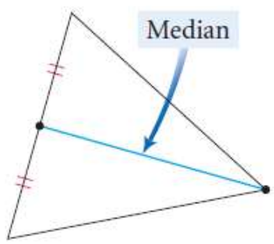
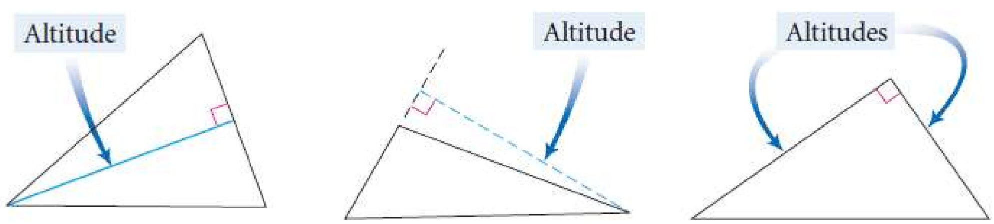
Lesson 4.8 – Special Triangle Conjectures

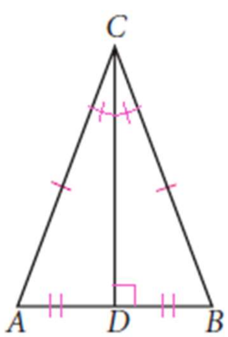
Median - The segment connecting the vertex of a triangle to the midpoint of its opposite side is a median.



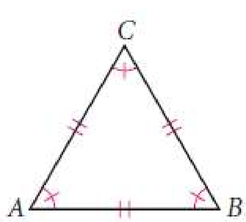
Altitude - An altitude of a triangle is a perpendicular segment from a vertex to the opposite side or to a line containing the opposite side.



Vertex Angle Bisector Conjecture - In an isosceles triangle, the bisector of the vertex angle is also the altitude and the median to the base.

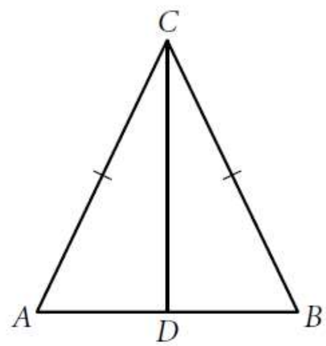


Equilateral/Equiangular Triangle Conjecture - Every equilateral triangle is equiangular, and, conversely, every equiangular triangle is equilateral.



Example 1: Use the figure.

$\overline{CD}$ is a median, perimeter $\Delta ABC = 60$, and $AC = 22$. $AD =$________



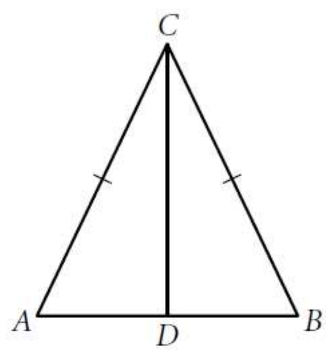
If $AC = 22$ then $BC = 22$ as well.

The perimeter $\Delta ABC = 60$. $\overline{AC}$ and $\overline{BC}$ make up 44 of that 60. That leaves side $AB = 16$.

To say that $\overline{CD}$ is a median means that $\overline{AD} \cong \overline{BD}$ and that D is a midpoint. So, $\boldsymbol{AD = 8}$.

Example 2: Use the figure.

$\overline{CD}$ is an angle bisector, and $m\angle A = 54°$. $m\angle ACD =$________



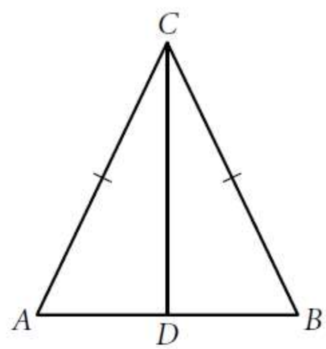
If $m\angle A = 54°$ then $m\angle B = 54°$ as well because they are base angles of an isosceles triangle.

The angles of ΔABC add to $180°$. $\angle A$ and $\angle B$ make up $108°$ of that $180°$. That leaves $m\angle ACB = 72°$.

To say that $\overline{CD}$ is an angle bisector means that $\angle ACD \cong \angle BCD$. So, $\boldsymbol{m\angle ACD = 36}°$.

Example 3: Use the figure.

$\overline{CD}$ is an altitude, perimeter $\Delta ABC = 42$, $m\angle ACD = 38°$, and $AD = 8$. $m\angle B$_______,
$CB =$________



To say that $\overline{CD}$ is an altitude means that $\angle ADC$ and $\angle CDB$ are right angles.

The angles of ΔADC add to 180°. $m\angle ACD = 38°$ and $m\angle ADC = 90°$ which makes up 128° of that 180°. That leaves $m\angle A = 52°$.

Since ΔABC is isosceles and $\angle A$ and $\angle B$ are the base angles, then $\boldsymbol{m\angle B = 52°}$.

If $AD = 8$ then $DB = 8$ as well because the altitude in an isosceles triangle is also the median.

The perimeter $\Delta ABC = 42$. $\overline{AB}$ makes up 16 of that 42. That leaves 26 to split between the two congruent sides. So, $\boldsymbol{CB = 13}$.

Example 4: Complete

ΔEQU is equilateral.

$m\angle E =$_________

We know that equilateral triangles are also equiangular. So, since triangle angles add to 180°, we can split the 180° evenly three ways and find that each angle is 60°.

$\boldsymbol{m\angle E = 60°}$

Example 5: Complete

ΔANG is equiangular and perimeter $\Delta ANG = 51$.

$AN =$__________

We know that equiangular triangles are also equilateral. So, since the sides of the triangle add to 51, we can split that 51 three ways and find that each side is 17.

$\boldsymbol{AN = 17}$