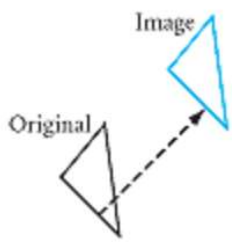
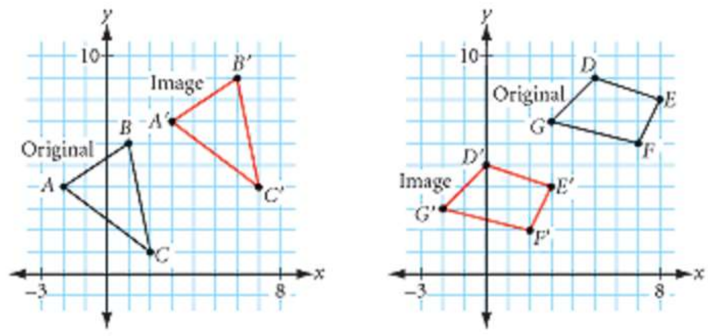
Lesson 7.1 – Rigid Transformations

Translation - sliding along a straight path without turning.

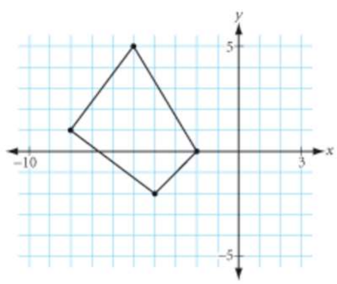


Translation

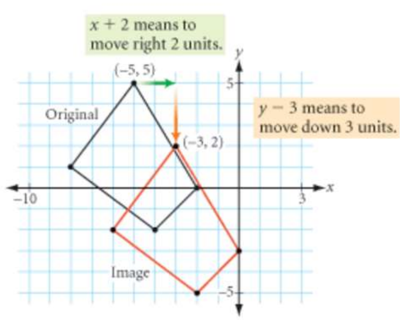


Translations on a coordinate grid

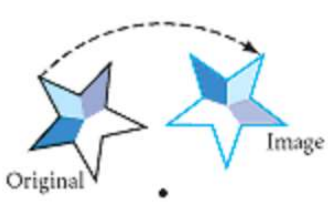
Transform the polygon at right using the rule $(x, y) \rightarrow (x + 2, y - 3)$. Describe the type and direction of the transformation, and write it as a vector.



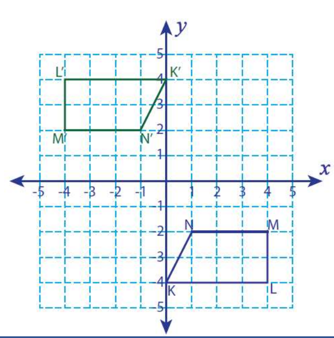
Apply the rule to each ordered pair. Every point of the polygon moves right 2 units and down 3 units. This is a translation by the vector $\langle 2, -3 \rangle$.



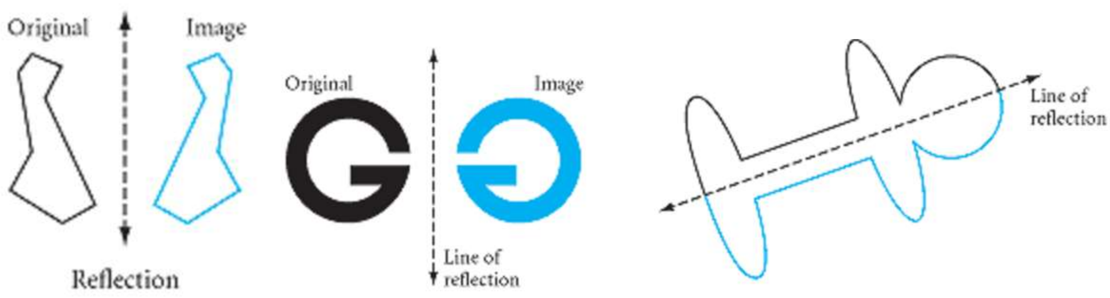
Rotation - all the points in the original figure rotate an identical number of degrees about a fixed center point.



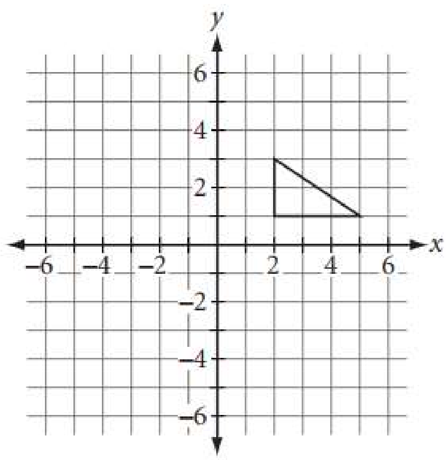
Rotation



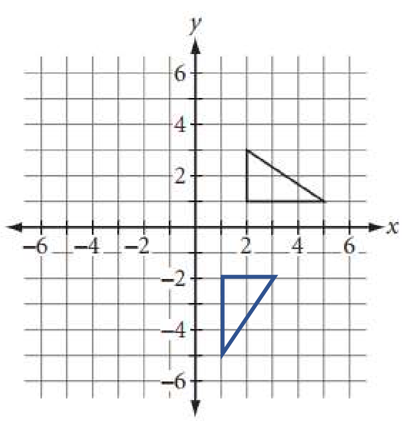
Reflection - produces a figure's "mirror image"



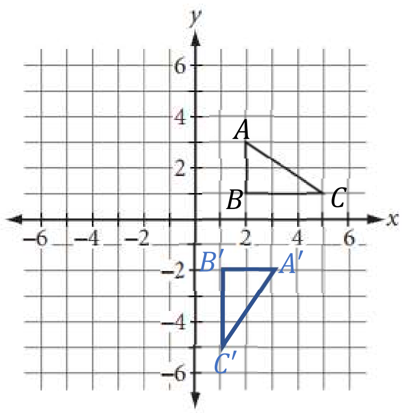
Example 1: Rotate the figure 90°. Write the ordered pair rule to represent the rotation.



To rotate the figure 90 degrees, we need to think about tracing it onto tracing paper, placing a pin in the origin (the point where the x- and y-axes cross) and rotating this figure clockwise 90 degrees. A clockwise rotation of 90 degrees would move this figure from the first quadrant into the fourth quadrant.



To determine the ordered pair rule, let's look at the change in the ordered pairs of each of the points. To efficiently do that, I'll label the angles. $\angle A$ rotated to $\angle A'$, $\angle B$ rotated to $\angle B'$, and $\angle C$ rotated to $\angle C'$.



$A(2,3) \rightarrow A'(3,-2)$

$B(2,1) \rightarrow B'(1,-2)$

$C(5,1) \rightarrow C'(1,-5)$

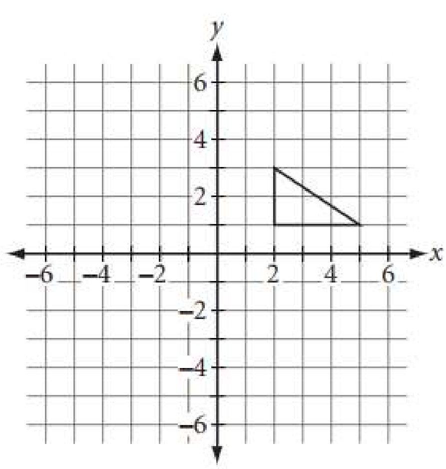
We can see that the x-coordinate of the original figure changed sign and moved to the y-coordinate spot. To change the sign of a coordinate, we place a negative in front of the variable.

So, our ordered pair rule starts as: $(x,y) \rightarrow (\quad ,-x)$

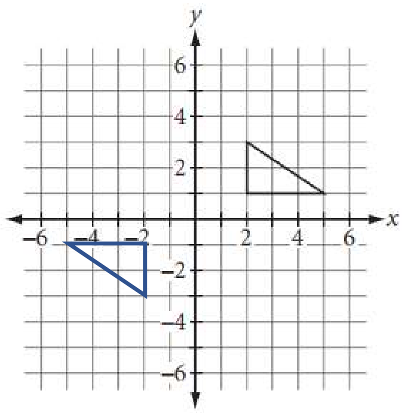
The y-coordinate of the original figure stays the same sign and moves into the x-coordinate spot.

This means our ordered pair rule is $\boldsymbol{(x,y) \rightarrow (y,-x)}$.

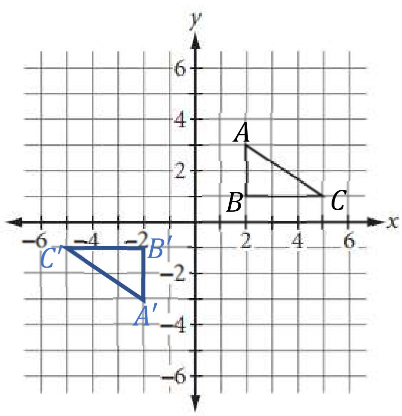
<u>Example 2: Rotate the figure 180°. Write the ordered pair rule to represent the rotation.</u>



To rotate the figure 180 degrees, we need to think about tracing it onto tracing paper, placing a pin in the origin (the point where the x- and y-axes cross) and rotating this figure clockwise 180 degrees. A clockwise rotation of 180 degrees would move this figure from the first quadrant into the third quadrant.



To determine the ordered pair rule, let's look at the change in the ordered pairs of each of the points. To efficiently do that, I'll label the angles. $\angle A$ rotated to $\angle A'$, $\angle B$ rotated to $\angle B'$, and $\angle C$ rotated to $\angle C'$.



$A(2, 3) \rightarrow A'(-2, -3)$

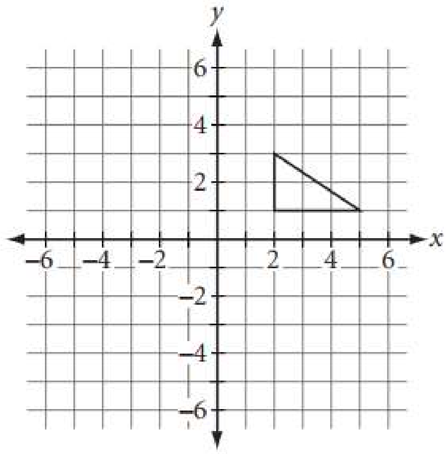
$B(2, 1) \rightarrow B'(-2, -1)$

$C(5, 1) \rightarrow C'(-5, -1)$

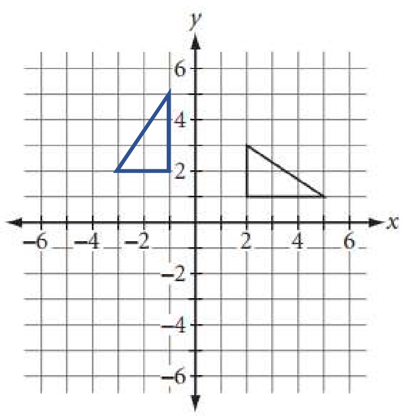
We can see that both the x-coordinate and the y-coordinate of the original figure changed sign, but not spot. To change the sign of a coordinate, we place a negative in front of the variable.

This means our ordered pair rule is $(\boldsymbol{x}, \boldsymbol{y}) \rightarrow (-\boldsymbol{x}, -\boldsymbol{y})$.

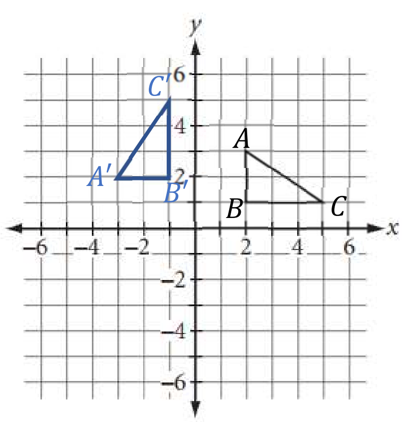
Example 3: Rotate the figure 270°. Write the ordered pair rule to represent the rotation.



To rotate the figure 270 degrees, we need to think about tracing it onto tracing paper, placing a pin in the origin (the point where the x- and y-axes cross) and rotating this figure clockwise 270 degrees. A clockwise rotation of 270 degrees would move this figure from the first quadrant into the second quadrant.



To determine the ordered pair rule, let's look at the change in the ordered pairs of each of the points. To efficiently do that, I'll label the angles. $\angle A$ rotated to $\angle A'$, $\angle B$ rotated to $\angle B'$, and $\angle C$ rotated to $\angle C'$.



$A(2,3) \rightarrow A'(-3,2)$

$B(2,1) \rightarrow B'(-1,2)$

$C(5,1) \rightarrow C'(-1,5)$

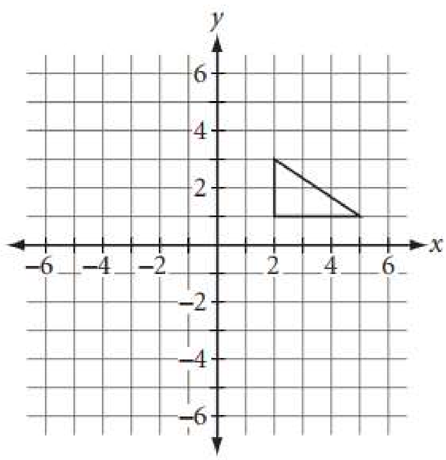
We can see that the x-coordinate of the original figure moved to the y-coordinate spot.

So, our ordered pair rule starts as: $(x,y) \rightarrow (\quad ,x)$

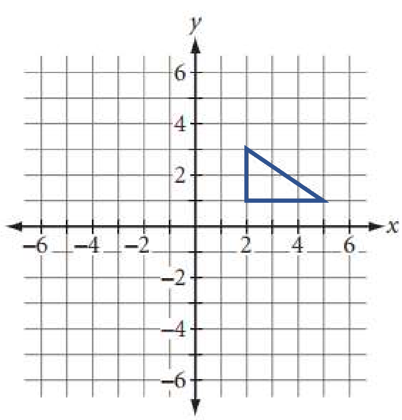
The y-coordinate of the original figure changed sign and moves into the x-coordinate spot. To change the sign of a coordinate, we place a negative in front of the variable.

This means our ordered pair rule is $\boldsymbol{(x,y) \rightarrow (-y,x)}$**.**

Example 4: Rotate the figure 360°. Write the ordered pair rule to represent the rotation.



To rotate the figure 360 degrees, it makes a full rotation and will be on top of itself.



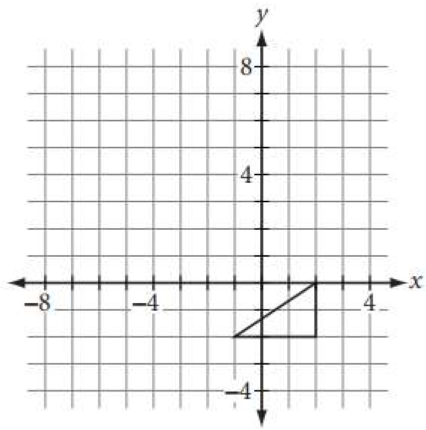
If a figure is on top of itself, the ordered pairs are the same for the original and the rotated image.

This means our ordered pair rule is $\boldsymbol{(x,y) \rightarrow (x,y)}$**.**

**The rules for rotations are the same regardless of where the figure starts.

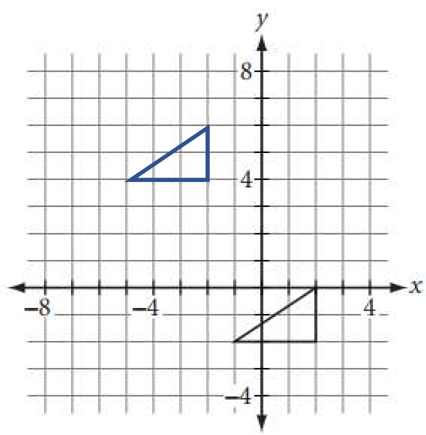
Example 5: Translate the triangle according to the ordered pair rule.

$(x, y) \rightarrow (x - 4, y + 6)$



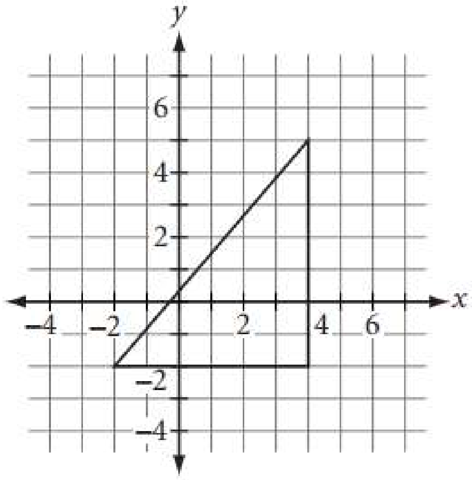
The $x - 4$ tells us to translate x-coordinates 4 units to the left (left because it is negative).
**Remember x-coordinates are horizontal changes.

The $y + 6$ tells us to translate y-coordinates 6 units up (up because it is positive). **Remember y-coordinates are vertical changes.



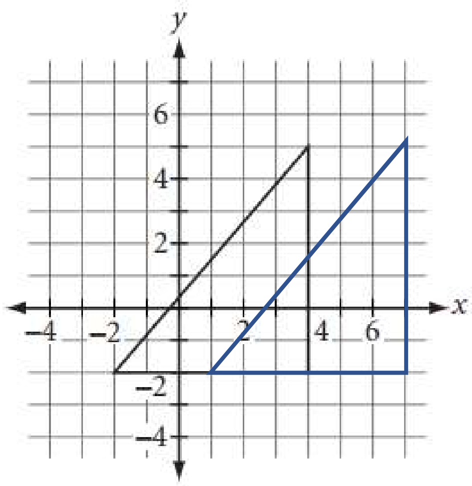
Example 6: Translate the triangle according to the ordered pair rule.

$(x, y) \rightarrow (x + 3, y)$



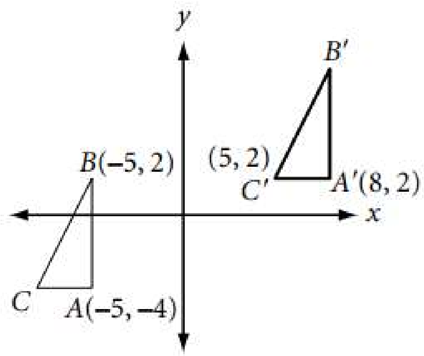
The $x + 3$ tells us to translate x-coordinates 3 units to the right (right because it is positive). **Remember x-coordinates are horizontal changes.

The y tells us to not move the y-coordinates up or down at all. **Remember y-coordinates are vertical changes.



Example 7: Complete the ordered pair rule that transforms each triangle to its image.

$(x, y) \rightarrow (\_\_\_\_\_\_, \_\_\_\_\_\_)$

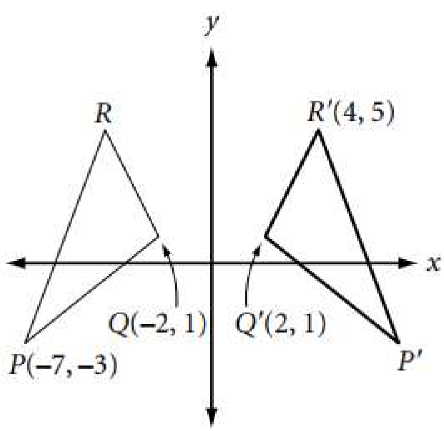


We can see that the triangle did not rotate or reflect, so this must be a translation. If we look at the ordered pairs that correspond (A and A'), we can see that the x-coordinate subtracted 13, and the y-coordinate subtracted 6.

So, the ordered pair rule is $(\boldsymbol{x}, \boldsymbol{y}) \rightarrow (\boldsymbol{x - 13}, \boldsymbol{y - 6})$**.**

Example 8: Complete the ordered pair rule that transforms each triangle to its image.

$(x, y) \rightarrow (\_\_\_\_\_\_, \_\_\_\_\_\_)$

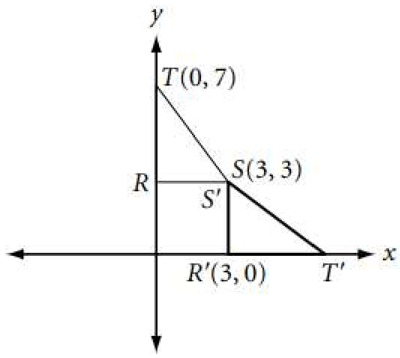


We can see that the triangle reflected over the y-axis. If we look at the ordered pairs that correspond (Q and Q'), we can see that the x-coordinate changed sign, and the y-coordinate stayed the same.

So, the ordered pair rule is $(\boldsymbol{x}, \boldsymbol{y}) \rightarrow (\boldsymbol{-x}, \boldsymbol{y})$**.**

Example 9: Complete the ordered pair rule that transforms each triangle to its image.

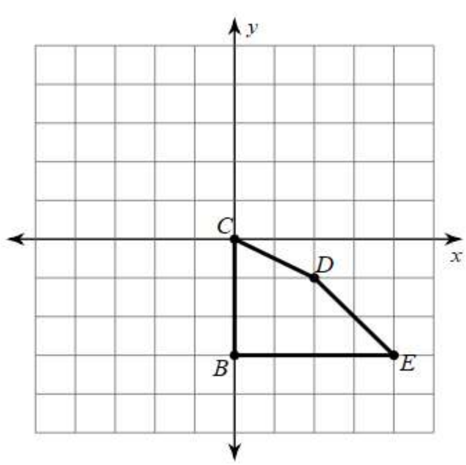
$(x, y) \rightarrow (\_\_\_\_\_, \_\_\_\_\_)$



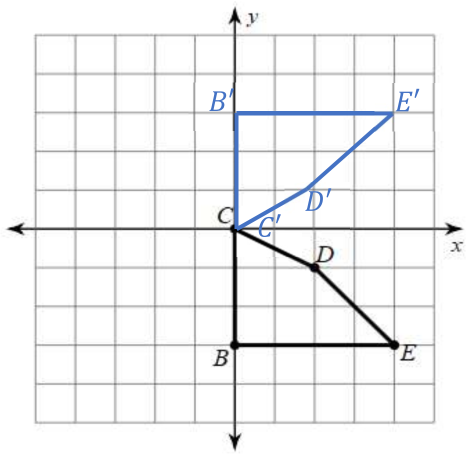
We can see that the triangle seems to have both rotated and reflected. If we look at the ordered pairs that correspond (S and S'), we can't learn much. However, we can imagine that the coordinates of T' are $(7, 0)$. Using those corresponding points, we can see that the x-coordinate and the y-coordinate switched spots.

So, the ordered pair rule is $\boldsymbol{(x, y) \rightarrow (y, x)}$.

Example 10: Reflect the quadrilateral across the x-axis. Write the ordered pair rule to represent the rotation.



To reflect the quadrilateral across the x-axis, we are essentially flipping it up over the x-axis,



To determine the ordered pair rule, let's look at the change in the ordered pairs of each of the points.

$B(0, -3) \rightarrow B'(0, 3)$

$C(0, 0) \rightarrow C'(0, 0)$

$D(2, -1) \rightarrow D'(2, 1)$

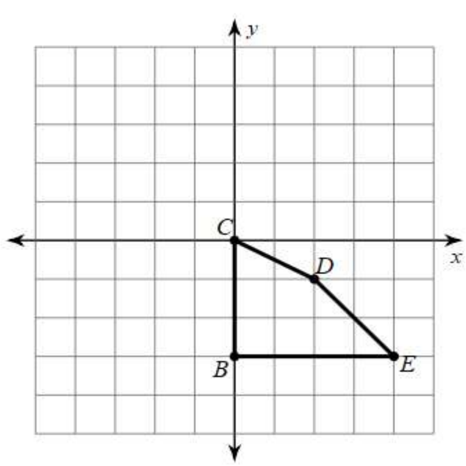
$E(4, -3) \rightarrow E'(4, 3)$

We can see that the x-coordinate of the original figure stayed the same.

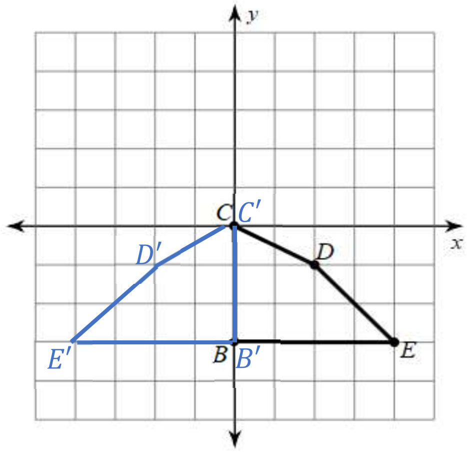
The y-coordinate of the original figure changed sign. To change the sign of a coordinate, we place a negative in front of the variable.

This means our ordered pair rule is $\boldsymbol{(x, y) \rightarrow (x, -y)}$.

Example 11: Reflect the quadrilateral across the y-axis. Write the ordered pair rule to represent the rotation.



To reflect the quadrilateral across the y-axis, we are essentially flipping it left over the y-axis,



To determine the ordered pair rule, let's look at the change in the ordered pairs of each of the points.

$B(0,-3) \rightarrow B'(0,-3)$

$C(0,0) \rightarrow C'(0,0)$

$D(2,-1) \rightarrow D'(-2,-1)$

$E(4,-3) \rightarrow E'(-4,-3)$

We can see that the y-coordinate of the original figure stayed the same.

The x-coordinate of the original figure changed sign. To change the sign of a coordinate, we place a negative in front of the variable.

This means our ordered pair rule is $\boldsymbol{(x, y) \rightarrow (-x, y)}$.