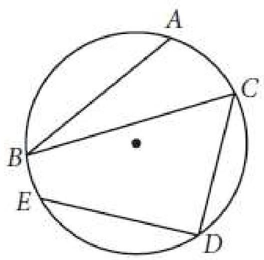
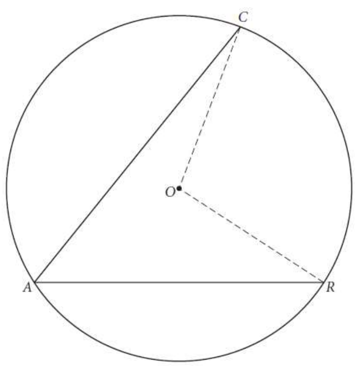
Lesson 6.3 – Arcs and Angles

Inscribed Angle - An inscribed angle has its vertex on the circle and its sides are chords.



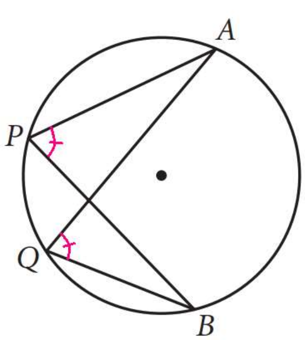
$\angle ABC$, $\angle BCD$, and $\angle CDE$ are inscribed angles.

Inscribed Angle Conjecture - The measure of an inscribed angle in a circle is one half the measure of the intercepted arc.



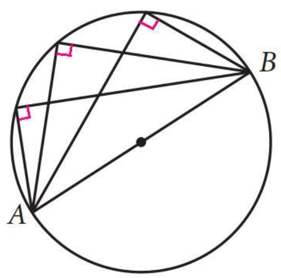
$$m\angle CAR = \frac{1}{2} m\,\widehat{CR}$$

Inscribed Angles Intercepting Arcs Conjecture - Inscribed angles that intercept the same arc are congruent.

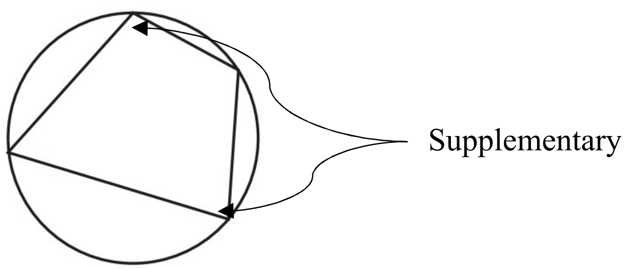


$\widehat{AD} \cong \widehat{BC}$

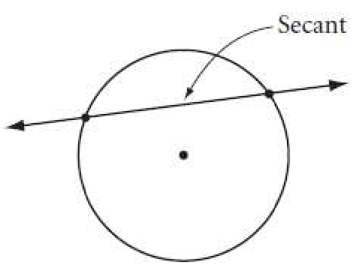
Angles Inscribed in a Semicircle Conjecture - Angles inscribed in a semicircle are right angles.



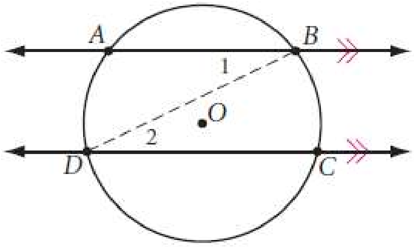
Cyclic Quadrilateral Conjecture - The opposite angles of a cyclic quadrilateral are supplementary.



Secant - A secant is a line that intersects a circle in two points.



Parallel Lines Intercepted Arcs Conjecture - Parallel lines intercept congruent arcs on a circle.



$\widehat{AD} \cong \widehat{BC}$

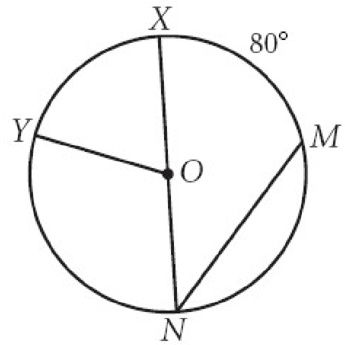
Example 1: Find the measure of each unknown.

$m\widehat{XM} = 80°$

$m\angle XNM =$______

$m\widehat{XN} =$________

$m\widehat{MN} =$________



$\angle XNM$ is the inscribed angle that intercepts arc $\widehat{XM}$. Inscribed angles are half the measure of the arcs they intercept.

$\boldsymbol{m\angle XNM = 40°}$

$\widehat{XN}$ is a semicircle (regardless of which way we go from X to N).

$\boldsymbol{m\widehat{XN} = 180°}$

$\widehat{XM} + \widehat{MN} = \widehat{XN}$. We know the measure of $\widehat{XM}$ and $\widehat{XN}$. $80 + \widehat{MN} = 180$.

$\boldsymbol{m\widehat{MN} = 100°}$

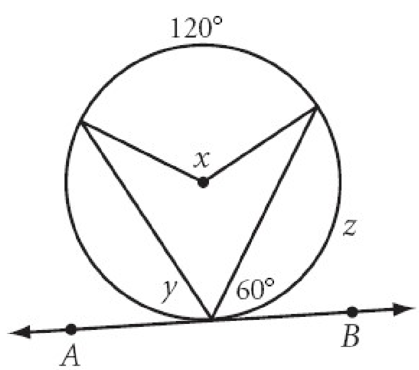
Example 2: Find the measure of each unknown.

$\overleftrightarrow{AB}$ is a tangent.

$x =$________

$y =$________

$z =$________



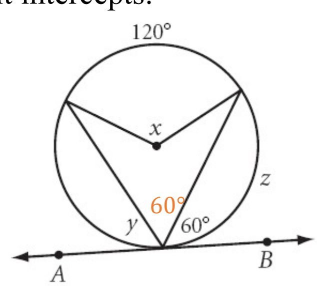
The measure of the central angle is equal to the measure of the arc it intercepts.

$\boldsymbol{x = 120°}$

The inscribed angle to the 120° arc is half the measure of the arc.

y and the two 60° angles must add to 180°.

$\boldsymbol{y = 60°}$

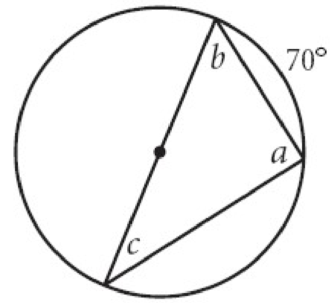


Because the three angles are congruent, all three arcs will be congruent.

$\boldsymbol{z = 120°}$

Example 3: Find the measure of each unknown.

$a =$________, $b =$________, $c =$________



a is the angle inscribed in a semicircle and must be a right angle.

$\boldsymbol{a = 90°}$

c is the angle inscribed to the $70°$ arc and must be half its measure.

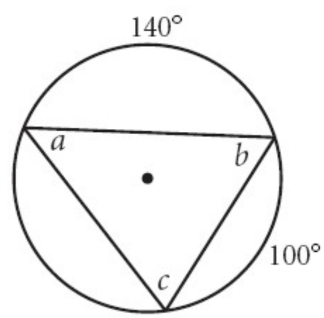
$\boldsymbol{c = 35°}$

a, b, and c form the three angles of a triangle and must add to $180°$. $180 - (90 + 35) = 55$.

$\boldsymbol{b = 55°}$

Example 4: Find the measure of each unknown.

$a =$________, $b =$________, $c =$________



a is the angle inscribed to the $100°$ arc and must be half its measure.

$\boldsymbol{a = 50°}$

c is the angle inscribed to the $140°$ arc and must be half its measure.

$\boldsymbol{c = 70°}$

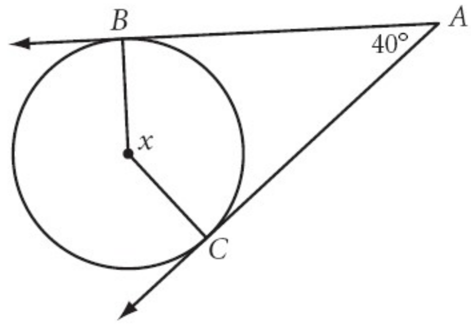
a, b, and c form the three angles of a triangle and must add to $180°$. $180 - (50 + 70) = 60$.

$\boldsymbol{b = 60°}$

Example 5: Find the measure of each unknown.

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are tangents.

$x =$________



We know that both of the angles that are formed by a radius to a tangent line are right angles.

The figure formed is a quadrilateral and must add to $360°$.

$360 - (90 + 90 + 40) = 140$

$\boldsymbol{x = 140°}$

Example 6: Find the measure of each unknown.

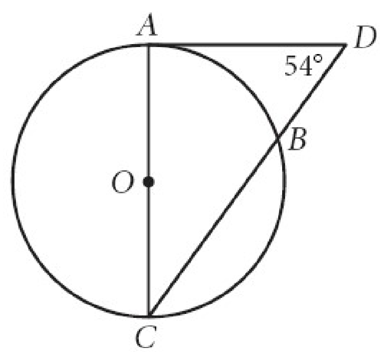
$\overline{AD}$ is a tangent. $\overline{AC}$ is a diameter.

$m\angle A =$________

$m\,\widehat{AB} =$________

$m\angle C =$________

$m\,\widehat{CB} =$________



$\angle A$ is formed by a radius to a tangent and must be a right angle.

$\boldsymbol{m\angle A = 90°}$

We can find the measure of $\angle C$ by solving the triangle ADC. $180 - (90 + 54) = 36$.

$\boldsymbol{m\angle C = 34°}$

$\angle C$ is the inscribed angle to $\widehat{AB}$, so the measure of $\widehat{AB}$ must be twice the measure of $\angle C$.

$\boldsymbol{m\,\widehat{AB} = 72°}$

$\widehat{AB}$ and $\widehat{CB}$ form a semicircle and must add to $180°$.

$\boldsymbol{m\,\widehat{CB} = 108°}$

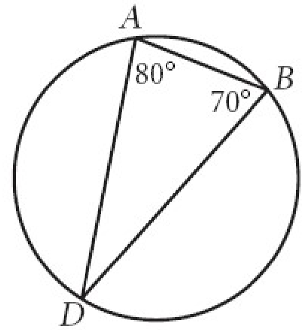
Example 7: Find the measure of each unknown.

$m\,\widehat{AD}$ =________

$m\angle D$ =________

$m\,\widehat{AB}$ =________

$m\,\widehat{DAB}$ =_______



$\angle B$ is the inscribed angle to $\widehat{AD}$, so the measure of $\widehat{AD}$ must be twice the measure of $\angle B$.

$m\,\widehat{AD} = 140°$

We can find the measure of $\angle D$ by solving the triangle. $180 - (80 + 70) = 30$.

$m\angle D = 30°$

$\angle D$ is the inscribed angle to $\widehat{AB}$, so the measure of $\widehat{AB}$ must be twice the measure of $\angle D$.

$m\,\widehat{AB} = 60°$

$\widehat{DAB}$ is formed by $\widehat{AD}$ and $\widehat{AB}$ added together.

$m\,\widehat{DAB} = 200°$

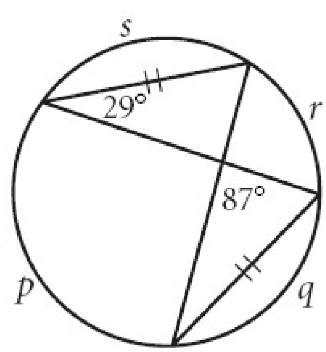
Example 8: Find the measure of each unknown.

p =________

q =________

r =________

s =________



The 29° angle is the inscribed angle to r. So, r must be twice the measure of the inscribed angle.

$\boldsymbol{r = 58°}$

Vertical angles are congruent and we can solve the triangle.

The 64° angle is the inscribed angle to p.

$\boldsymbol{p = 128°}$

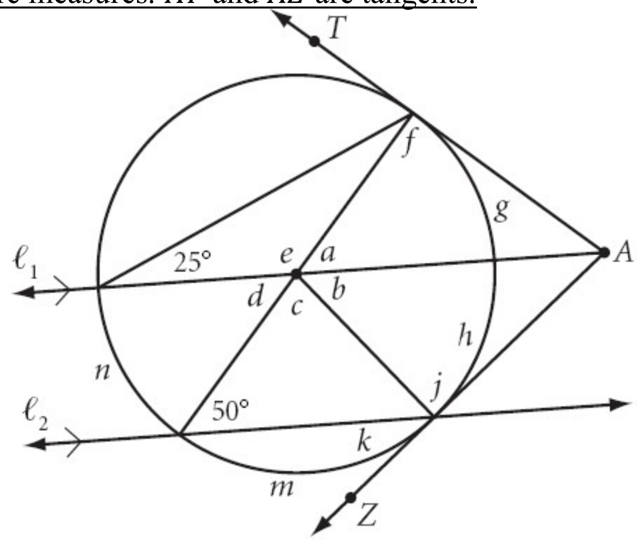
We know that all arcs of a circle should add to $360°$ and we know that s and q should be congruent since the chords that form them are congruent. $\frac{360-(58+12\ )}{2} = 87$.

$\boldsymbol{s = 87°}$

$\boldsymbol{q = 87°}$

<u>Example 9: Find the lettered angle and arc measures. $\overrightarrow{AT}$ and $\overrightarrow{AZ}$ are tangents.</u>

$a =$________
$b =$________
$c =$________
$d =$________
$e =$________
$f =$________
$g =$________
$h =$________
$j =$________
$k =$________
$m =$________
$n =$________



a is a corresponding angle to the 50° angle.

$\boldsymbol{a = 50°}$

g is the arc formed by the central angle a, so it must be congruent to a.

$\boldsymbol{g = 50°}$

d is a vertical angle to a.

$\boldsymbol{d = 50°}$

n is the arc formed by the central angle d, so it must be congruent to d.

$\boldsymbol{n = 50°}$

h and n are arcs intercepted by congruent lines and must be congruent.

$\boldsymbol{h = 50°}$

h is the arc formed by the central angle b, so b must be congruent to h.

$\boldsymbol{b = 50°}$

f is formed by a radius to a tangent and must be a right angle.

$\boldsymbol{f = 90°}$

j is formed by a radius to a tangent and must be a right angle.

$\boldsymbol{j = 90°}$

e forms a linear pair with a.

$\boldsymbol{e = 130°}$

d, b, and c form a line and must add to $180°$.

$\boldsymbol{c = 80°}$

m is the arc formed by the central angle c, so it must be congruent to c.

$\boldsymbol{m = 80°}$

k, j, and the unmarked angle between them form a line and must add to $180°$. The unmarked angle is the other base angle of an isosceles triangle (because the radii of the circle must be congruent.) So, the unmarked angle is $50°$. $180 - (50 + 90) = 40$.

$\boldsymbol{k = 40°}$