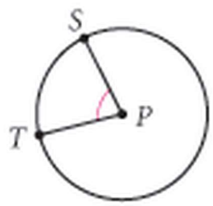
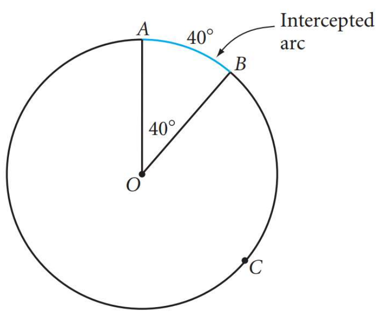
Lesson 6.1 – Tangent Properties

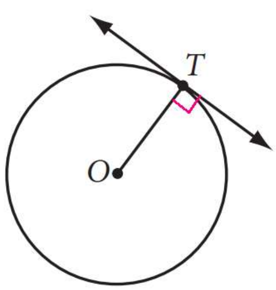
Central Angle - A central angle is an angle whose vertex is the center of the circle and whose sides pass through the circle.



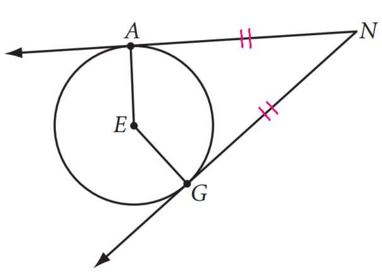
Intercepted Arc - $\angle BOA$ is said to intercept $\widehat{AB}$ because the arc is within the angle. The measure of an arc is defined to be the measure of its central angle.



Tangent Conjecture - A tangent to a circle is perpendicular to the radius drawn to the point of tangency.



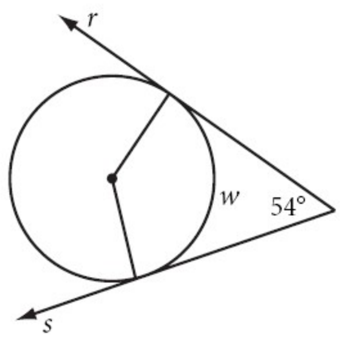
Tangent Segments Conjecture - Tangent segments to a circle from a point outside the circle are congruent.



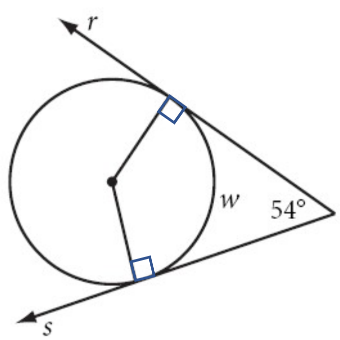
Example 1: Find the missing measure.

Rays r and s are tangents.

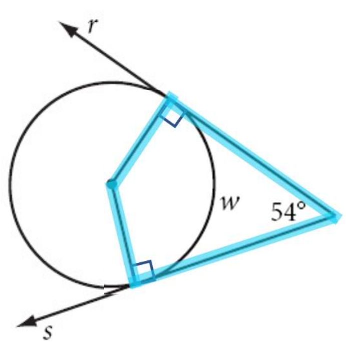
$w =$________



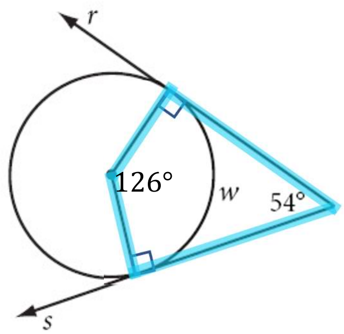
A tangent to a radius makes a right angle.



We can then look at the quadrilateral.



Remember than angles of a quadrilateral add to 360°. $360 - (90 + 90 + 54) = 360 - 234 = 126$

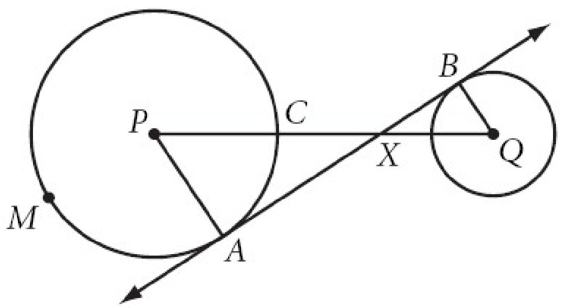


Since the measure of an arc must be the same as the measure of the central angle that forms it, $\boldsymbol{w = 126°}$.

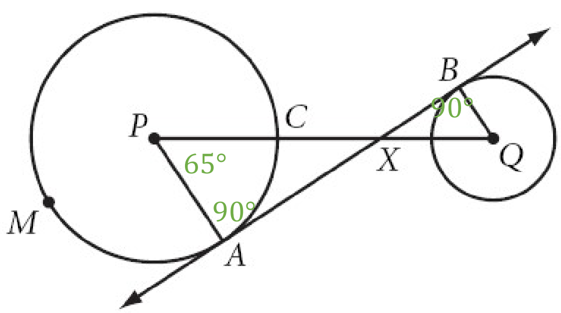
Example 2: Find the missing measure.

$\overleftrightarrow{AB}$ is tangent to both circles and $m\,\widehat{AMC} = 295°$.

$m\angle BQX =$________

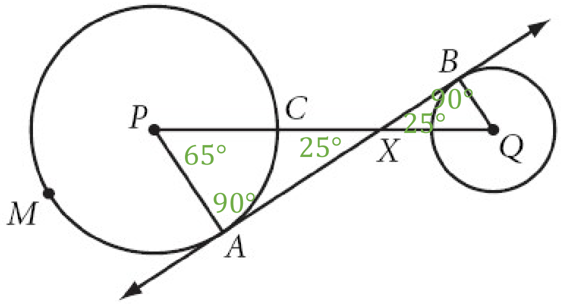


If $m\,\widehat{AMC} = 295°$, then $m\,\widehat{AC} = 65°$. Since arc measures are equal to the measures of the central angles that form $m\angle APC = 65°$. We also know that $\overline{AP}$ and $\overline{QB}$ are radii to tangent $\overleftrightarrow{AB}$. So, $m\angle PAX = 90°$ and $m\angle QBX = 90°$.



We can solve ΔAPX.

$180° - (65° + 90°) = 25°$. Vertical angles are congruent.



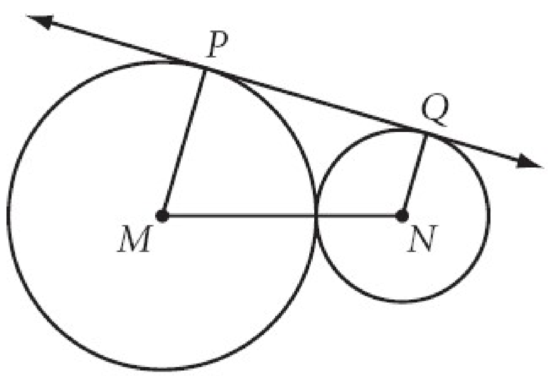
We can now solve ΔBQX. $180° - (25° + 90°) = 65°$.

$m\angle BQX = 65°$

Example 3: Complete.

$\overleftrightarrow{PQ}$ is tangent to two externally tangent noncongruent circles, M and N.

a. $m\angle NQP =$________, $m\angle MPQ =$________
b. What kind of quadrilateral is $MNQP$? Explain your reasoning.



$\overline{MP}$ is a radius that intersects tangent line $\overleftrightarrow{PQ}$ at the point of tangency, so they must be perpendicular.

$\boldsymbol{m\angle MPQ = 90°}$

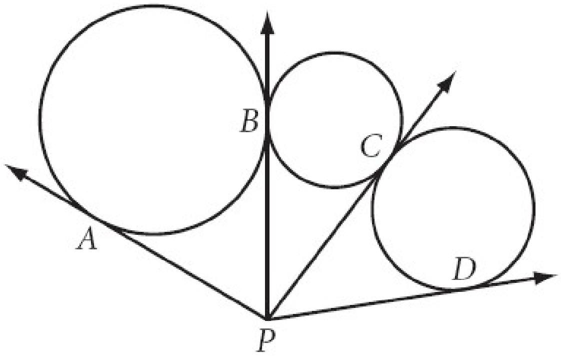
$\overline{NQ}$ is a radius that intersects tangent line $\overleftrightarrow{PQ}$ at the point of tangency, so they must be perpendicular.

$\boldsymbol{m\angle NQP = 90°}$

Since corresponding angles would be congruent, we know that $\overline{MP} || \overline{NQ}$. Since the circles are not congruent, we know that $\overline{MN} \nparallel \overline{PQ}$. **So, the quadrilateral has one pair of parallel sides and is a trapezoid.**

Example 4: Explain why $\overline{PA} \cong \overline{PD}$.

$\overrightarrow{PA}, \overrightarrow{PB}, \overrightarrow{PC}$, and $\overrightarrow{PD}$ are tangents.



Since $\overline{PA}$ and $\overline{PB}$ are tangent to the same circle from the same point outside the circle, we know that $\overline{PA} \cong \overline{PB}$.

Since $\overline{PB}$ and $\overline{PC}$ are tangent to the same circle from the same point outside the circle, we know that $\overline{PB} \cong \overline{PC}$.

Since $\overline{PC}$ and $\overline{PD}$ are tangent to the same circle from the same point outside the circle, we know that $\overline{PC} \cong \overline{PD}$.

So, $\overline{PA} \cong \overline{PB} \cong \overline{PC} \cong \overline{PD}$.