Solving Literal Equations

Example 1:

Solve for y.

$3x - 4y = 7$

When solving literal equations, it is important to focus on what the problem is asking us to solve for. In order to solve for that variable, we need to isolate it on one side of the equation. In this problem, we are being asked to solve for y.

$3x - 4y = 7$

If I now consider the things on the same side of the equation as y, I notice that there is a $3x$ and 4. The 4 is multiplied into y. The $3x$ is added into the y-term. So, using my reversed order of operations, I know that I need to create an additive identity with the $3x$ term first.

$3x - 4y = 7$

$-3x \qquad -3x$

$-4y = 7 - 3x$ **Notice that I am not combining $7 - 3x$. They are not like terms and cannot be combined.

It is important after each step to re-evaluate what we are being asked to solve for.

$-4y = 7 - 3x$

The only thing left on the same side of the equation as y is the -4 which is multiplied into y. So, we will need to create a multiplicative identity.

$$\frac{-4y}{-4} = \frac{7 - 3x}{-4}$$

$$y = \frac{7 - 3x}{-4}$$

When we have two terms in the numerator and only one in the denominator, we can split the terms into two separate fractions with the same denominator.

$$y = \frac{7}{-4} - \frac{3x}{-4}$$

$$y = -1\frac{3}{4} + \frac{3x}{4}$$

Once the variable we were asked to solve for is isolated, and we have simplified the other side of the equation, we know that the problem is solved: $\boldsymbol{y = -1\frac{3}{4} + \frac{3x}{4}}$.

Example 2:

Solve for n.

$15 = 3n + 6p$

When solving literal equations, it is important to focus on what the problem is asking us to solve for. In order to solve for that variable, we need to isolate it on one side of the equation. In this problem, we are being asked to solve for n.

$15 = 3n + 6p$

If I now consider the things on the same side of the equation as n, I notice that there is a 3 and $6p$. The 3 is multiplied into n. The $6p$ is added into the n-term. So, using my reversed order of operations, I know that I need to create an additive identity with the $6p$ term first.

$15 = 3n + 6p$

$-6p \qquad -6p$

$15 - 6p = 3n$ **Notice that I am not combining $15 - 6p$. They are not like terms and cannot be combined.

It is important after each step to re-evaluate what we are being asked to solve for.

$15 - 6p = 3n$

The only thing left on the same side of the equation as n is the 3 which is multiplied into n. So, we will need to create a multiplicative identity.

$$\frac{15 - 6p}{3} = \frac{3n}{3}$$

$$\frac{15 - 6p}{3} = n$$

When we have two terms in the numerator and only one in the denominator, we can split the terms into two separate fractions with the same denominator.

$$\frac{15}{3} - \frac{6p}{3} = n$$

$5 - 2p = n$

Once the variable we were asked to solve for is isolated, and we have simplified the other side of the equation, we know that the problem is solved: $\boldsymbol{n = 5 - 2p}$.

Example 3:

Solve for k.

$$\frac{k-2}{5} = 11j$$

We are being asked to solve for (isolate) k.

$$\frac{k-2}{5} = 11j$$

Remember that in a problem like this, we assume that the numerator is in parentheses.

$$\frac{(k-2)}{5} = 11j$$

Our reverse order of operations tells us to leave what is in parentheses until last. Outside of the parentheses is a division by 5, so we must handle that first.

$$5 \cdot \frac{(k-2)}{5} = 11j \cdot 5$$

$(k-2) = 55j$ **I can multiply unlike terms because $11j \cdot 5 = 11 \cdot j \cdot 5 = 11 \cdot 5 \cdot j$

$= 55j$

$$k - 2 = 55j$$

It is important after each step to re-evaluate what we are being asked to solve for.

$$k - 2 = 55j$$

The only thing left on the same side of the equation as k is the 2 which is subtracted from k. So, we will need to create an additive identity.

$$k - 2 = 55j$$

$$+2 \quad\quad +2$$

$k = 55j + 2$ **Notice that I am not combining $55j + 2$. They are not like terms and cannot be combined.

Once the variable we were asked to solve for is isolated, and we have simplified the other side of the equation, we know that the problem is solved: $\boldsymbol{k = 55j + 2}$.

Example 4:

Solve for d: $d + 5c = 3d - 1$

We are being asked to solve for (isolate) d.

$d + 5c = 3d - 1$

Since we have a term with d on each side of the equation, we have to us the exact methods that we used when solving multi-step equations with the variable on both sides to get the variables all onto one side of the equation. I will solve this equation two ways.

$d + 5c = 3d - 1$

$-3d \quad\quad -3d$

$-2d + 5c = -1$

Notice that we can combine $d - 3d$ because they are like terms.

$-2d + 5c = -1$

$-2d + 5c = -1$

$\quad -5c \quad -5c$

$-2d = -1 - 5c$

Don't combine $-1 - 5c$. They are not like terms.

$-2d = -1 - 5c$

$-2d = -1 - 5c$

$$\frac{-2d}{-2} = \frac{-1-5c}{-2}$$

$$d = \frac{-1-5c}{-2}$$

$$d = \frac{-1-5c}{-2}$$

$$d = \frac{-1}{-2} - \frac{5c}{-2}$$

$$d = \frac{1}{2} + 2\frac{1}{2}c$$

$d + 5c = 3d - 1$

$-d \quad\quad -d$

$\quad 5c = 2d - 1$

Notice that we can combine $3d - d$ because they are like terms.

$5c = 2d - 1$

$5c = 2d - 1$

$+1 \quad\quad +1$

$5c + 1 = 2d$

Don't combine $5c + 1$. They are not like terms.

$5c + 1 = 2d$

$5c + 1 = 2d$

$$\frac{5c+1}{2} = \frac{2d}{2}$$

$$\frac{5c+1}{2} = d$$

$$\frac{5c+1}{2} = d$$

$$\frac{5c}{2} + \frac{1}{2} = d$$

$$2\frac{1}{2}c + \frac{1}{2} = d$$

You should notice that we have the same solution, regardless of which way we solve the equation.

Once the variable we were asked to solve for is isolated, and we have simplified the other side of the equation, we know that the problem is solved**:** $\boldsymbol{d = \frac{1}{2} + 2\frac{1}{2}c}$**.**

Example 5:

Solve the equation or formula for the variable specified. $-3x + b = 6x$; for x

We are being asked to solve for (isolate) x.

$-3x + b = 6x$

Since we have a term with x on each side of the equation, we have to us the exact methods that we used when solving multi-step equations with the variable on both sides to get the variables all onto one side of the equation. I will solve this equation two ways.

$-3x + b = 6x$

$-6x \qquad -6x$

$-9x + b = 0$

Notice that we can combine $-3x - 6x$ because they are like terms.

$-9x + b = 0$

$-9x + b = 0$

$\quad -b \quad -b$

$-9x = -b$

There is no need to write the 0 in $0 - b$.

$-9x = -b$

$-9x = -b$

$$\frac{-9x}{-9} = \frac{-b}{-9}$$

$$x = \frac{b}{9}$$

$$x = \frac{b}{9}$$

$-3x + b = 6x$

$+3x \qquad +3x$

$\qquad b = 9x$

Notice that we can combine $6x + 3x$ because they are like terms.

$b = 9x$

$b = 9x$

$$\frac{b}{9} = \frac{9x}{9}$$

$$\frac{b}{9} = x$$

$$\frac{b}{9} = x$$

You should notice that we have the same solution, regardless of which way we solve the equation. As when we were solving multi-step equations, leaving one side of the equation with a zero is often the least efficient way of solving, but it can be done.

Once the variable we were asked to solve for is isolated, and we have simplified the other side of the equation, we know that the problem is solved: $\boldsymbol{x = \frac{b}{9}}$.

Example 6:

Solve the equation or formula for the variable specified. $4z + b = 2z + c$; for z

We are being asked to solve for (isolate) z.

$4z + b = 2z + c$

Since we have a term with z on each side of the equation, we have to us the exact methods that we used when solving multi-step equations with the variable on both sides to get the variables all onto one side of the equation. I will solve this equation two ways.

$4z + b = 2z + c$	$4z + b = 2z + c$
$-2z \quad -2z$	$-4z \quad -4z$
$2z + b = c$	$b = -2z + c$
$2z + b = c$	$b = -2z + c$
$-b \quad -b$	$b = -2z + c$
$2z = c - b$	$-c \quad -c$
$2z = c - b$	$b - c = -2z$
$\frac{2z}{2} = \frac{c-b}{2}$	$b - c = -2z$
$z = \frac{c-b}{2}$	$\frac{b-c}{-2} = \frac{-2z}{-2}$
$z = \frac{c}{2} - \frac{b}{2}$	$\frac{b-c}{-2} = z$
	$\frac{b}{-2} - \frac{c}{-2} = z$
	$-\frac{b}{2} + \frac{c}{2} = z$

You should notice that we have the same solution, regardless of which way we solve the equation. The terms are simply switched.

Once the variable we were asked to solve for is isolated, and we have simplified the other side of the equation, we know that the problem is solved**:** $\boldsymbol{z = \frac{c}{2} - \frac{b}{2}}$.

Example 7:

Solve the equation or formula for the variable specified. $\frac{y+a}{b} = c$; for y

We are being asked to solve for (isolate) y.

$$\frac{y + a}{b} = c$$

Remember that in a problem like this, we assume that the numerator is in parentheses.

$$\frac{(y + a)}{b} = c$$

Our reverse order of operations tells us to leave what is in parentheses until last. Outside of the parentheses is a division by b, so we must handle that first.

$$b \cdot \frac{(y + a)}{b} = c \cdot b$$

$(y + a) = bc$ **We usually put multiple multiplied variables in alphabetical order: $c \cdot b = b \cdot c = bc$

$$y + a = bc$$

$$\begin{aligned} y + a &= bc \\ -a \quad &- a \end{aligned}$$

$y = bc - a$ **Notice that I am not combining $bc - a$. They are not like terms and cannot be combined.

Once the variable we were asked to solve for is isolated, and we have simplified the other side of the equation, we know that the problem is solved: $\boldsymbol{y = bc - a}$.

Example 8:

Solve the equation or formula for the variable specified. $p = a(b + c)$; for a

We are being asked to solve for (isolate) a.

$$p = a(b + c)$$

Remember that there is an implied multiplication between the variable and the parentheses.

$p = \boldsymbol{a} \cdot (b + c)$

We can treat the $(b + c)$ as a single term to be divided out.

$$\frac{p}{(b+c)} = \frac{a \cdot (b+c)}{(b+c)}$$

$$\frac{p}{(b+c)} = a$$

We do not need the parentheses in the denominator since there is nothing else in the denominator.

$$\frac{p}{b+c} = a$$

Once the variable we were asked to solve for is isolated, and we have simplified the other side of the equation, we know that the problem is solved: $\boldsymbol{\frac{p}{b+c} = a}$.

**Note: Unlike numerators, we cannot split an added denominator into two fractions.