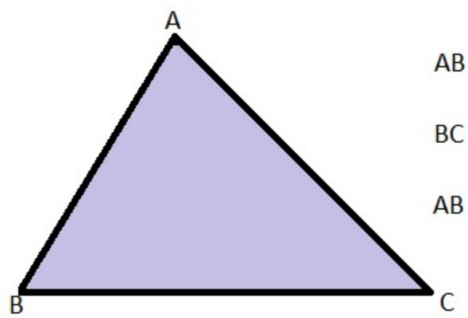
Lesson 4.3 – Triangle Inequalities

Triangle Inequality Conjecture - The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



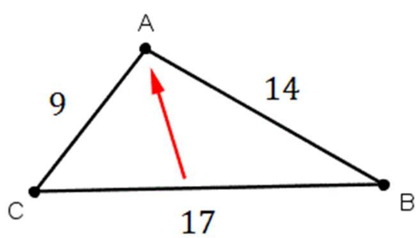
AB + BC > AC

BC + AC > AB

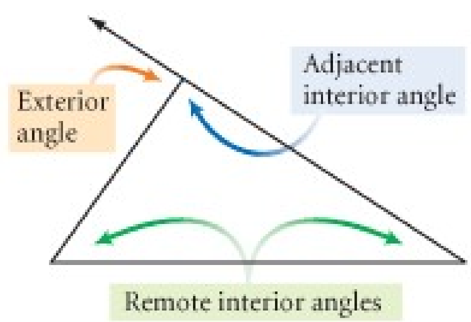
AB + AC > BC

Side-Angle Inequality Conjecture - In a triangle, if one side is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

$\angle A$ must be the largest angle.



Triangle Exterior Angle Conjecture - The measure of an exterior angle of a triangle is equal to the sum of the measures of the remote interior angles.



Example 1: Determine whether it is possible to draw a triangle with the sides of the given measures.

16 cm, 30 cm, 45 cm

We know that the sum of the lengths of any two sides must be greater than the length of the third side. If we check that the shorter two sides add to more than the longest side, that is sufficient to determine if we can draw a triangle.

The shorter sides are 16 cm and 30 cm.

$16 + 30 = 46$ and that sum is greater than the longest side (45 cm) $46 > 45$.

Since the sum of the shorter two sides is greater than the length of the longest side, we can draw a triangle with side lengths 16 cm, 30 cm, and 45 cm.

Example 2: Determine whether it is possible to draw a triangle with the sides of the given measures.

9 km, 17 km, 28 km

We know that the sum of the lengths of any two sides must be greater than the length of the third side. If we check that the shorter two sides add to more than the longest side, that is sufficient to determine if we can draw a triangle.

The shorter sides are 9 km and 17 km.

$9 + 17 = 26$ and that sum is not greater than the longest side (28 km) $26 \ngtr 28$.

Since the sum of the shorter two sides is not greater than the length of the longest side, we can not draw a triangle with side lengths 9 km, 17 km, and 28 km, Example 3: Complete

If 17 and 36 are the lengths of two sides of a triangle, what is the range of possible values for the length of the third side?

We know that the sum of the lengths of any two sides must be greater than the length of the third side. There are two options for the third side of this triangle:

Option 1: The third side is the longest side in the triangle

If the third side is the longest side, then 17 and 36 must add to more than the third side.

$$17 + 36 > x$$

$$53 > x$$

Option 2: The third side is not the longest side in the triangle

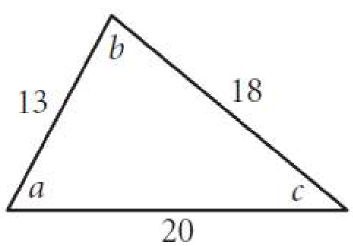
If the third side is not the longest side, then 36 is the longest side and 17 and the third side must add to more than 36.

$$17 + x > 36$$

$$x > 19$$

So, the third side must be less than 53 and more than 19. We can write that as $19 < x < 53$.

Example 4: Arrange the unknown measures in order from greatest to least.



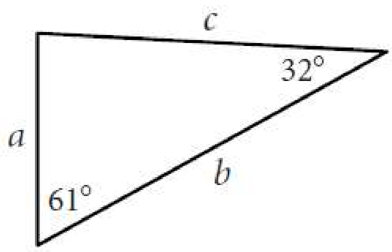
The largest angle is opposite the longest side. The longest side is 20. The angle directly opposite 20 is b. b is the greatest measure angle.

The next largest side is 18. The angle directly opposite 18 is a. So, a is the next largest side.

13 is the shortest side. The angle directly opposite 13 is c. c is the smallest angle measure.

$\boldsymbol{b, a, c}$

Example 5: Arrange the unknown measures in order from greatest to least.



We need to start by finding the measure of the missing angle. All angles of a triangle add to $180°$. If we call the missing angle x, $61° + 32° + x = 180°$ and $x = 87°$.

The longest side is opposite the largest angle. The largest angle is $87°$. The side directly opposite that angle is b. b is the longest side.

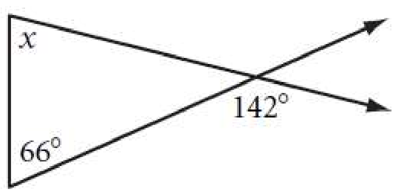
The next largest angle is $61°$. The side directly opposite that angle is c. So, c is the next longest side.

The smallest angle is $32°$. The side directly opposite that angle is a. a is the shortest side.

$\boldsymbol{b, c, a}$

Example 6: Find the missing measure.

$x =$ _______

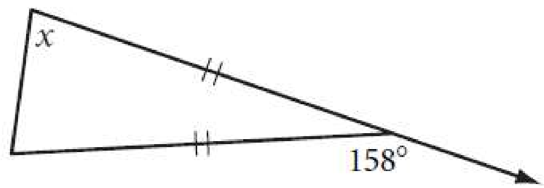


The exterior angle in a triangle is equal to the sum of the remote interior angles.

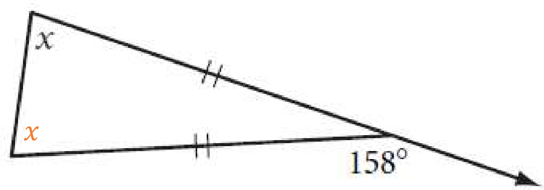
$x + 66° = 142°$	$142°$ is an exterior angle. $66°$ and x are the remote interior angles.
$\boldsymbol{x = 76°}$	Subtract 66 from both sides

Example 7: Find the missing measure.

$x =$ ______



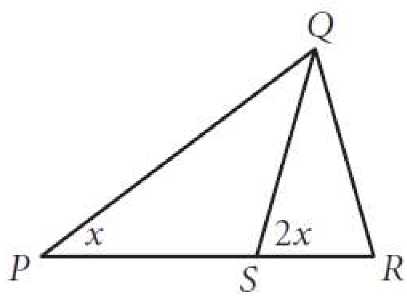
The triangle is isosceles. The two angles opposite the congruent sides must also be congruent. So…



The exterior angle in a triangle is equal to the sum of the remote interior angles.

$x + x = 158°$	158° is an exterior angle. The two x's are the remote interior angles.
$2x = 158°$	Combine like terms
$\boldsymbol{x = 79°}$	Divide by two on both sides

Example 8: Explain why ΔPQS is isosceles.



$\angle QSR$ is an exterior angle to ΔPQS. The remote interior angles are $\angle SPQ$ and $\angle PQS$.

The exterior angle in a triangle is equal to the sum of the remote interior angles.

$m\angle SPQ + m\angle PQS = m\angle QSR$

$x + m\angle PQS = 2x$ Replace angle measures with known values

$m\angle PQS = x$ Subtract x from both sides

Since the triangle has two congruent angles ($m\angle PQS = x = m\angle SPQ$), we know that ΔPQS must be isosceles.