<u>Solving Multi-Step Equations with the Variable on One Side</u>

Remember that the **order of operations** is the rule that lets you know which operations to perform first in numerical expressions.

- Grouping symbols (Parentheses) ( ) or [ ]
- Powers (Exponents) x^n
- Multiply and Divide $\cdot$ $\div$
  from left to right
- Add and Subtract $+$ $-$
  from left to right

When solving multi-step equations with the variable on one side, we use the order of operations in reverse to determine which things to handle first in the solving of an equation.

So, we will solve equations using this order of operations:

- Add and Subtract $+$ $-$
  from left to right
- Multiply and Divide $\cdot$ $\div$
  from left to right
- Powers (Exponents) x^n
- Grouping symbols (Parentheses) ( ) or [ ]

Remember that our goal is always to create the additive identity of zero for addition or subtraction and the multiplicative identity of one for multiplication or division.

<u>Example 1:</u>

$7m - 17 = 60$

As we look at this equation, we can see that there are two numbers on the same side of the equation as the variable (7 and -17). In order to determine what number we should handle first, we should look at our reversed order of operations. The reversed order of operations says we should handle addition and subtraction first. That will be followed by taking care of any multiplication or division.

This means we should handle the -17 first since it is subtracted from the variable and then we will handle the 7 since it is multiplied into the variable.

Remember that in cases of addition and subtraction, we are trying to create the additive identity (zero). So, to make -17 equal zero, we should add 17 to both sides.

$7m - 17 = 60$

$\quad +17 \quad + 17$

$7m = 77$

**The $7m$ doesn't change because we didn't do anything to that. The $-17 + 17$ becomes a zero that we don't need to write since it doesn't change anything. We add on the right-hand side; $60 + 17 = 77$.

Now, we have to handle the 7. In cases of multiplication and division, we are trying to create the multiplicative identity (one). To make the $7 \cdot$ equal one, we will need to divide by 7 on both sides.

$7m = 77$

$\frac{7m}{7} = \frac{77}{7}$

$m = 11$

**Remember that the $\frac{7}{7} = 1$ so we don't need to write that and we calculate the right-hand side; $\frac{77}{7} = 11$.

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$7m - 17 = 60$

$7(11) - 17 = 60$

$77 - 17 = 60$

$60 = 60$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{m = 11}$.

Example 2:

$2a - 6 = 4$

As we look at this equation, we can see that there are two numbers on the same side of the equation as the variable (2 and -6). In order to determine what number we should handle first, we should look at our reversed order of operations. The reversed order of operations says we should handle addition and subtraction first. That will be followed by taking care of any multiplication or division.

This means we should handle the -6 first since it is subtracted from the variable and then we will handle the 2 since it is multiplied into the variable.

Remember that in cases of addition and subtraction, we are trying to create the additive identity (zero). So, to make -6 equal zero, we should add 6 to both sides.

$2a - 6 = 4$

$\quad +6 \quad +6$

$2a = 10$ **The $2a$ doesn't change because we didn't do anything to that. The $-6 + 6$ becomes a zero that we don't need to write since it doesn't change anything. We add on the right-hand side; $4 + 6 = 10$.

Now, we have to handle the 2. In cases of multiplication and division, we are trying to create the multiplicative identity (one). To make the $2 \cdot$ equal one, we will need to divide by 2 on both sides.

$2a = 10$

$\frac{2a}{2} = \frac{10}{2}$

$a = 5$ **Remember that the $\frac{2}{2} = 1$ so we don't need to write that and we calculate the right-hand side; $\frac{10}{2} = 5$.

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$2a - 6 = 4$

$2(5) - 6 = 4$

$10 - 6 = 4$

$4 = 4$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{a = 5}$.

<u>Example 3:</u>

$8 = 3r + 7$

The fact that the variable is on the right-hand side of the equation doesn't change how we solve it. Just isolate the variable on the right side.

As we look at this equation, we can see that there are two numbers on the same side of the equation as the variable (3 and 7). In order to determine what number we should handle first, we should look at our reversed order of operations. The reversed order of operations says we should handle addition and subtraction first. That will be followed by taking care of any multiplication or division.

This means we should handle the 7 first since it is added to the variable and then we will handle the 3 since it is multiplied into the variable.

Remember that in cases of addition and subtraction, we are trying to create the additive identity (zero). So, to make 7 equal zero, we should subtract 7 from both sides.

$8 = 3r + 7$

$-7 \qquad -7$

$1 = 3r$

**The $3r$ doesn't change because we didn't do anything to that. The $7 - 7$ becomes a zero that we don't need to write since it doesn't change anything. We add on the left-hand side; $8 - 7 = 1$.

Now, we have to handle the 3. In cases of multiplication and division, we are trying to create the multiplicative identity (one). To make the $3 \cdot$ equal one, we will need to divide by 3 on both sides.

$1 = 3r$

$\frac{1}{3} = \frac{3r}{3}$

$\frac{1}{3} = r$

**Remember that the $\frac{3}{3} = 1$ so we don't need to write that and we calculate the left-hand side; $\frac{1}{3} = \frac{1}{3}$. It is a common misconception to make $\frac{1}{3} = 3$. Remember that if the numerator is smaller than the denominator, the solution cannot be a whole number.

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$8 = 3r + 7$

$8 = 3\left(\frac{1}{3}\right) + 7$

$$3\left(\frac{1}{3}\right) = \frac{3}{1}\left(\frac{1}{3}\right) = \frac{3}{3} = 1$$

$8 = 1 + 7$

$8 = 8$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{r = \frac{1}{3}}$.

Example 4:

$$\frac{t}{8} + 21 = 14$$

As we look at this equation, we can see that there are two numbers on the same side of the equation as the variable (8 and 21). In order to determine what number we should handle first, we should look at our reversed order of operations. The reversed order of operations says we should handle addition and subtraction first. That will be followed by taking care of any multiplication or division.

This means we should handle the 21 first since it is subtracted from the variable and then we will handle the 8 since it is divided from the variable.

Remember that in cases of addition and subtraction, we are trying to create the additive identity (zero). So, to make 21 equal zero, we should subtract 21 from both sides.

$$\frac{t}{8} + 21 = 14$$

$$-21 \quad -21$$

$$\frac{t}{8} = -7$$

Now, we have to handle the 8. In cases of multiplication and division, we are trying to create the multiplicative identity (one). To make the $/8$ equal one, we will need to multiply by 8 on both sides.

$$\frac{t}{8} = -7$$

$$8 \cdot \frac{t}{8} = -7 \cdot 8$$

$$t = -56$$

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$$\frac{t}{8} + 21 = 14$$

$$\frac{-56}{8} + 21 = 14$$

$$-7 + 21 = 14$$

$$14 = 14 \quad \checkmark$$

Since our solution checks out, we know we have the correct solution: $\boldsymbol{t = -56}$.

Example 5:

$$\frac{p - 15}{9} = -6$$

This problem looks a little bit different than the other problems. When we see a large fraction bar like in this problem, we assume that there are parentheses around both the numerator and denominator.

$$\frac{(p - 15)}{(9)} = -6$$

We don't need the parentheses around the denominator since there is only one thing in the denominator.

$$\frac{(p - 15)}{9} = -6$$

The reversed order of operations says we should handle addition and subtraction first. That will be followed by taking care of any multiplication or division. We should handle anything in parentheses last. If we ignore what is in the parentheses, the only thing outside of the parentheses (on the same side of the equation as the variable) is a division by 9. So, we should handle that first.

To make the $/9$ equal one, we will need to multiply by 9 on both sides.

$$\frac{(p - 15)}{9} = -6$$

$$9 \cdot \frac{(p - 15)}{9} = -6 \cdot 9$$

$$(p - 15) = -54$$

We no longer need the parentheses around the left-hand side of the equation since there is nothing else left on that side except what is in the parentheses.

$$p - 15 = -54$$

Now, we need to make -15 equal zero, we should add 15 to both sides.

$p - 15 = -54$

$\quad +15 \quad + 15$

$p = -39$

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$\frac{p - 15}{9} = -6$

$\frac{-39 - 15}{9} = -6$

$\frac{-54}{9} = -6$

$-6 = -6 \qquad \checkmark$

Since our solution checks out, we know we have the correct solution: $\boldsymbol{p = -39}$.

<u>Example 6:</u>

$4 = \frac{k - 12}{5}$

When we see a large fraction bar like in this problem, we assume that there are parentheses around both the numerator and denominator. However, we don't need to write the parentheses around the denominator since there is only one thing in the denominator.

$4 = \frac{(k - 12)}{5}$

The reversed order of operations says we should handle anything in parentheses last. If we ignore what is in the parentheses, the only thing outside of the parentheses (on the same side of the equation as the variable) is a division by 5. So, we should handle that first.

To make the $/5$ equal one, we will need to multiply by 5 on both sides.

$4 = \frac{(k - 12)}{5}$

$5 \cdot 4 = \frac{(k - 12)}{5} \cdot 5$

$20 = (k - 12)$

We no longer need the parentheses around the right-hand side of the equation since there is nothing else left on that side except what is in the parentheses.

$20 = k - 12$

Now, we need to make -12 equal zero, we should add 12 to both sides.

$20 = k - 12$

$+12 \quad\quad + 12$

$32 = k$

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$$4 = \frac{k - 12}{5}$$

$$4 = \frac{32 - 12}{5}$$

$$4 = \frac{20}{5}$$

$4 = 4$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{k = 32}$.

Example 7:

$$\frac{n + 1}{-2} - 1 = 14$$

When we see a large fraction bar like in this problem, we assume that there are parentheses around both the numerator and denominator. We don't need the parentheses around the denominator since there is only one thing in the denominator.

$$\frac{(n + 1)}{-2} - 1 = 14$$

We should handle anything in parentheses last. If we ignore what is in the parentheses, there are two numbers outside of the parentheses (on the same side of the equation as the variable); a division by -2 and a subtraction of 1. So, we should handle the subtraction first according to our reversed order of operations. Then, we will handle the division.

To make -1 equal zero, we should add 1 to both sides.

$$\frac{(n+1)}{-2}-1=14$$

$$+1 \quad +1$$

$$\frac{(n+1)}{-2}=15$$

To make the $/-2$ equal one, we will need to multiply by -2 on both sides.

$$\frac{(n+1)}{-2}=15$$

$$-2\cdot\frac{(n+1)}{-2}=15\cdot-2$$

$$(n+1)=-30$$

We no longer need the parentheses around the left-hand side of the equation since there is nothing else left on that side except what is in the parentheses.

$$n+1=-30$$

Now, we need to make 1 equal zero, we should subtract 1 from both sides.

$$n+1=-30$$

$$-1 \quad -1$$

$$n=-31$$

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$$\frac{n+1}{-2}-1=14$$

$$\frac{-31+1}{-2}-1=14$$

$$\frac{-30}{-2}-1=14$$

$$15-1=14$$

$$14=14 \quad \checkmark$$

Since our solution checks out, we know we have the correct solution: $\boldsymbol{n=-31}$.

<u>Example 8:</u>

$$4g-2=-6$$

The reversed order of operations says we should handle addition and subtraction first. That will be followed by taking care of any multiplication or division.

This means we should handle the -2 first since it is subtracted from the variable. To make -2 equal zero, we should add 2 to both sides.

$4g - 2 = -6$

$\quad +2 \quad +2$

$4g = -4$

To make the $4 \cdot$ equal one, we will need to divide by 4 on both sides.

$4g = -4$

$\frac{4g}{4} = \frac{-4}{4}$

$g = -1$

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$4g - 2 = -6$

$4(-1) - 2 = -6$

$-4 - 2 = -6$

$-6 = -6 \quad \checkmark$

Since our solution checks out, we know we have the correct solution: $\boldsymbol{g = -1}$.

<u>Example 9:</u>

$18 = 5p + 3$

The reversed order of operations says we should handle addition and subtraction first. That will be followed by taking care of any multiplication or division.

This means we should handle the 3 first since it is added to the variable. To make 3 equal zero, we should subtract 3 from both sides.

$18 = 5p + 3$

$-3 \quad\quad -3$

$15 = 5p$

To make the $5 \cdot$ equal one, we will need to divide by 5 on both sides.

$15 = 5p$

$\frac{15}{5} = \frac{5p}{5}$

$3 = p$

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$18 = 5p + 3$

$18 = 5(3) + 3$

$18 = 15 + 3$

$18 = 18$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{p = 3}$.

<u>Example 10:</u>

$9 = 1 + \frac{m}{7}$

The reversed order of operations says we should handle addition and subtraction first. That will be followed by taking care of any multiplication or division.

This means we should handle the 1 first since it is added to the variable. To make 1 equal zero, we should subtract 1 from both sides.

$9 = 1 + \frac{m}{7}$

$-1 \ \ -1$

$8 = \frac{m}{7}$

To make the $/7$ equal one, we will need to multiply by 7 on both sides.

$8 = \frac{m}{7}$

$7 \cdot 8 = \frac{m}{7} \cdot 7$

$56 = m$

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$9 = 1 + \frac{m}{7}$

$9 = 1 + \frac{56}{7}$

$9 = 1 + 8$

$9 = 9$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{m = 56}$.

<u>Example 11:</u>

$\frac{3}{2}a - 8 = 11$

The reversed order of operations says we should handle addition and subtraction first. That will be followed by taking care of any multiplication or division.

This means we should handle the -8 first since it is subtracted from the variable. To make -8 equal zero, we should add 8 to both sides.

$\frac{3}{2}a - 8 = 11$

$\quad +8 \quad +8$

$\frac{3}{2}a = 19$

To make the $\frac{3}{2}\cdot$ equal one, we will need to divide by $\frac{3}{2}$ on both sides.

$\frac{3}{2}a = 19$

$\frac{\frac{3}{2}a}{\frac{3}{2}} = \frac{19}{\frac{3}{2}}$

$$19 \div \frac{3}{2} = \frac{19}{1} \div \frac{3}{2} = \frac{19}{1} \cdot \frac{2}{3} = \frac{38}{3} = 12\frac{2}{3}$$

$a = 12\frac{2}{3}$

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$$\frac{3}{2}a - 8 = 11$$

$$\frac{3}{2}\left(12\frac{2}{3}\right) - 8 = 11$$

$$\frac{3}{2}\left(12\frac{2}{3}\right) = \frac{3}{2}\left(\frac{38}{3}\right) = \frac{114}{6} = 19$$

$$19 - 8 = 11$$

$$11 = 11 \quad \checkmark$$

Since our solution checks out, we know we have the correct solution: $\boldsymbol{a = 12\frac{2}{3}}$.

Example 12:

$$20 = \frac{n-3}{8}$$

When we see a large fraction bar like in this problem, we assume that there are parentheses around both the numerator and denominator. However, we don't need to write the parentheses around the denominator since there is only one thing in the denominator.

$$20 = \frac{(n-3)}{8}$$

The reversed order of operations says we should handle anything in parentheses last. If we ignore what is in the parentheses, the only thing outside of the parentheses (on the same side of the equation as the variable) is a division by 8. So, we should handle that first.

To make the $/8$ equal one, we will need to multiply by 8 on both sides.

$$20 = \frac{(n-3)}{8}$$

$$8 \cdot 20 = \frac{(n-3)}{8} \cdot 8$$

$$160 = (n-3)$$

We no longer need the parentheses around the right-hand side of the equation since there is nothing else left on that side except what is in the parentheses.

$$160 = n - 3$$

Now, we need to make -3 equal zero, we should add 3 to both sides.

$160 = n - 3$

$+3 \qquad + 3$

$163 = n$

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$$20 = \frac{n-3}{8}$$

$$20 = \frac{163-3}{8}$$

$$20 = \frac{160}{8}$$

$20 = 20$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{n = 163}$.

Example 13:

$$\frac{b+4}{-2} + 3 = -14$$

When we see a large fraction bar like in this problem, we assume that there are parentheses around both the numerator and denominator. We don't need the parentheses around the denominator since there is only one thing in the denominator.

$$\frac{(b+4)}{-2} + 3 = -14$$

We should handle anything in parentheses last. If we ignore what is in the parentheses, there are two numbers outside of the parentheses (on the same side of the equation as the variable); a division by -2 and an addition of 3. So, we should handle the addition first according to our reversed order of operations. Then, we will handle the division.

To make 3 equal zero, we should subtract 3 from both sides.

$$\frac{(b+4)}{-2} + 3 = -14$$

$-3 \quad - 3$

$$\frac{(b+4)}{-2} = -17$$

To make the $/-2$ equal one, we will need to multiply by -2 on both sides.

$$\frac{(b+4)}{-2} = -17$$

$$-2 \cdot \frac{(b+4)}{-2} = -17 \cdot -2$$

$$(b+4) = 34$$

We no longer need the parentheses around the left-hand side of the equation since there is nothing else left on that side except what is in the parentheses.

$$b + 4 = 34$$

Now, we need to make 4 equal zero, we should subtract 4 from both sides.

$$b + 4 = 34$$

$$\quad -4 \quad -4$$

$$b = 30$$

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$$\frac{b+4}{-2} + 3 = -14$$

$$\frac{30+4}{-2} + 3 = -14$$

$$\frac{34}{-2} + 3 = -14$$

$$-17 + 3 = -14$$

$$-14 = -14 \quad \checkmark$$

Since our solution checks out, we know we have the correct solution: $\boldsymbol{b = 30}$.

<u>Example 14:</u>

Twelve decreased by twice a number equals -34. Write an equation for this situation and then find the number.

We need to turn the sentence into an equation.

$$\underbrace{\text{Twelve}}_{12}\ \underbrace{\text{decreased by}}_{-}\ \underbrace{\text{twice}}_{2\cdot}\ \underbrace{\text{a number}}_{n}\ \underbrace{\text{equals}}_{=}\ \underset{-34}{-34}$$

$12 - 2n = -34$

The reversed order of operations says we should handle addition and subtraction first. That will be followed by taking care of any multiplication or division.

To make 12 equal zero, we should subtract 12 from both sides.

$12 - 2n = -34$

$-12 \qquad\quad -12$

$-2n = -46$ **Remember that the negative belongs to the two!

To make the $-2\cdot$ equal one, we will need to divide by -2 on both sides.

$-2n = -46$

$$\frac{-2n}{-2} = \frac{-46}{-2}$$

$n = 23$

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$12 - 2n = -34$

$12 - 2(23) = -34$

$12 - 46 = -34$

$-34 = -34$ ✓

Since our solution checks out, we know we have the correct solution. **The number is 23.**

<u>Example 15:</u>

The English alphabet contains 2 more than twice as many letters as the Hawaiian alphabet. How many letters are there in the Hawaiian alphabet?

We need to turn the sentence into an equation.

$$\underbrace{\text{The English alphabet}}_{E}\ \underbrace{\text{contains}}_{=}\ \underset{2}{2}\ \underbrace{\text{more than}}_{+}\ \underbrace{\text{twice as many}}_{2\cdot}\ \text{letters as}\ \underbrace{\text{the Hawaiian alphabet}}_{H}$$

$E = 2 + 2H$

We know that the English alphabet has 26 letters, so let's substitute that number in for E.

$26 = 2 + 2H$

The reversed order of operations says we should handle addition and subtraction first. That will be followed by taking care of any multiplication or division.

To make 2 equal zero, we should subtract 2 from both sides.

$26 = 2 + 2H$

$-2 \ -2$

$24 = 2H$

To make the $2\cdot$ equal one, we will need to divide by2 on both sides.

$24 = 2H$

$\frac{24}{2} = \frac{2H}{2}$

$12 = H$

Once the variable is isolated, we have our solution. We should always double-check our solution, though.

$12 - 46 = -34$

$26 = 2 + 2(12)$

$26 = 2 + 24$

$26 = 26$ ✓

Since our solution checks out, we know we have the correct solution. **The Hawaiian alphabet contains 12 letters.**