

Solving Multi-Step Inequalities Notes

Remember the rules for solving equations. They also apply for solving inequalities. We work backwards through the order of operations. Things that are added and subtracted to the variable must go away before anything that is multiplied to or divided from the variable.

Example 1: Inequality Involving a Negative Coefficient

$$-7b + 19 < -16$$

$$\begin{array}{r} -19 \\ -19 \end{array}$$

**Subtract 19 from both sides to get rid of the +19

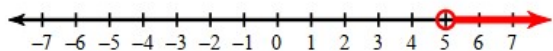
$$-7b < -35$$

$$\frac{-7b}{-7} < \frac{-35}{-7}$$

**Divide by -7 on both sides to get rid of the multiplication by -7

$$b > 5$$

**We had to flip the inequality because the number we divided by was a negative number.



Example 2: Inequality Involving a Positive Coefficient

$$6h - 10 \geq 26$$

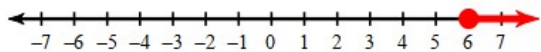
$$\begin{array}{r} +10 \\ +10 \end{array}$$

$$6h \geq 36$$

$$\frac{6h}{6} \geq \frac{36}{6}$$

$$h \geq 6$$

**We do not flip this inequality because the number we divided by was positive.



Example 3: Inequality Involving a Fractional Coefficient

$$-3 \geq \frac{2}{3}r + 3$$

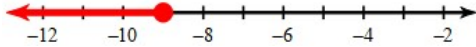
$$\begin{array}{r} -3 \\ -3 \end{array}$$

$$-6 \geq \frac{2}{3}r$$

$$\frac{-6}{\frac{2}{3}} \geq \frac{\frac{2}{3}r}{\frac{3}{2}}$$

$$**\frac{-6}{\frac{2}{3}} = -\frac{6}{1} \cdot \frac{3}{2} = -\frac{18}{2} = -9$$

$$-9 \geq r$$



Example 4: Inequality with a Variable on Both Sides

$$7b + 11 > 9b - 13$$

$$-7b \quad -7b$$

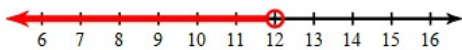
$$11 > 2b - 13$$

$$+13 \quad +13$$

$$24 > 2b$$

$$\frac{24}{2} > \frac{2b}{2}$$

$$12 > b$$



Example 5: Inequality with the Distributive Property

$$-6 \leq 3(5v - 2)$$

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Two blue curved arrows originate from the coefficient 3 and point to the terms 5v and -2 inside the parentheses, illustrating the distributive property.

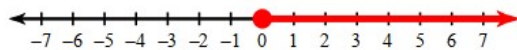
$$-6 \leq 15v - 6$$

$$+6 \quad +6$$

$$0 \leq 15v$$

$$\frac{0}{15} \leq \frac{15v}{15}$$

$$0 \leq v$$



Example 6: Inequality with Distributive Property and Variable on Both Sides

$$-5(g + 4) > 3(g - 4)$$

$$-5(g + 4) > 3(g - 4)$$

$$-5g - 20 > 3g - 12$$

$$-3g \quad -3g$$

$$-8g - 20 > -12$$

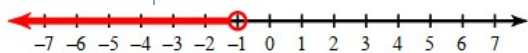
$$+20 \quad +20$$

$$-8g > 8$$

$$\frac{-8g}{-8} > \frac{8}{-8}$$

$$g < -1$$

**Remember to flip the inequality because of the division by a negative number.



Example 7: Multi-Step Inequality

$$3d - 2(8d - 9) > 3 - (2d + 7)$$

**Whenever there is just a negative in front of the parentheses, we treat it like a negative one to be distributed.

$$3d - 2(8d - 9) > 3 - 1(2d + 7)$$

$$3d - 2(8d - 9) > 3 - 1(2d + 7)$$

$$3d - 16d + 18 > 3 - 2d - 7$$

$$-13d + 18 > -4 - 2d$$

**Combine the $3d - 16d$ on the left side and the $3 + -7$ on the right side

$$+2d \quad +2d$$

$$-11d + 18 > -4$$

$$-18 \quad -18$$

$$-11d > -22$$

$$\frac{-11d}{-11} > \frac{-22}{-11}$$

$$d < 2$$



Example 8: No Solution

$$3 - 8x \geq 9 + 2(1 - 4x)$$

$$3 - 8x \geq 9 + 2 - 8x \quad \text{**Distribute the 2}$$

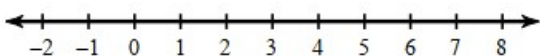
$$3 - 8x \geq 11 - 8x \quad \text{**Combine the 9 + 2}$$

$$+8x \quad +8x$$

$$3 \geq 11$$

**The variable disappears, so we evaluate whether this is a true statement. 3 is not greater than 11, so this is a false statement and this is a “no solution” problem.

No Solution



**When there is no solution, we cannot shade the number line because there are no solutions. So, just leave the number line blank.

Example 9: All Real Numbers

$$8(t + 2) - 3(t - 4) > 5(t - 7) + 8$$

$$8t + 16 - 3t + 12 > 5t - 35 + 8 \quad \text{**Distribute the 8, -3, and 5}$$

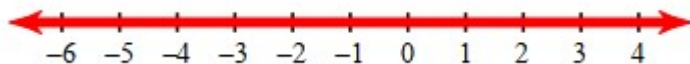
$$5t + 28 > 5t - 27 \quad \text{**Combine the } 8t - 3t \text{ and } +16 + 12 \text{ on the left side and the } -35 + 8 \text{ on the right side}$$

$$-5t \quad -5t$$

$$28 > -27$$

**The variable disappears, so we evaluate whether this is a true statement. 28 is greater than -27, so this is a true statement and this is an “all real numbers” problem.

All Real Numbers



**When all real numbers are solutions, we must shade the entire number line because all the numbers on the number line are solutions to the inequality.