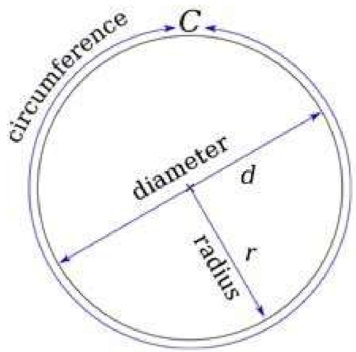
Lesson 6.6 – Circumference/Diameter Ratio Problems

Circumference Conjecture - If C is the circumference and d is the diameter of a circle, then there is a number π such that $C = \pi d$. If $d = 2r$ where r is the radius, then $C = 2\pi r$.



Example 1: Complete.

Alfonzo's Pizzeria bakes olive pieces in the outer crust of its 20-inch (diameter) pizza. There is at least one olive pieces per inch of crust. How many olive pieces will you get in one slice of pizza? Assume the pizza is cut into eight slices.

The outer crust of the pizza is the circumference of the pizza and we are told that the diameter of the pizza is 20 inches.

$C = \pi d$

$C = \pi(20)$

$C \approx 62.8319 \ldots$ **We are approximating, wo we multiply $20 \cdot \pi$ in our calculator.

We aren't going to put part of an olive piece on the pizza. Since the problem tells us that there is at least one olive piece per inch of crust and the crust is 62.8 inches, we will have 63 olive pieces on the pizza.

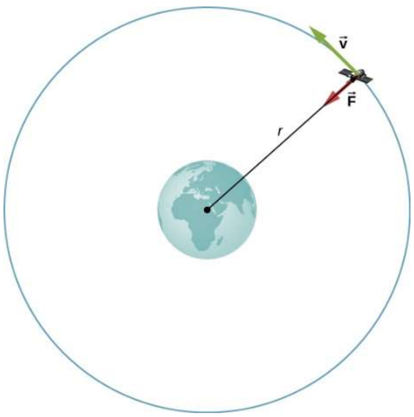
We are told that the pizza is cut into 8 slices.

$\frac{63}{8} \approx 7.875$

We will have at least 7.8 olive pieces on each slice of pizza.

Example 2: Complete.

A satellite in geostationary orbit stays over the same spot on Earth. The satellite completes one orbit in the same time Earth rotates once about its axis (23.93 hours). If the satellite's orbit has radius 4.23×10^7 m, calculate the satellite's orbital speed (tangential velocity) in meters per second.



In a problem like this, it is important to start with appropriate units. We are asked for speed in meters per second. We are given the distance (radius) in meters, so that's in the correct unit. However, time is given in hours and we need to convert that to seconds.

23.93 hours $\cdot$ 60 minutes/hour $= 1435.8$ minutes

1435.8 minutes $\cdot$ 60 seconds/minute $= 86{,}148$ seconds

Now, we know that the total distance that the satellite travels will be the circumference. So, let's calculate that.

We know that the diameter of a circle is twice as long as the radius. So, $d = 2(4.23 \times 10^7) = 8.46 \times 10^7$

$C = \pi d$

$C = \pi(8.46 \times 10^7)$

$C \approx 265{,}778{,}738.5$ **We are approximating, wo we multiply in our calculator.

Speed is calculated by dividing distance traveled by time.

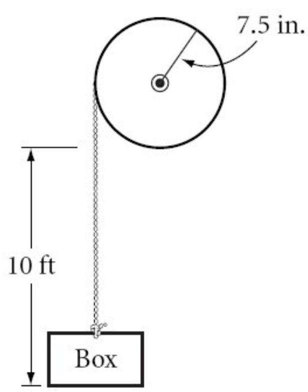
$$\frac{365{,}778{,}738.5}{86{,}148} \approx 3085.14$$

Because these numbers are so large, there is no reason to worry about the decimal part of the speed.

The satellite is travelling at 3085 meters per second.

Example 3: Complete.

To use the machine, you turn the crank, which turns the pulley wheel, which winds the rope and lifts the box. Through how many rotations must you turn the crank to life the box 10 feet?



The first thing you should notice about this problem is that the radius is given to us in inches and the distance we are asked to lift the box is 10 feet. Those two things need to be in the same unit. I'm choosing to convert the 10 feet to inches, but you could choose the other option.

$10 \text{ feet} \cdot 12 \text{ inches/foot} = 120 \text{ inches}$

Every time we rotate the crank the rope will be wound the distance of the circumference of the wheel.

We know that the diameter of a circle is twice as long as the radius. So, $d = 2(7.5) = 15$

$C = \pi d$

$C = \pi(15)$

$C \approx 47.1239$ **We are approximating, wo we multiply in our calculator.

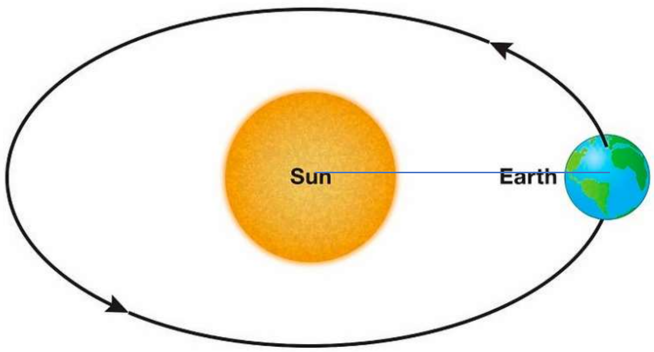
Each rotation of the wheel winds up just over 47 inches of rope. We need to wind up 120 inches.

$\frac{120}{47.12} \approx 2.55$

We need to rotate the crank 2.5 rotations to lift the box 10 feet.

Example 4: Complete.

As you sit in your chair, you are whirling through space with Earth as it moves around the sun. If the average distance from Earth to the sun is 1.4957×10^{11} m and Earth completes one revolution every 364.25 days, what is your "sitting" speed in space relative to the sun? Give your answer in km/h, rounded to the nearest 100 km/h.



In a problem like this, it is important to start with appropriate units. We are asked for speed in kilometers per hour. We are given the distance (radius) in meters, so we need to convert meters to kilometers. Time is given in days and we need to convert that to hours.

1.4957×10^{11} meters $\div$ 1000 meters/kilometer $= 1.497 \times 10^{8}$ kilometers

364.25 days $\cdot$ 24 hours/day $= 8742$ hours

The total distance we travel is the circumference of the circle.

The average distance from Earth to the sun is the radius. We know that the diameter of a circle is twice as long as the radius. So, $d = 2(1.497 \times 10^{8}) = 2.994 \times 10^{8}$

$C = \pi d$

$C = \pi(2.994 \times 10^{8})$

$C \approx 940{,}592{,}840.48$ **We are approximating, wo we multiply in our calculator.

Speed is calculated by dividing distance traveled by time.

$$\frac{940{,}592{,}840.48}{8742} \approx 107595$$

We need to round this number to the nearest 100.

Our "sitting" speed is 107,600 km/hr.