^4)^2 (-2)^6 y^6 && \text{Power of a Product} \\
 &= 9x^2 y^8 (64)y^6 && \text{Power of a Power} \\
 &= 9(64)x^2 \cdot y^8 \cdot y^6 && \text{Commutative Property} \\
 &= 576x^2 y^{14} && \text{Product of Powers}
 \end{aligned}$$

Check your progress:

1) $\left(\frac{1}{2}a^2b^2\right)^3 [(-4b)^2]^2$

$$\left(\frac{1}{2}\right)^3 (a^2)^3 (b^2)^3 (-4b)^{2 \cdot 2}$$

$$= \left(\frac{1}{8}\right) (a^{2 \cdot 3}) (b^{2 \cdot 3}) (-4b)^4$$

$$= \left(\frac{1}{8}\right) (a^6) (b^6) (-4)^4 (b^4) = \left(\frac{1}{8}\right) (a^6) (b^6) (256) (b^4)$$

$$= \left(\frac{1}{8} \cdot 256\right) (a^6) (b^6 \cdot b^4) = 32a^6 b^{6+4} = 32a^6 b^{10}$$

Practice

Determine whether the expression is a monomial. Write *yes* or *no*. Explain.

- 1) $5 - 7d$ No. The expression involves the difference, not the product.
- 2) $\frac{4a}{3b}$ No. The expression involves a quotient of variables.
- 3) n Yes. A single variable is a monomial.

Simplify.

4) $x(x^4)(x^6)$

$$x^{1+4+6} = x^{11}$$

5) $(4a^4b)(9a^2b^3)$

$$(4 \cdot 9)(a^4 \cdot a^2)(b \cdot b^3) = 36(a^{4+2})(b^{1+3}) \\ = 36a^6b^4$$

6) $[(2^3)^2]^3$

$$[2^{3 \cdot 2}]^3 = [2^6]^3 = 2^{6 \cdot 3} \\ = 2^{18}$$

7) $[(3^2)^2]^2$

$$[3^{2 \cdot 2}]^2 = [3^4]^2 = 3^{4 \cdot 2} \\ = 3^8$$

8) $(3y^5z)^2$

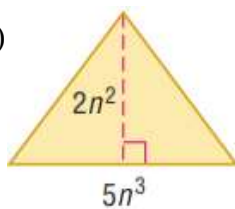
$$(3^2)(y^{5 \cdot 2})(z^2) = 9y^{10}z^2$$

9) $(-2f^2g)^3$

$$(-2)^3(f^{2 \cdot 3})(g^3) = -8f^6g^3$$

Express the area of the triangle as a monomial. Reminder: area of a triangle is $A = \frac{1}{2}bh$.

10)



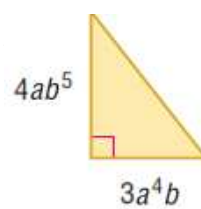
$$A = \frac{1}{2}(5n^3)(2n^2)$$

$$= \frac{1}{2}(5 \cdot 2)(n^3 \cdot n^2)$$

$$= \frac{1}{2}(10)(n^{3+2})$$

$$= 5n^5$$

11)



$$= \frac{1}{2}(3a^4b)(4ab^5)$$

$$A = \frac{1}{2}(3 \cdot 4)(a^4 \cdot a)(b \cdot b^5)$$

$$= \frac{1}{2}(12)(a^{4+1})(b^{1+5})$$

$$= 6a^5b^6$$

Simplify.

12) $(-2v^3w^4)^3(-3vw^3)^2$

$$(-2)^3(v^3)^3(w^4)^3(-3)^2(v)^2(w^3)^2$$

$$= -8(v^{3 \cdot 3})(w^{4 \cdot 3})(9)(v^2)(w^{3 \cdot 2})$$

$$= (-8)(v^9)(w^{12})(9)(v^2)(w^6)$$

$$= (-8 \cdot 9)(v^9 \cdot v^2)(w^{12} \cdot w^6)$$

$$= (-72)(v^{9+2})(w^{12+6})$$

$$= -72v^{11}w^{18}$$

13) $(5x^2y)^2(2xy^3z)^3(4xyz)$

$$(5)^2(x^2)^2(y)^2(2)^3(x)^3(y^3)^3(z)^3(4xyz)$$

$$= (25)(x^{2 \cdot 2})(y^2)(8)(x^3)(y^{3 \cdot 3})(z^3)(4xyz)$$

$$= (25)(x^4)(y^2)(8)(x^3)(y^9)(z^3)(4xyz)$$

$$= (25 \cdot 8 \cdot 4)(x^4 \cdot x^3 \cdot x)(y^2 \cdot y^9 \cdot y)(z^3 \cdot z)$$

$$= 800(x^{4+3+1})(y^{2+9+1})(z^{3+1})$$

$$= 800x^8y^{12}z^4$$