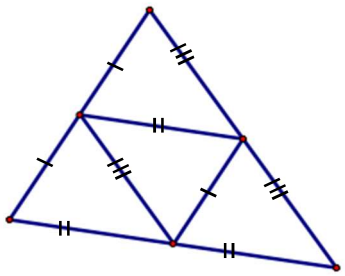
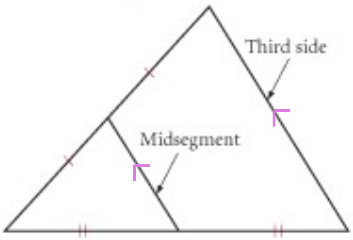
Lesson 5.4 – Properties of Midsegments

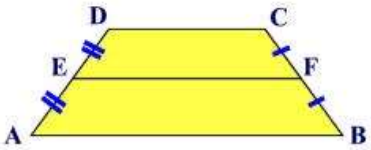
Three Midsegments Conjecture - The three midsegments of a triangle divide it into four congruent triangles.



Triangle Midsegment Conjecture - A midsegment of a triangle is parallel to the third side and half the length of the third side.



Trapezoid Midsegment Conjecture - A midsegment of a trapezoid is parallel to the bases and is equal in length to the average of the lengths of the bases.



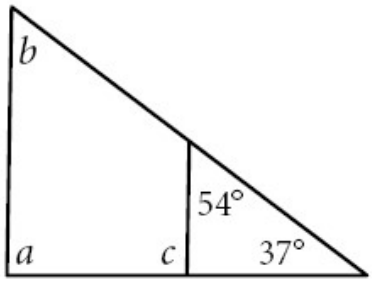
$\overline{EF} \parallel \overline{DC}$; $\overline{EF} \parallel \overline{AB}$

$$EF = \frac{1}{2}(DC + AB)$$

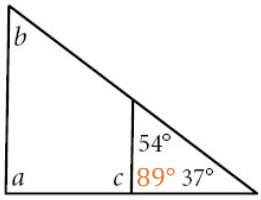
Example 1: Find the missing measures.

The figure shows a midsegment

$a =$_____, $b =$_____, $c =$_____



Let's start by finding the missing angle measure in the small triangle to the right.



Because the midsegment is parallel to the third side of the triangle, we know that a and the 89° angle are corresponding angles on parallel lines and must be congruent.

$\boldsymbol{a = 89°}$

The midsegment is parallel to the third side and that means that b is a corresponding angle on parallel lines to the 54° angle.

$\boldsymbol{b = 54°}$

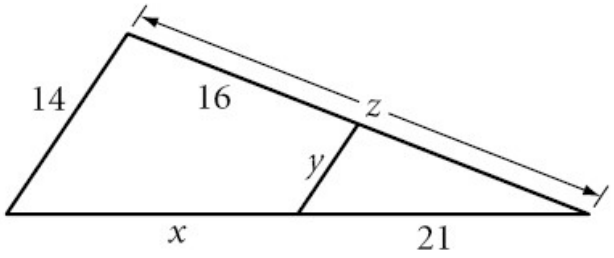
c is a linear pair with the 89° angle. $180 - 89 = 91$

$\boldsymbol{c = 91°}$

Example 2: Find the missing measures.

The figure shows a midsegment

$x =$_____, $y =$_____, $z =$_____



Since we know that the figure shows a midsegment, we know that the midsegment bisects the two sides it intersects. So, x must be the same length as 21.

$\boldsymbol{x = 21}$

The midsegment of a triangle is parallel to the third side and half the length of the third side. Since the third side has a length of 14, y must have a length of $\frac{14}{2} = 7$.

$\boldsymbol{y = 7}$

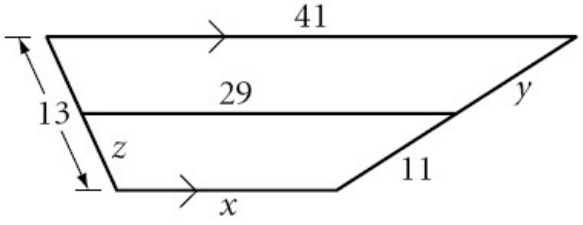
We know that the figure shows a midsegment, so the right side of the top segment must be 16. That means the entire length of the segment (z) is $16 + 16 = 32$.

$\boldsymbol{z = 32}$

Example 3: Find the missing measures.

The figure shows a midsegment

$x =$_____, $y =$_____, $z =$_____



x and 41 are the lengths of the bases of the trapezoid. 29 is the length of the midsegment. We know that the length of the midsegments should be equal to the average of the bases.

**Remember that to average two things, we add them up and divide by 2.

$$\frac{x+41}{2}=29$$

$$2\cdot\frac{x+41}{2}=29\cdot 2$$

$$x+41=58$$

$$-41 \quad -41$$

$$x=17$$

Since the figure shows a midsegment, we know that y should be the same measure as the segment below it (11).

$$y=11$$

The figure shows a midsegment, so we know that the midsegment divides the side with length 13 in half.

$$\frac{13}{2}=6.5$$

$$z=6.5$$

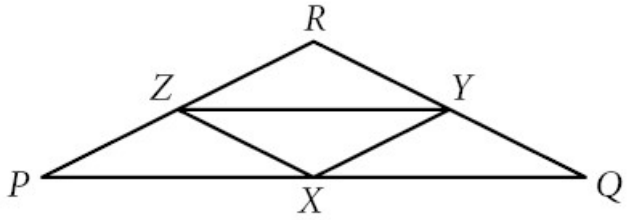
Example 4: Find the missing measures.

X, Y, and Z are midpoints. Perimeter $\Delta PQR=132, RQ=55$, and $PZ=20$.

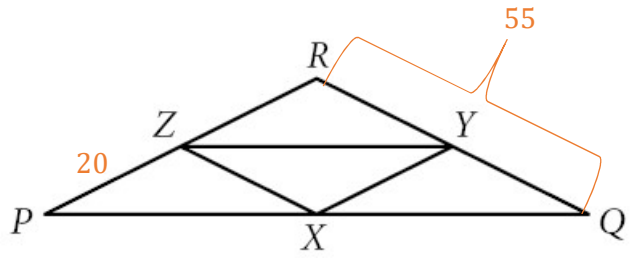
Perimeter $\Delta XYZ=$_____

$PQ=$_____

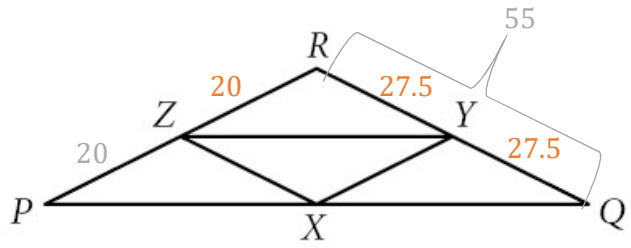
$ZX=$_____



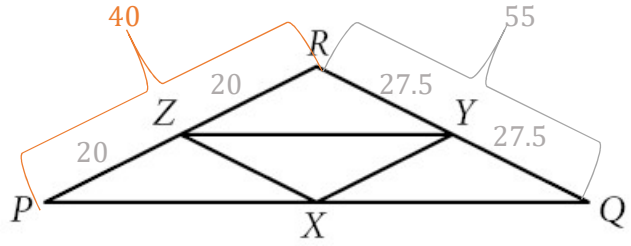
Let's start by labeling the two sides given to us.



Since we know that X and Y are midpoints, we know that $PZ = ZR$. We also know that RY and YQ are half of RQ.



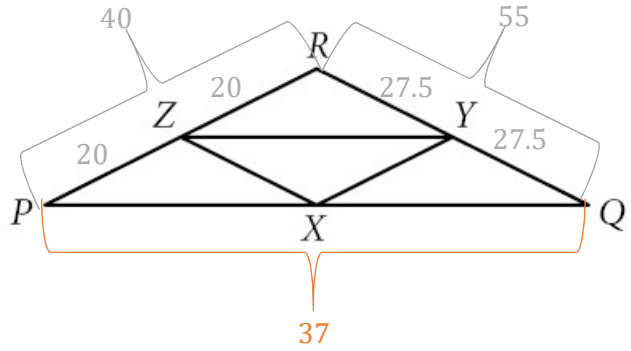
We know that we can add PZ and ZR to find PR.



The perimeter of $\Delta PQR = 132$. So, we can subtract 40 and 55 from that perimeter to find the length of side PQ.

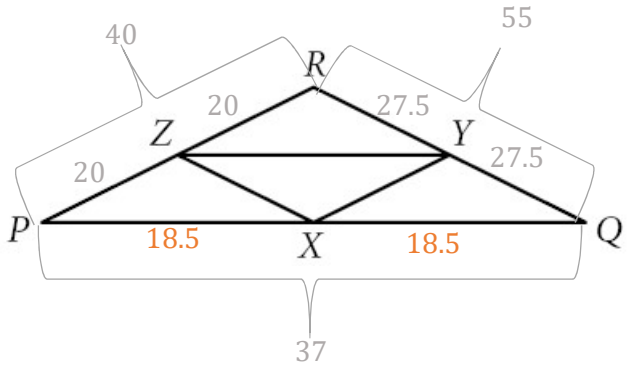
$132 - (40 + 55) = 132 - 95 = 37$

$\boldsymbol{PQ = 37}$

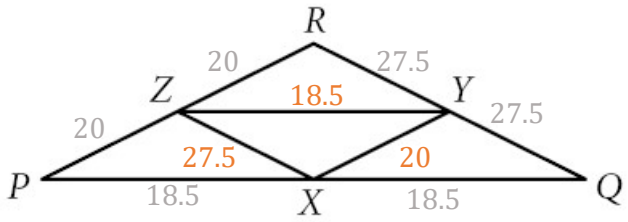


Since X is a midpoint, we know that PX and XQ are half of PQ.

$\frac{37}{2} = 18.5$



The three midsegments conjecture shows us that $\overline{ZX} \cong \overline{RY} \cong \overline{YQ}$, $\overline{XY} \cong \overline{PZ} \cong \overline{ZR}$, and $\overline{ZY} \cong \overline{PX} \cong \overline{XQ}$.



$\boldsymbol{ZX = 27.5}$

To find perimeter ΔXYZ, we need to add the three sides of the triangle.

$27.5 + 18.5 + 20 = 66$

Perimeter $\boldsymbol{\Delta XYZ = 66}$