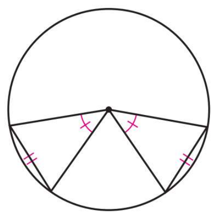
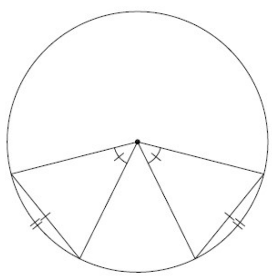
Lesson 6.2 – Chord Properties

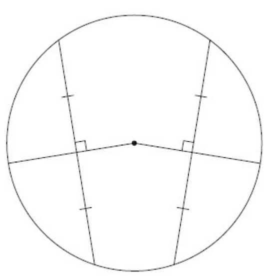
Chord Central Angles Conjecture - If two chords in a circle are congruent, then they determine two central angles that are congruent.



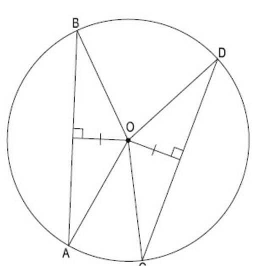
Chord Arc Conjecture - If two chords in a circle are congruent, then their intercepted arcs are congruent.



Perpendicular to a Chord Conjecture - The perpendicular segment from the center of a circle to a chord is the bisector of the chord.



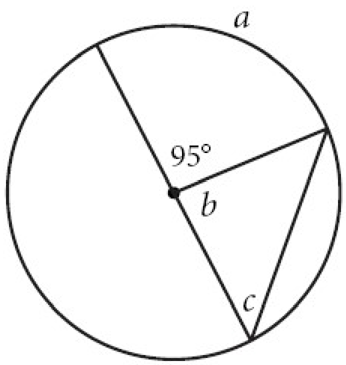
Chord Distance to a Center Conjecture - Two congruent chords in a circle are equidistant from the center of the circle.



$\overline{AB} \cong \overline{CD}$

Example 1: Find the measure of each unknown or write "cannot be determined."

$a =$________, $b =$________, $c =$________



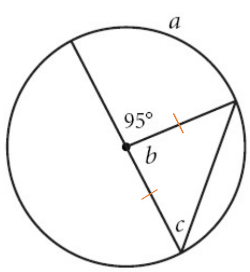
Arc measures are equal to the measures of the central angles that form them. The central angle that forms a is the 95° angle.

$\boldsymbol{a = 95°}$

b and the 95° angle are a linear pair of angles and must add to 180°. $180 - 95 = 85$.

$\boldsymbol{b = 85°}$

Since all radii in a circle must be congruent, we know that the triangle is isosceles.

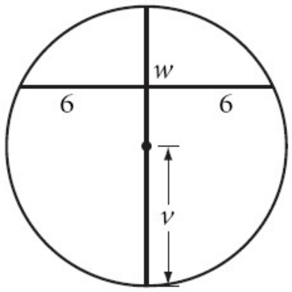


That means that b is the vertex angle in the isosceles triangle. c is a base angle and must be congruent to the other angle in the triangle that is unmarked. $\frac{180-85}{2} = 47.5$.

$\boldsymbol{c = 47.5°}$

Example 2: Find the measure of each unknown or write "cannot be determined."

$w =$_______, $v =$________



The perpendicular to a chord conjecture tells us that is the perpendicular segment from the center of a circle that will bisect a chord. Since the chord is bisected by a segment that passes through the center of the circle we know that the segments must be perpendicular.

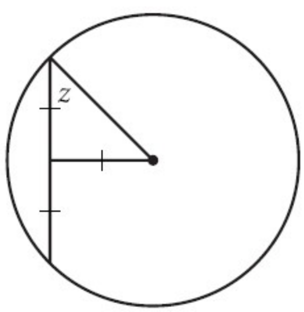
$\boldsymbol{w = 90°}$

We know that the diameter needs to be the longest chord in a circle. We also know that the radius (v) needs to be half the measure of the diameter. The only indication of length we have is that the chord that is not the diameter is 12 units long. We know the diameter should be longer than that, but we don't know how long it is.

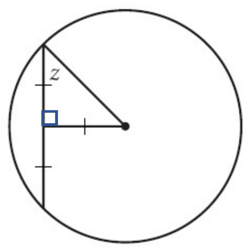
$\boldsymbol{v =}$ **cannot be determined**

Example 3: Find the measure of each unknown or write "cannot be determined."

$z =$_______



The perpendicular to a chord conjecture tells us that is the perpendicular segment from the center of a circle that will bisect a chord. Since the chord is bisected by a segment that passes through the center of the circle we know that the segments must be perpendicular.

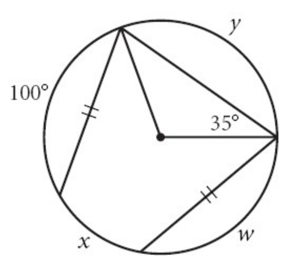


The vertex angle of the isosceles triangle is $90°$. z is one of the base angles of the isosceles triangle. $\frac{180}{2} = 45$.

$\boldsymbol{z = 45°}$

<u>Example 4: Find the measure of each unknown or write "cannot be determined."</u>

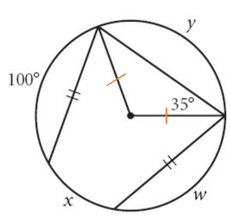
$w =$_______, $x =$________, $y =$________



The chord arcs tell us that if two chords in a circle are congruent, then their intercepted arcs are congruent. So, the two chords that are marked congruent cut off congruent arcs.

$\boldsymbol{w = 100°}$

The radii of a circle are congruent, so we have an isosceles triangle where the base angles both measure $35°$. The vertex angle measures $180 - 2(35) = 110°$. That vertex angle is also the central angle that forms y.



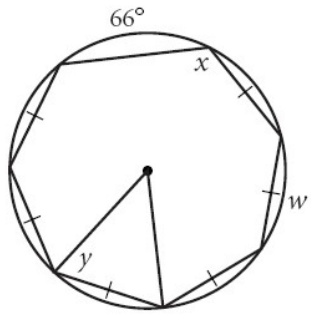
$\boldsymbol{y = 110°}$

The arcs of a circle should add to $360°$. We know the measures of three of the four arcs. $360 - (100 + 100 + 110) = 50$.

$\boldsymbol{x = 50°}$

Example 5: Find the measure of each unknown or write "cannot be determined."

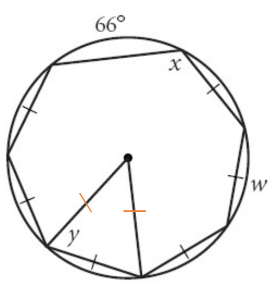
$w =$_______, $x =$________, $y =$________



Congruent chords intercept congruent arcs. All arcs of a circle add to $360°$. We know one of the arcs of the circle measures $66°$ and the other six arcs are all congruent. $\frac{360-6}{6} = 49$.

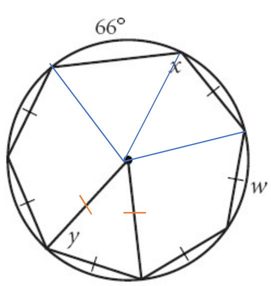
$\boldsymbol{w = 49°}$

The central angles are congruent to the measures of the arcs they intercept. So, the vertex angle of the isosceles triangle is $49°$. y is one of the base angles of the isosceles triangle. $\frac{180-49}{2} =$ 65.5.

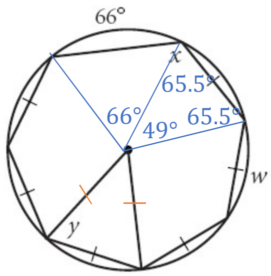


$\boldsymbol{y = 65.5°}$

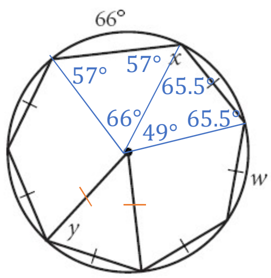
If we create two more triangles, we can use similar reasoning to find the value of x.



One triangle is congruent to the triangle we already solved. The other triangle has a central angle equal to $66°$.



We can solve for the remaining angles in the triangle. $\frac{180-}{2} = 57$.

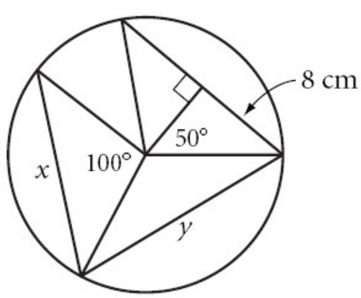


The value of x is the sum of the $57°$ angle and $65.5°$ angle.

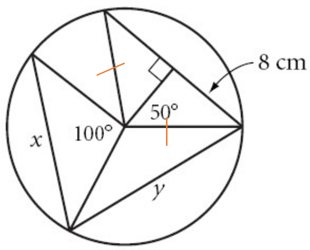
$\boldsymbol{x = 122.5°}$

<u>Example 6: Find the measure of each unknown or write "cannot be determined."</u>

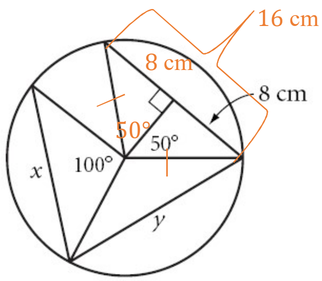
$x =$_______, $y =$________



Radii of a circle are congruent.



The altitude of an isosceles triangle is also the vertex angle bisector and the median.



Since the central angle that forms the chord with length 16 cm is equal to the central angle of the chord that forms x, we know that the chords are congruent.

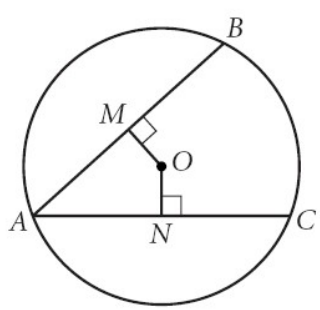
$\boldsymbol{x = 16}$ **cm**

Since there is not any further information about angles, we cannot determine the central angle that forms y.

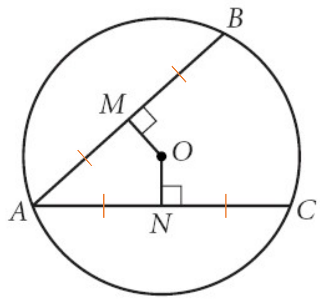
$\boldsymbol{y =}$**cannot be determined**

<u>Example 7: Complete</u>

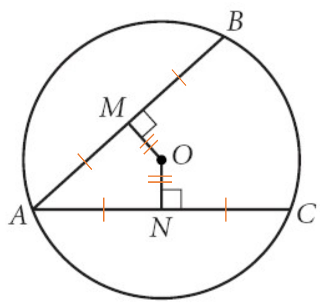
$\overline{AB} \cong \overline{AC}$. $AMON$ is a ________________. Justify your answer.



Since $\overline{OM}$ and $\overline{ON}$ are perpendicular and pass through the center, we know that they bisect $\overline{AB}$ and $\overline{AC}$.



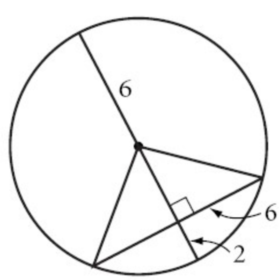
Since $\overline{AB}$ and $\overline{AC}$ are congruent and the perpendicular segment from the center measures distance, we know that $\overline{OM}$ and $\overline{ON}$ are also congruent.



So, we know that ***AMON* is a kite**.

<u>Example 8: Complete</u>

What's wrong with this picture?

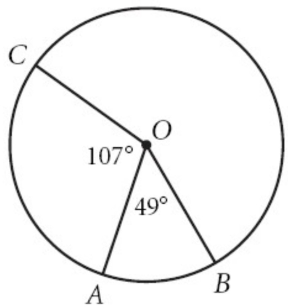


The segment that is perpendicular and passes through the center is the bisector, so the chord that is not the diameter is bisected. That means that the length of that chord should be 12 units. However, the radius is 6 units long and that would make the length of the diameter 12 units. We know that the diameter should be the longest chord in a circle.

The diameter is not the longest chord in the circle.

Example 9: Complete

$m\,\widehat{AB}$ =_______, $m\,\widehat{ABC}$ =________, $m\,\widehat{BAC}$ =________, $m\,\widehat{ACB}$ =________



$\angle AOB$ is the central angle that forms $\widehat{AB}$, so they are congruent.

$\boldsymbol{m\,\widehat{AB} = 49°}$

$\widehat{ABC}$ is the entire circle except the $107°$ arc. So, $360 - 107 = 253$.

$\boldsymbol{m\,\widehat{ABC} = 253°}$

$\widehat{BAC}$ is formed by $\widehat{AB} + \widehat{AC}$. $107 + 49 = 156$.

$\boldsymbol{m\,\widehat{BAC} = 156°}$

$\widehat{ACB}$ is the entire circle except the $49°$ arc. So, $360 - 49 = 311$.

$\boldsymbol{m\,\widehat{ACB} = 311°}$