Solving Equations with the Variable on Each Side

Example 1:

$-2 + 10p = 8p - 1$

In these situations, our goal is to have the variables on one side of the equation, not both. So, we can do that by creating the additive identity on either side of the equation for those variable terms. Let's do this problem two different ways so that you can see that it doesn't matter which variable I choose to create the additive identity for.

If I want to create the additive identity for $10p$, I must subtract $10p$ from both sides of the equation.

$-2 + 10p = 8p - 1$

$-10p \quad - 10p$

$-2 = -2p - 1$

Notice that I'm taking the entire variable term and creating the additive identity so that the $(10p - 10p)$ term becomes a zero on the left-hand side. On the right-hand side, I place the $-10p$ underneath the term that it is a like term with $(8p)$ and I calculate $8p - 10p$.

From here, the solving should become a familiar problem.

$-2 = -2p - 1$

$+1 \qquad + 1$

$-1 = -2p$

$\frac{-1}{-2} = \frac{-2p}{-2}$

$\frac{1}{2} = p$

If I want to create the additive identity for $8p$, I must subtract $8p$ from both sides of the equation.

$-2 + 10p = 8p - 1$

$-8p \quad - 8p$

$-2 + 2p = -1$

Notice that I'm taking the entire variable term and creating the additive identity so that the $(8p - 8p)$ term becomes a zero on the right-hand side. On the left-hand side, I place the $-8p$ underneath the term that it is a like term with $(10p)$ and I calculate $10p - 8p$.

From here, the solving should become a familiar problem.

$-2 + 2p = -1$

$+2 \qquad + 2$

$2p = 1$

$\frac{2p}{2} = \frac{1}{2}$

$p = \frac{1}{2}$

Notice that I get the same solution either way that I solve this equation. The solving process just looks a little different in each case.

Let's check our solution to make sure we have a correct solution.

$-2 + 10p = 8p - 1$

$-2 + 10\left(\frac{1}{2}\right) = 8\left(\frac{1}{2}\right) - 1$

$10\left(\frac{1}{2}\right) = \frac{10}{1}\left(\frac{1}{2}\right) = \frac{10}{2} = 5$ $8\left(\frac{1}{2}\right) = \frac{8}{1}\left(\frac{1}{2}\right) = \frac{8}{2} = 4$

$-2 + 5 = 4 - 1$

$3 = 3$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{p = \frac{1}{2}}$.

Example 2:

$3w + 2 = 7w$

In these situations, our goal is to have the variables on one side of the equation, not both. So, we can do that by creating the additive identity on either side of the equation for those variable terms. Let's do this problem two different ways so that you can see that it doesn't matter which variable I choose to create the additive identity for.

If I want to create the additive identity for $3w$, I must subtract $3w$ from both sides of the equation.

$3w + 2 = 7w$

$-3w \quad - 3w$

$2 = 4w$

From here, the solving should become a familiar problem.

$2 = 4w$

$\frac{2}{4} = \frac{4w}{4}$

$\frac{1}{2} = w$

If I want to create the additive identity for $7w$, I must subtract $7w$ from both sides of the equation.

$3w + 2 = 7w$

$-7w \quad - 7w$

$-4w + 2 = 0$

From here, the solving should become a familiar problem.

$-4w + 2 = 0$

$-2 \quad - 2$

$-4w = -2$

$\frac{-4w}{-4} = \frac{-2}{-4}$

$w = \frac{1}{2}$

Notice that I get the same solution either way that I solve this equation. The solving process is a little faster using the method on the left because I never leave one side of the equation with a zero. Leaving one side of the equation with a zero is typically the least efficient method of solving.

Let's check our solution to make sure we have a correct solution.

$3w + 2 = 7w$

$3\left(\frac{1}{2}\right) + 2 = 7\left(\frac{1}{2}\right)$

$3\left(\frac{1}{2}\right) = \frac{3}{1}\left(\frac{1}{2}\right) = \frac{3}{2} = 1\frac{1}{2}$ $\quad$ $7\left(\frac{1}{2}\right) = \frac{7}{1}\left(\frac{1}{2}\right) = \frac{7}{2} = 3\frac{1}{2}$

$1\frac{1}{2} + 2 = 3\frac{1}{2}$

$3\frac{1}{2} = 3\frac{1}{2}$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{w = \frac{1}{2}}$.

<u>Example 3:</u>

$\frac{s}{2} + 1 = \frac{1}{4}s - 6$

Before we start this problem, let's write it out a little differently. We know that $\frac{s}{2} = \frac{1s}{2} = \frac{1}{2}s$. So, if we re-write the equation with a $\frac{1}{2}s$ instead of an $\frac{s}{2}$, the two sides of the equation will look a little more similar.

$\frac{1}{2}s + 1 = \frac{1}{4}s - 6$

Let's do this problem two different ways so that you can see that it doesn't matter which variable I choose to create the additive identity for.

If I want to create the additive identity for $\frac{1}{2}s$, I must subtract $\frac{1}{2}s$ from both sides of the equation.

$\frac{1}{2}s + 1 = \frac{1}{4}s - 6$

$-\frac{1}{2}s \quad -\frac{1}{2}s$

If I want to create the additive identity for $\frac{1}{4}s$, I must subtract $\frac{1}{4}s$ from both sides of the equation.

$\frac{1}{2}s + 1 = \frac{1}{4}s - 6$

$-\frac{1}{4}s \quad -\frac{1}{4}s$

$\frac{1}{4}s - \frac{1}{2}s = \frac{1}{4}s - \frac{1 \cdot 2}{2 \cdot 2}s = \frac{1}{4}s - \frac{2}{4}s = -\frac{1}{4}s$

$$1 = -\frac{1}{4}s - 6$$

From here, the solving should become a familiar problem.

$1 = -\frac{1}{4}s - 6$

$+6 \qquad +6$

$7 = -\frac{1}{4}s$

$\frac{7}{-\frac{1}{4}} = \frac{-\frac{1}{4}s}{-\frac{1}{4}}$

$7 \div -\frac{1}{4} = \frac{7}{1} \div -\frac{1}{4} = \frac{7}{1} \cdot -\frac{4}{1} = -\frac{28}{1} = -28$

$-28 = s$

$\frac{1}{2}s - \frac{1}{4}s = \frac{1 \cdot 2}{2 \cdot 2}s - \frac{1}{4}s = \frac{2}{4}s - \frac{1}{4}s = \frac{1}{4}s$

$\frac{1}{4}s + 1 = -6$

From here, the solving should become a familiar problem.

$\frac{1}{4}s + 1 = -6$

$-1 \quad -1$

$\frac{1}{4}s = -7$

$\frac{\frac{1}{4}s}{\frac{1}{4}} = \frac{-7}{\frac{1}{4}}$

$-7 \div \frac{1}{4} = -\frac{7}{1} \div \frac{1}{4} = -\frac{7}{1} \cdot \frac{4}{1} = -\frac{28}{1} = -28$

$s = -28$

I get the same solution regardless of which way I solve the equation.

Let's check our solution to make sure we have a correct solution.

$\frac{s}{2} + 1 = \frac{1}{4}s - 6$

$\frac{-28}{2} + 1 = \frac{1}{4}(-28) - 6$

$\frac{1}{4}(-28) = \frac{1}{4}\left(\frac{-28}{1}\right) = -\frac{28}{4} = -7$

$-14 + 1 = -7 - 6$

$-13 = -13$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{s = -28}$

Example 4:

$4(2r - 8) = \frac{1}{7}(49r + 70)$

Before we can start thinking about creating an additive identity for the variable on one side, we need to consider our reverse order of operations. That tells us that we have to leave anything in parentheses until the end. If I look at what is outside the parentheses, I'm not really sure what to do with that. There are a couple of things I could do, but the easiest method is to get rid of the parentheses. I can do that using the distributive property on both sides of the equation.

$4(2r-8)=\frac{1}{7}(49r+70)$

$$4(2r-8)=4\cdot 2r-4\cdot 8=8r-32$$

$$\frac{1}{7}(49r+70)=\frac{1}{7}\cdot 49r+\frac{1}{7}\cdot 70=7r+10$$

$8r-32=7r+10$

From here, I can choose to solve the equation by creating an additive identity for either variable.

$8r-32=7r+10$

$-7r \qquad -7r$

$r-32=10$

$+32 \quad +32$

$r=42$

$8r-32=7r+10$

$-8r \qquad -8r$

$-32=-r+10$

$-10 \qquad -10$

$-42=-r$

$\frac{-42}{-1}=\frac{-r}{-1}$

$42=r$

Let's check our solution to make sure we have a correct solution.

$4(2r-8)=\frac{1}{7}(49r+70)$

$4(2(42)-8)=\frac{1}{7}(49(42)+70)$

$4(84-8)=\frac{1}{7}(2058+70)$

$4(76)=\frac{1}{7}(2128)$

$$\frac{1}{7}\left(\frac{2128}{1}\right)=\frac{2128}{7}=304$$

$304=304$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{r=42}$.

Example 5:

$8s - 10 = 3(6 - 2s)$

The first thing we should do on this problem is get rid of the parentheses using the distributive property. I only need to use the distributive property on the right-hand side of the equation since there are no parentheses on the left-hand side.

$8s - 10 = 3(6 - 2s)$

$$3(6 - 2s) = 3 \cdot 6 - 3 \cdot 2s = 18 - 6s$$

$8s - 10 = 18 - 6s$

From here, I can choose to solve the equation by creating an additive identity for either variable.

$8s - 10 = 18 - 6s$	$8s - 10 = 18 - 6s$
$+6s \qquad + 6s$	$-8s \qquad - 8s$
$14s - 10 = 18$	$-10 = 18 - 14s$
$+10 \quad + 10$	$-18 \quad - 18$
$14s = 28$	$-28 = -14s$
$\frac{14s}{14} = \frac{28}{14}$	$\frac{-28}{-14} = \frac{-14s}{-14}$
$s = 2$	$2 = s$

Let's check our solution to make sure we have a correct solution.

$8s - 10 = 3(6 - 2s)$

$8(2) - 10 = 3(6 - 2(2))$

$16 - 10 = 3(6 - 4)$

$6 = 3(2)$

$6 = 6$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{s = 2}$.

Example 6:

$7(n - 1) = -2(3 + n)$

Let's start by getting rid of the parentheses using the distributive property on both sides of the equation.

$7(n-1) = -2(3+n)$

$$7(n-1) = 7 \cdot n - 7 \cdot 1 = 7n - 7$$

$$-2(3+n) = -2 \cdot 3 + -2 \cdot n = -6 - 2n$$

$7n - 7 = -6 - 2n$

From here, I can choose to solve the equation by creating an additive identity for either variable.

$7n - 7 = -6 - 2n$	$7n - 7 = -6 - 2n$
$+2n \qquad + 2n$	$-7n \qquad - 7n$
$9n - 7 = -6$	$-7 = -6 - 9n$
$+7 \quad + 7$	$+6 \quad + 6$
$9n = 1$	$-1 = -9n$
$\frac{9n}{9} = \frac{1}{9}$	$\frac{-1}{-9} = \frac{-9n}{-9}$
$n = \frac{1}{9}$	$\frac{1}{9} = n$

Let's check our solution to make sure we have a correct solution.

$7(n-1) = -2(3+n)$

$7\left(\frac{1}{9} - 1\right) = -2\left(3 + \frac{1}{9}\right)$

$$\frac{1}{9} - 1 = \frac{1}{9} - \frac{1}{1} = \frac{1}{9} - \frac{1 \cdot 9}{1 \cdot 9} = \frac{1}{9} - \frac{9}{9} = -\frac{8}{9}$$

$$3 + \frac{1}{9} = \frac{3}{1} + \frac{1}{9} = \frac{3 \cdot 9}{1 \cdot 9} + \frac{1}{9} = \frac{27}{9} + \frac{1}{9} = \frac{28}{9}$$

$7\left(-\frac{8}{9}\right) = -2\left(\frac{28}{9}\right)$

$$7\left(-\frac{8}{9}\right) = \frac{7}{1}\left(-\frac{8}{9}\right) = -\frac{56}{9} = -6\frac{2}{9}$$

$$-2\left(\frac{28}{9}\right) = -\frac{2}{1}\left(\frac{28}{9}\right) = -\frac{56}{9} = -6\frac{2}{9}$$

$-6\frac{2}{9} = -6\frac{2}{9}$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{n = \frac{1}{9}}$.

Example 7:

$2m + 5 = 5(m - 7) - 3m$

Let's start by getting rid of the parentheses using the distributive property.

$2m + 5 = 5(m - 7) - 3m$

$$5(m - 7) = 5 \cdot m - 5 \cdot 7 = 5m - 35$$

$2m + 5 = 5m - 35 - 3m$

Since we have like terms on the right-hand side of the equation, we should combine those.

$2m + 5 = 5m - 35 - 3m$

$$5m - 3m = 2m$$

$2m + 5 = 2m - 35$

From here, I can choose to solve the equation by creating an additive identity for either variable.

$2m + 5 = 2m - 35$

$-2m \qquad -2m$

$5 = -35$

$2m + 5 = 2m - 35$

$-2m \qquad -2m$

$5 = -35$

In either situation, the variable is now zero. This means that our variable terms are equal on both sides of the equation. What we can plug into this equation to make it true depends on what we are left with.

In this case we are left with the equation $5 = -35$, which is not true. Five does not equal negative thirty-five. This means that no value will ever satisfy the conditions of this equation. This equation has no solution.

No Solution

Example 8:

$3(r + 1) - 5 = 3r - 2$

Let's start by getting rid of the parentheses using the distributive property.

$3(r + 1) - 5 = 3r - 2$

$$3(r + 1) = 3 \cdot r + 3 \cdot 1 = 3r + 3$$

$3r + 3 - 5 = 3r - 2$

Since we have like terms on the leftt-hand side of the equation, we should combine those.

$3r + 3 - 5 = 3r - 2$

$$+3 - 5 = -2$$

$3r - 2 = 3r - 2$

From here, I can choose to solve the equation by creating an additive identity for either variable.

$3r - 2 = 3r - 2$

$-3r \quad\ -3r$

$-2 = -2$

$3r - 2 = 3r - 2$

$-3r \quad\ -3r$

$-2 = -2$

In either situation, the variable is now zero. This means that our variable terms are equal on both sides of the equation. What we can plug into this equation to make it true depends on what we are left with.

In this case we are left with the equation $-2 = -2$, which is true. Negative two does equal negative two. This means that *any* value will ever satisfy the conditions of this equation. This equation has all numbers as its solution.

All Real Numbers

Example 9:

$7x + 5(x - 1) = -5 + 12x$

Let's start by getting rid of the parentheses using the distributive property.

$7x + 5(x - 1) = -5 + 12x$

$$5(x - 1) = 5 \cdot x - 5 \cdot 1 = 5x - 5$$

$7x + 5x - 5 = -5 + 12x$

Since we have like terms on the left-hand side of the equation, we should combine those.

$7x + 5x - 5 = -5 + 12x$

$$7x + 5x = 12x$$

$12x - 5 = -5 + 12x$

From here, I can choose to solve the equation by creating an additive identity for either variable.

$12x - 5 = -5 + 12x$	$12x - 5 = -5 + 12x$
$-12x \qquad\qquad -12x$	$-12x \qquad\qquad -12x$
$-5 = -5$	$-5 = -5$

In either situation, the variable is now zero. This means that our variable terms are equal on both sides of the equation. What we can plug into this equation to make it true depends on what we are left with.

In this case we are left with the equation $-5 = -5$, which is true. Negative five does equal negative five. This means that *any* value will ever satisfy the conditions of this equation. This equation has all numbers as its solution.

All Real Numbers

Example 10:

$6(y - 5) = 2(10 + 3y)$

Let's start by getting rid of the parentheses using the distributive property.

$6(y - 5) = 2(10 + 3y)$

$$6(y - 5) = 6 \cdot y - 6 \cdot 5 = 6y - 30$$

$$2(10 + 3y) = 2 \cdot 10 + 2 \cdot 3y = 20 + 6y$$

$6y - 30 = 20 + 6y$

From here, I can choose to solve the equation by creating an additive identity for either variable.

$6y - 30 = 20 + 6y$	$6y - 30 = 20 + 6y$
$-6y \qquad\qquad -6y$	$-6y \qquad\qquad -6y$
$-30 = 20$	$-30 = 20$

In either situation, the variable is now zero. This means that our variable terms are equal on both sides of the equation. What we can plug into this equation to make it true depends on what we are left with.

In this case we are left with the equation $-30 = 20$, which is not true. Negative thirty does not equal twenty. This means that no value will ever satisfy the conditions of this equation. This equation has no solution.

No Solution

Example 11:

$20c + 5 = 5c + 65$

$20c + 5 = 5c + 65$	$20c + 5 = 5c + 65$
$-5c \qquad -5c$	$-20c \qquad -20c$
$15c + 5 = 65$	$5 = -15c + 65$
$-5 \quad -5$	$-65 \qquad -65$
$15c = 60$	$-60 = -15c$
$\frac{15c}{15} = \frac{60}{15}$	$\frac{-60}{-15} = \frac{-15c}{-15}$
$c = 4$	$4 = c$

Let's check our solution to make sure we have a correct solution.

$20c + 5 = 5c + 65$

$20(4) + 5 = 5(4) + 65$

$80 + 5 = 20 + 65$

$85 = 85$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{c = 4}$.

Example 12:

$\frac{3}{8} - \frac{1}{4}t = \frac{1}{2}t - \frac{3}{4}$

$\frac{3}{8} - \frac{1}{4}t = \frac{1}{2}t - \frac{3}{4}$	$\frac{3}{8} - \frac{1}{4}t = \frac{1}{2}t - \frac{3}{4}$
$-\frac{1}{2}t \quad -\frac{1}{2}t$	$+\frac{1}{4}t \quad +\frac{1}{4}t$
$-\frac{1}{4}t - \frac{1}{2}t = -\frac{1}{4}t - \frac{1 \cdot 2}{2 \cdot 2}t$	$\frac{1}{2}t + \frac{1}{4}t = \frac{1 \cdot 2}{2 \cdot 2}t + \frac{1}{4}t$
$= -\frac{1}{4}t - \frac{2}{4}t = -\frac{3}{4}t$	$= \frac{2}{4}t + \frac{1}{4}t = \frac{3}{4}t$
$\frac{3}{8} - \frac{3}{4}t = -\frac{3}{4}$	$\frac{3}{8} = \frac{3}{4}t - \frac{3}{4}$

$$\frac{3}{8} - \frac{3}{4}t = -\frac{3}{4}$$

$$-\frac{3}{8} \qquad -\frac{3}{8}$$

$$-\frac{3}{4} - \frac{3}{8} = -\frac{3 \cdot 2}{4 \cdot 2} - \frac{3}{8}$$

$$= -\frac{6}{8} - \frac{3}{8} = -\frac{9}{8}$$

$$-\frac{3}{4}t = -\frac{9}{8}$$

$$\frac{-\frac{3}{4}t}{-\frac{3}{4}} = \frac{-\frac{9}{8}}{-\frac{3}{4}}$$

$$-\frac{9}{8} \div -\frac{3}{4} = -\frac{9}{8} \cdot -\frac{4}{3} = \frac{36}{24} = \frac{3}{2} = 1\frac{1}{2}$$

$$t = 1\frac{1}{2}$$

$$\frac{3}{8} = \frac{3}{4}t - \frac{3}{4}$$

$$+\frac{3}{4} \qquad +\frac{3}{4}$$

$$\frac{3}{8} + \frac{3}{4} = \frac{3}{8} + \frac{3 \cdot 2}{4 \cdot 2}$$

$$= \frac{3}{8} + \frac{6}{8} = \frac{9}{8}$$

$$\frac{9}{8} = \frac{3}{4}t$$

$$\frac{\frac{9}{8}}{\frac{3}{4}} = \frac{\frac{3}{4}t}{\frac{3}{4}}$$

$$\frac{9}{8} \div \frac{3}{4} = \frac{9}{8} \cdot \frac{4}{3} = \frac{36}{24} = \frac{3}{2} = 1\frac{1}{2}$$

$$1\frac{1}{2} = t$$

Let’s check our solution to make sure we have a correct solution.

$$\frac{3}{8} - \frac{1}{4}t = \frac{1}{2}t - \frac{3}{4}$$

$$\frac{3}{8} - \frac{1}{4}\left(1\frac{1}{2}\right) = \frac{1}{2}\left(1\frac{1}{2}\right) - \frac{3}{4}$$

$$\frac{1}{4}\left(1\frac{1}{2}\right) = \frac{1}{4}\left(\frac{3}{2}\right) = \frac{3}{8} \qquad \frac{1}{2}\left(1\frac{1}{2}\right) = \frac{1}{2}\left(\frac{3}{2}\right) = \frac{3}{4}$$

$$\frac{3}{8} - \frac{3}{8} = \frac{3}{4} - \frac{3}{4}$$

$$0 = 0 \ \checkmark$$

Since our solution checks out, we know we have the correct solution: $\boldsymbol{t = 1\frac{1}{2}}$.

<u>Example 13:</u>

$$3(a - 5) = -6$$

Let’s use the distributive property on the left-hand side of the equation to get rid of the parentheses.

$3(a-5)=-6$

$3(a-5)=3\cdot a-3\cdot 5=3a-15$

$3a-15=-6$

$+15 \quad +15$

$3a=9$

$\frac{3a}{3}=\frac{9}{3}$

$a=3$

Let's check our solution to make sure we have a correct solution

.$3(a-5)=-6$

$3(3-5)=-6$

$3(-2)=-6$

$-6=-6$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{a=3}$.

<u>Example 14:</u>

$6=3+5(d-2)$

Let's use the distributive property on the right-hand side of the equation to get rid of the parentheses.

$6=3+5(d-2)$

**Note: It is a common misconception to want to add the $3+5$ prior to distributing. Remember that order of operations says you cannot add before you multiply.

$5(d-2)=5\cdot d-5\cdot 2=5d-10$

$6=3+5d-10$

$3-10=-7$

$6=-7+5d$

$+7 \quad +7$

$13=5d$

$$\frac{13}{5} = \frac{5d}{5}$$

$$2\frac{3}{5} = d$$

Let's check our solution to make sure we have a correct solution

$$6 = 3 + 5(d - 2)$$

$$6 = 3 + 5\left(2\frac{3}{5} - 2\right)$$

$$6 = 3 + 5\left(\frac{3}{5}\right)$$

$$6 = 3 + 3$$

$$6 = 6 \checkmark$$

Since our solution checks out, we know we have the correct solution: $\boldsymbol{d = 2\frac{3}{5}}$.

<u>Example 15:</u>

$$5 + 2(n + 1) = 2n$$

Let's use the distributive property on the left-hand side of the equation to get rid of the parentheses.

$$5 + 2(n + 1) = 2n$$

**Note: It is a common misconception to want to add the $5 + 2$ prior to distributing. Remember that order of operations says you cannot add before you multiply.

$$2(n + 1) = 2 \cdot n + 2 \cdot 1 = 2n + 2$$

$$5 + 2n + 2 = 2n$$

$$5 + 2 = 7$$

$$7 + 2n = 2n$$

$$\quad -2n \quad -2n$$

$$7 = 0$$

The variable is zero on both sides of the equation. We are left with the equation $7 = 0$, which is not true. No value will ever satisfy the conditions of this equation.

No Solution

Example 16:

$7 - 3r = r - 4(2 + r)$

Let's use the distributive property on the right-hand side of the equation to get rid of the parentheses.

$7 - 3r = r - 4(2 + r)$

**Note: The negative belongs to the four. So, I will distribute a negative 4.

$-4(2 + r) = -4 \cdot 2 + -4 \cdot r = -8 - 4r$

$7 - 3r = r - 8 - 4r$

$r - 4r = -3r$

$7 - 3r = -3r - 8$

$+3r \quad + 3r$

$7 = -8$

The variable is zero on both sides of the equation. We are left with the equation $7 = -8$, which is not true. No value will ever satisfy the conditions of this equation.

No Solution

Example 17:

$14v + 6 = 2(5 + 7v) - 4$

Let's use the distributive property on the right-hand side of the equation to get rid of the parentheses.

$14v + 6 = 2(5 + 7v) - 4$

**Note: The negative belongs to the four. So, I will distribute a negative 4.

$2(5 + 7v) = 2 \cdot 5 + 2 \cdot 7v = 10 + 14v$

$14v + 6 = 10 + 14v - 4$

$10 - 4 = 6$

$14v + 6 = 6 + 14v$

$-14v \qquad - 14v$

$6 = 6$

The variable is zero on both sides of the equation. We are left with the equation $6 = 6$, which is true. *Any* value will ever satisfy the conditions of this equation.

All Real Numbers

Example 18:

$5h - 7 = 5(h - 2) + 3$

Let's use the distributive property on the right-hand side of the equation to get rid of the parentheses.

$5h - 7 = 5(h - 2) + 3$

$5(h - 2) = 5 \cdot h - 5 \cdot 2 = 5h - 10$

$5h - 7 = 5h - 10 + 3$

$-10 + 3 = -7$

$5h - 7 = 5h - 7$

$-5h \quad -5h$

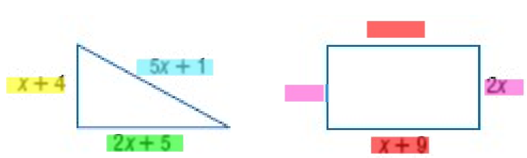
$-7 = -7$

The variable is zero on both sides of the equation. We are left with the equation $-7 = -7$, which is true. *Any* value will ever satisfy the conditions of this equation

All Real Numbers

Example 19:

Find the value of x so that the figures have the same perimeter.



Perimeter is the lengths of all of the sides of the figure added together.

Perimeter of the Triangle:

$x + 4 + 2x + 5 + 5x + 1$

$x + 4 + 2x + 5 + 5x + 1$

Perimeter of the Rectangle:

$2x + 2x + x + 9 + x + 9$

$2x + 2x + x + 9 + x + 9$

$x + 2x + 5x = 8x$	$2x + 2x + x + x = 6x$
$4 + 5 + 1 = 10$	$9 + 9 = 18$
$8x + 10$	$6x + 18$

Now that we have simplified the perimeters as much as possible, we need to find the value that will make the perimeters equal to each other.

$8x + 10 = 6x + 18$

$-6x \qquad -6x$

$2x + 10 = 18$

$\quad -10 \quad -10$

$2x = 8$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

The value of x that will make the figures have the same perimeter is $x = 4$.

<u>Example 20:</u>

Justify each step.

$6n + 7 = 8n - 13$

$6n + 7 - 6n = 8n - 13 - 6n$ a) ?

$7 = 2n - 13$ b) ?

$7 + 13 = 2n - 13 + 13$ c) ?

$20 = 2n$ d) ?

$\frac{20}{2} = \frac{2n}{2}$ e) ?

$10 = n$ f) ?

To complete this problem, we have to look at each step and determine what the solver of the equation did to get from the previous step to this step.

$6n + 7 = 8n - 13$

$6n + 7 - 6n = 8n - 13 - 6n$ a) **Subtract $6n$ from each side of the equation**

$7 = 2n - 13$ b) **Combine like terms**

$7 + 13 = 2n - 13 + 13$ c) **Add 13 to each side of the equation**

$20 = 2n$ d) **Combine like terms**

$\frac{20}{2} = \frac{2n}{2}$ e) **Divide by two on each side of the equation**

$10 = n$ f) **Combine like terms**

<u>Example 21:</u>

Determine whether each solution is correct. If the solution is not correct, find the error and give the correct solution.

$2(g + 5) = 22$	$5d = 2d - 18$	$-6z + 13 = 7z$
$2g + 5 = 22$	$5d - 2d = 2d - 18 - 2d$	$-6z + 13 - 6z = 7z - 6z$
$2g + 5 - 5 = 22 - 5$	$3d = -18$	$13 = z$
$2g = 17$	$\frac{3d}{3} = \frac{-18}{3}$	
$\frac{2g}{2} = \frac{17}{2}$	$d = -6$	
$g = 8.5$		

The first solution on the left is incorrect. In the second row down the solver should have distributed the two through the set of parentheses. The correct solution should look like:

$2(g + 5) = 22$

$2g + 10 = 22$

$2g + 10 - 10 = 22 - 10$

$2g = 12$

$\frac{2g}{2} = \frac{12}{2}$

$g = 6$

The equation in the middle is solved correctly.

The equation on the right is solved incorrectly. In the second row down the solver should have added $6z$ to both sides to create the additive identity. The correct solution should look like:

$-6z + 13 = 7z$

$-6z + 13 + 6z = 7z + 6z$

$13 = 13z$

$\frac{13}{13} = \frac{13z}{13}$

$1 = z$