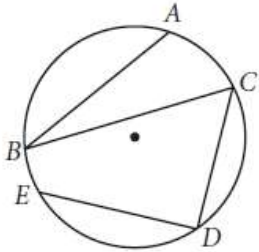


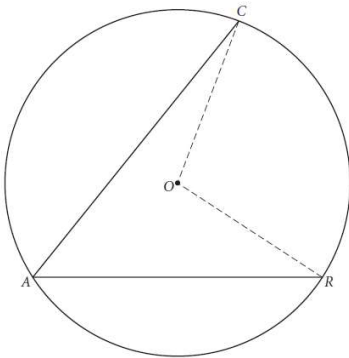
Lesson 6.3 – Arcs and Angles

Inscribed Angle - An inscribed angle has its vertex on the circle and its sides are chords.



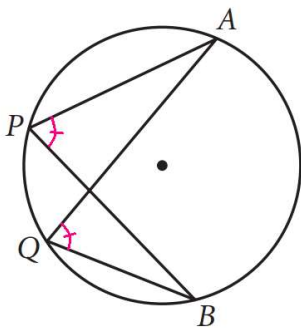
$\angle ABC$, $\angle BCD$, and $\angle CDE$ are inscribed angles.

Inscribed Angle Conjecture - The measure of an inscribed angle in a circle is one half the measure of the intercepted arc.

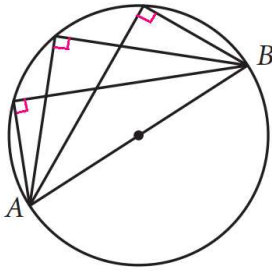


$$m\angle CAR = \frac{1}{2}m\widehat{CR}$$

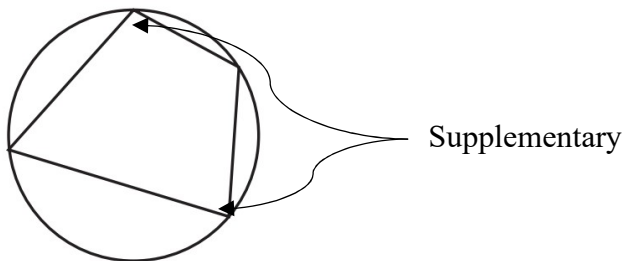
Inscribed Angles Intercepting Arcs Conjecture - Inscribed angles that intercept the same arc are congruent.



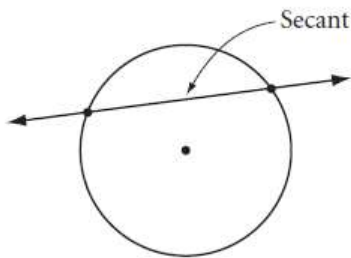
Angles Inscribed in a Semicircle Conjecture - Angles inscribed in a semicircle are right angles.



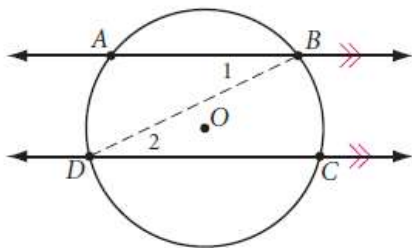
Cyclic Quadrilateral Conjecture - The opposite angles of a cyclic quadrilateral are supplementary.



Secant - A secant is a line that intersects a circle in two points.



Parallel Lines Intercepted Arcs Conjecture - Parallel lines intercept congruent arcs on a circle.



$$\widehat{AD} \cong \widehat{BC}$$

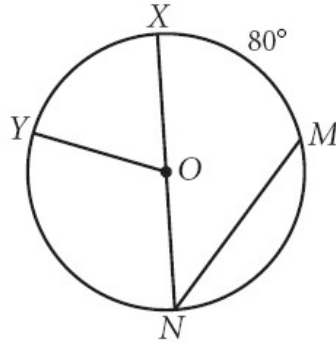
Example 1: Find the measure of each unknown.

$$m\widehat{XM} = 80^\circ$$

$$m\angle XNM = \underline{\hspace{2cm}}$$

$$m\widehat{XN} = \underline{\hspace{2cm}}$$

$$m\widehat{MN} = \underline{\hspace{2cm}}$$



$\angle XNM$ is the inscribed angle that intercepts arc \widehat{XM} . Inscribed angles are half the measure of the arcs they intercept.

$$m\angle XNM = 40^\circ$$

\widehat{XN} is a semicircle (regardless of which way we go from X to N).

$$m\widehat{XN} = 180^\circ$$

$\widehat{XM} + \widehat{MN} = \widehat{XN}$. We know the measure of \widehat{XM} and \widehat{XN} . $80 + \widehat{MN} = 180$.

$$m\widehat{MN} = 100^\circ$$

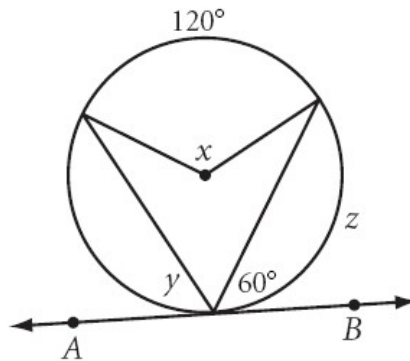
Example 2: Find the measure of each unknown.

\overleftrightarrow{AB} is a tangent.

$$x = \underline{\hspace{2cm}}$$

$$y = \underline{\hspace{2cm}}$$

$$z = \underline{\hspace{2cm}}$$



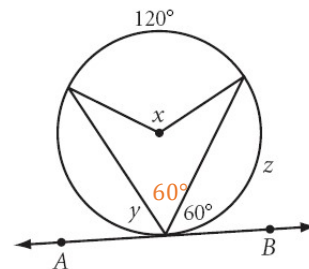
The measure of the central angle is equal to the measure of the arc it intercepts.

$$x = 120^\circ$$

The inscribed angle to the 120° arc is half the measure of the arc.

y and the two 60° angles must add to 180° .

$$y = 60^\circ$$

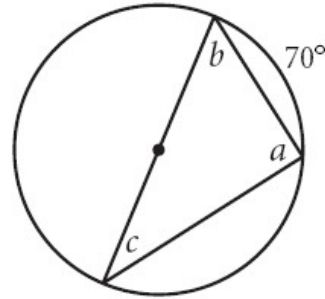


Because the three angles are congruent, all three arcs will be congruent.

$$z = 120^\circ$$

Example 3: Find the measure of each unknown.

$$a = \underline{\hspace{2cm}}, b = \underline{\hspace{2cm}}, c = \underline{\hspace{2cm}}$$



a is the angle inscribed in a semicircle and must be a right angle.

$$a = 90^\circ$$

c is the angle inscribed to the 70° arc and must be half its measure.

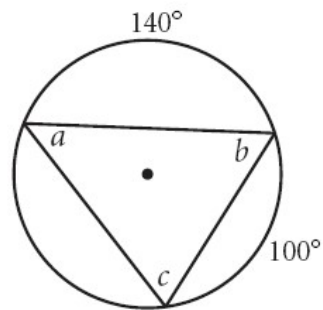
$$c = 35^\circ$$

$a, b,$ and c form the three angles of a triangle and must add to 180° . $180 - (90 + 35) = 55$.

$$b = 55^\circ$$

Example 4: Find the measure of each unknown.

$$a = \underline{\hspace{2cm}}, b = \underline{\hspace{2cm}}, c = \underline{\hspace{2cm}}$$



a is the angle inscribed to the 100° arc and must be half its measure.

$$a = 50^\circ$$

c is the angle inscribed to the 140° arc and must be half its measure.

$$c = 70^\circ$$

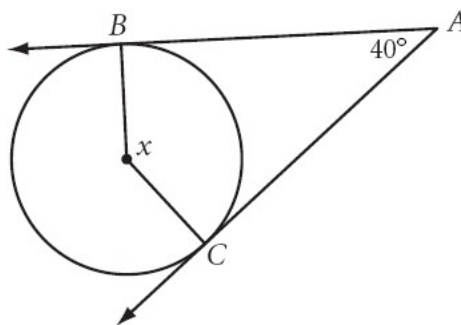
$a, b,$ and c form the three angles of a triangle and must add to 180° . $180 - (50 + 70) = 60$.

$$b = 60^\circ$$

Example 5: Find the measure of each unknown.

\overrightarrow{AB} and \overrightarrow{AC} are tangents.

$x =$ _____



We know that both of the angles that are formed by a radius to a tangent line are right angles.

The figure formed is a quadrilateral and must add to 360° .

$$360 - (90 + 90 + 40) = 140$$

$$x = \mathbf{140^\circ}$$

Example 6: Find the measure of each unknown.

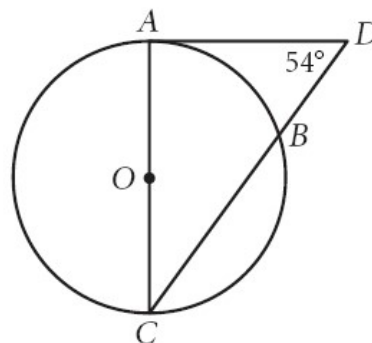
\overline{AD} is a tangent. \overline{AC} is a diameter.

$m\angle A =$ _____

$m\widehat{AB} =$ _____

$m\angle C =$ _____

$m\widehat{CB} =$ _____



$\angle A$ is formed by a radius to a tangent and must be a right angle.

$$m\angle A = \mathbf{90^\circ}$$

We can find the measure of $\angle C$ by solving the triangle ADC . $180 - (90 + 54) = 36$.

$$m\angle C = \mathbf{34^\circ}$$

$\angle C$ is the inscribed angle to \widehat{AB} , so the measure of \widehat{AB} must be twice the measure of $\angle C$.

$$m\widehat{AB} = \mathbf{72^\circ}$$

\widehat{AB} and \widehat{CB} form a semicircle and must add to 180° .

$$m\widehat{CB} = \mathbf{108^\circ}$$

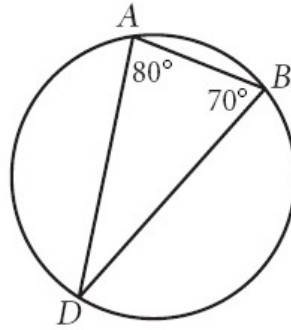
Example 7: Find the measure of each unknown.

$$m\widehat{AD} = \underline{\hspace{2cm}}$$

$$m\angle D = \underline{\hspace{2cm}}$$

$$m\widehat{AB} = \underline{\hspace{2cm}}$$

$$m\widehat{DAB} = \underline{\hspace{2cm}}$$



$\angle B$ is the inscribed angle to \widehat{AD} , so the measure of \widehat{AD} must be twice the measure of $\angle B$.

$$m\widehat{AD} = 140^\circ$$

We can find the measure of $\angle D$ by solving the triangle. $180 - (80 + 70) = 30$.

$$m\angle D = 30^\circ$$

$\angle D$ is the inscribed angle to \widehat{AB} , so the measure of \widehat{AB} must be twice the measure of $\angle D$.

$$m\widehat{AB} = 60^\circ$$

\widehat{DAB} is formed by \widehat{AD} and \widehat{AB} added together.

$$m\widehat{DAB} = 200^\circ$$

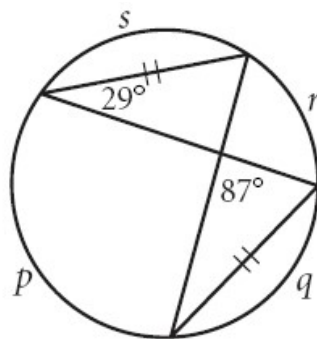
Example 8: Find the measure of each unknown.

$$p = \underline{\hspace{2cm}}$$

$$q = \underline{\hspace{2cm}}$$

$$r = \underline{\hspace{2cm}}$$

$$s = \underline{\hspace{2cm}}$$



The 29° angle is the inscribed angle to r . So, r must be twice the measure of the inscribed angle.

$$r = 58^\circ$$

Vertical angles are congruent and we can solve the triangle.

The 64° angle is the inscribed angle to p .

$$p = 128^\circ$$

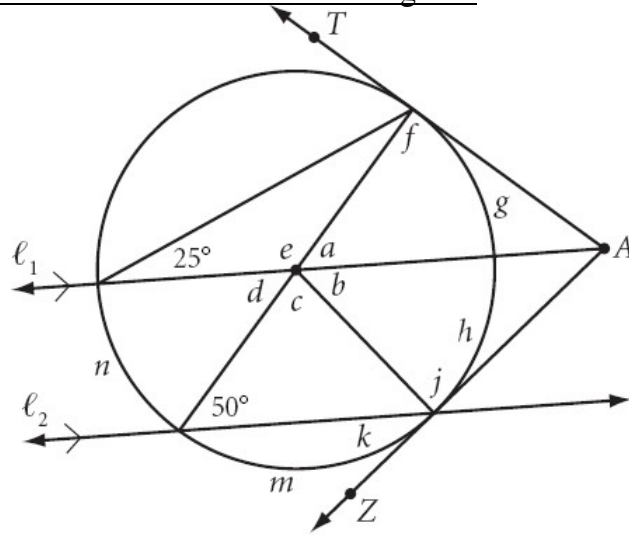
We know that all arcs of a circle should add to 360° and we know that s and q should be congruent since the chords that form them are congruent. $\frac{360-(58+128)}{2} = 87$.

$s = 87^\circ$

$q = 87^\circ$

Example 9: Find the lettered angle and arc measures. \overrightarrow{AT} and \overrightarrow{AZ} are tangents.

- $a =$ _____
- $b =$ _____
- $c =$ _____
- $d =$ _____
- $e =$ _____
- $f =$ _____
- $g =$ _____
- $h =$ _____
- $j =$ _____
- $k =$ _____
- $m =$ _____
- $n =$ _____



a is a corresponding angle to the 50° angle.

$a = 50^\circ$

g is the arc formed by the central angle a , so it must be congruent to a .

$g = 50^\circ$

d is a vertical angle to a .

$d = 50^\circ$

n is the arc formed by the central angle d , so it must be congruent to d .

$n = 50^\circ$

h and n are arcs intercepted by congruent lines and must be congruent.

$h = 50^\circ$

h is the arc formed by the central angle b , so b must be congruent to h .

$b = 50^\circ$

f is formed by a radius to a tangent and must be a right angle.

$f = 90^\circ$

j is formed by a radius to a tangent and must be a right angle.

$$j = 90^\circ$$

e forms a linear pair with a .

$$e = 130^\circ$$

$d, b,$ and c form a line and must add to 180° .

$$c = 80^\circ$$

m is the arc formed by the central angle c , so it must be congruent to c .

$$m = 80^\circ$$

$k, j,$ and the unmarked angle between them form a line and must add to 180° . The unmarked angle is the other base angle of an isosceles triangle (because the radii of the circle must be congruent.) So, the unmarked angle is 50° . $180 - (50 + 90) = 40$.

$$k = 40^\circ$$