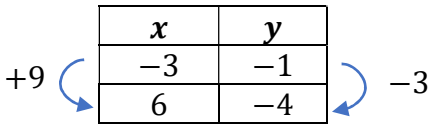
Writing Equations in Slope-Intercept Form Given Two Points

**Remember that slope-intercept form is $y = mx + b$ where m is slope and b is the y-intercept.

Example 1: Write an Equation Given Two Points

Write an equation of a line that passes through $(-3, -1)$ and $(6, -4)$

The first thing we have to do when given two points is calculate slope.



$$\frac{\Delta y}{\Delta x} = \frac{-3}{9} = -\frac{1}{3}$$

The slope (m) is $-\frac{1}{3}$.

$y = mx + b$

We can replace m with the slope we just calculated.

$$y = -\frac{1}{3}x + b$$

We can choose to use either of the two points: ***You do not need to plug both in. You pick one and use that one. I'm using both to show that it doesn't matter which you choose.***

If we use $(-3, -1)$:

Replace x with -3.

Replace y with -1.

$-1 = -\frac{1}{3}(-3) + b$

Multiply the $-\frac{1}{3} \cdot -3$.

$-1 = 1 + b$

Solve for b:

$-1 = 1 + b$

$-1 \quad -1$

$-2 = b$

If we use $(6, -4)$:

Replace x with 6.

Replace y with -4.

$-4 = -\frac{1}{3}(6) + b$

Multiply the $-\frac{1}{3} \cdot 6$.

$-4 = -2 + b$

Solve for b:

$-4 = -2 + b$

$+2 \quad +2$

$-2 = b$

Notice that we get the same y-intercept regardless of which point we choose to plug in for x and y.

Our final answer is when we write the equation of the line with both the slope and y-intercept replaced.

$y = -\frac{1}{3}x - 2$ **This is our solution

Example 2: Write an Equation Given Two Points

Write an equation of a line that passes through $(5, 1)$ and $(8, -2)$

The first thing we have to do when given two points is calculate slope.

	x	y	
$+3$	5	1	-3
	8	-2	

$$\frac{\Delta y}{\Delta x} = \frac{-3}{3} = -1$$

The slope (m) is -1.

$y = mx + b$

We can replace m with the slope we just calculated.

$y = -1x + b$

We can choose to use either of the two points: ***You do not need to plug both in. You pick one and use that one. I'm using both to show that it doesn't matter which you choose.***

If we use $(5, 1)$:

Replace x with 5.

Replace y with 1.

$1 = -1(5) + b$

Multiply the $-1 \cdot 5$.

$1 = -5 + b$

If we use $(8, -2)$:

Replace x with 8.

Replace y with -2.

$-2 = -1(8) + b$

Multiply the $-1 \cdot 8$.

$-2 = -8 + b$

Solve for b:

$1 = -5 + b$

$+5 \ +5$

$6 = b$

Solve for b:

$-2 = -8 + b$

$+8 \ \ +8$

$6 = b$

Notice that we get the same y-intercept regardless of which point we choose to plug in for x and y.

Our final answer is when we write the equation of the line with both the slope and y-intercept replaced.

$y = -1x + 6$ ** A $-1x$ is the same as $-x$

$y = -x + 6$ **This is our solution

Example 3: Write an Equation Given Two Points

Write an equation of a line that passes through $(6, 0)$ and $(0, 4)$

The first thing we have to do when given two points is calculate slope.

x	y
6	0
0	4

-6 $+4$

$$\frac{\Delta y}{\Delta x} = \frac{4}{-6} = -\frac{2}{3}$$

The slope (m) is $-\frac{2}{3}$.

$y = mx + b$

We can replace m with the slope we just calculated.

$$y = -\frac{2}{3}x + b$$

We can choose to use either of the two points: ***You do not need to plug both in. You pick one and use that one. I'm using both to show that it doesn't matter which you choose.***

If we use $(6, 0)$:

Replace x with 6.

Replace y with 0.

If we use $(0, 4)$:

Replace x with 0.

Replace y with 4.

$0 = -\frac{2}{3}(6) + b$

Multiply the $-\frac{2}{3} \cdot 6$.

$0 = -4 + b$

Solve for b:

$0 = -4 + b$

$+4 \quad +4$

$4 = b$

$4 = -\frac{2}{3}(0) + b$

Multiply the $-\frac{2}{3} \cdot 0$.

$4 = 0 + b$

Solve for b:

$4 = 0 + b$

$4 = b$

Notice that we get the same y-intercept regardless of which point we choose to plug in for x and y.

Our final answer is when we write the equation of the line with both the slope and y-intercept replaced.

$y = -\frac{2}{3}x + 4$ **This is our solution

Example 4: Write an Equation Given Two Points

Write an equation of a line that passes through $(5, 2)$ and $(-7, -4)$

The first thing we have to do when given two points is calculate slope.

x	y
5	2
-7	-4

-12 (change in x) $\quad -6$ (change in y)

$$\frac{\Delta y}{\Delta x} = \frac{-6}{-12} = \frac{1}{2}$$

The slope (m) is $\frac{1}{2}$.

$y = mx + b$

We can replace m with the slope we just calculated.

$y = \frac{1}{2}x + b$

We can choose to use either of the two points: ***You do not need to plug both in. You pick one and use that one. I'm using both to show that it doesn't matter which you choose.***

If we use $(5, 2)$:

Replace x with 5.

Replace y with 2.

$2 = \frac{1}{2}(5) + b$

Multiply the $\frac{1}{2} \cdot 5$.

$2 = 2\frac{1}{2} + b$

Solve for b:

$2 = 2\frac{1}{2} + b$

$-2\frac{1}{2} \quad -2\frac{1}{2}$

$-\frac{1}{2} = b$

If we use $(-7, -4)$:

Replace x with -7.

Replace y with -4.

$-4 = \frac{1}{2}(-7) + b$

Multiply the $\frac{1}{2} \cdot -7$.

$-4 = -3\frac{1}{2} + b$

Solve for b:

$-4 = -3\frac{1}{2} + b$

$+3\frac{1}{2} \quad +3\frac{1}{2}$

$-\frac{1}{2} = b$

Notice that we get the same y-intercept regardless of which point we choose to plug in for x and y.

Our final answer is when we write the equation of the line with both the slope and y-intercept replaced.

$y = \frac{1}{2}x - \frac{1}{2}$ **This is our solution

<u>Example 5: Write an Equation Given Two Points</u>

Write an equation of a line that passes through $(2, -3)$ and $(8, -3)$

The first thing we have to do when given two points is calculate slope.

	x	y	
$+6$	2	-3	$+0$
	8	-3	

$\frac{\Delta y}{\Delta x} = \frac{0}{6} = 0$

The slope (m) is 0.

Slopes of zero define horizontal lines whose equations are of the form $y = \#$

In both of the points $(2, -3)$ and $(8, -3)$, -3 is the y-value. So, our equation must be:

$y = -3$

Example 6: Write an Equation Given Two Points

Write an equation of a line that passes through $(6, -1)$ and $(6, 2)$

The first thing we have to do when given two points is calculate slope.

	x	y	
$+0$	6	-1	$+3$
	6	2	

$$\frac{\Delta y}{\Delta x} = \frac{3}{0} = undefined$$

The slope (m) is undefined.

Undefined slopes are vertical lines whose equations are $x = \#$.

In both of the points $(6, -1)$ and $(6, 2)$, 6 is the x-value. So, our equation must be:

$x = 6$