Multiplying and Dividing Rational Numbers

Multiplying and Dividing Integers Rules:

- If two signs are the same, the product or quotient is positive.
- If twp signs are different, then the product or quotient is negative.

Different ways of expressing multiplication:

- 5×4
- $(5)(4)$
- $5 \cdot 4$
- $5(4)$

Different ways of expression division:

- $8 \div 4$
- $\frac{8}{4}$
- $8/4$

Example 1: Evaluate

$(-3)(2)$

The 3 is negative and the 2 is positive. Since the two signs are different, the product will be negative.

$-(3 \cdot 2) = -6$

$(-3)(2) = -6$

Example 2: Evaluate

$(-7)(-8)$

The 7 is negative and the 8 is also negative. Since the two signs are the same, the product will be positive.

$+(7 \cdot 8) = +56$

$(-7)(-8) = 56$

Example 3: Evaluate

$8(-3)$

The 8 is positive and the 3 is negative. Since the two signs are different, the product will be negative.

$-(8 \cdot 3) = -24$

$8(-3) = -24$

Example 4: Evaluate

$-5 \cdot 8$

The 5 is negative and the 8 is positive. Since the two signs are different, the product will be negative.

$-(5 \cdot 8) = -40$

$-5 \cdot 8 = -40$

Example 5: Evaluate

$-9 \cdot -6$

The 9 is negative and the 6 is also negative. Since the two signs are the same, the product will be positive.

$+(9 \cdot 6) = +54$

$-9 \cdot -6 = 54$

Example 6: Evaluate

$$\frac{-9}{3}$$

The 9 is negative and the 3 is positive. Since the two signs are different, the quotient will be negative.

$-(9 \div 3) = -3$

$$\frac{-9}{3} = -3$$

Example 7: Evaluate

$84 \div -4$

The 84 is positive and the 4 is negative. Since the two signs are different, the quotient will be negative.

$-(84 \div 4) = -21$

$84 \div -4 = -21$

Example 8: Evaluate

$-15 \div -3$

The 15 is negative and the 3 is also negative. Since the two signs are the same, the quotient will be positive.

$+(15 \div 3) = +5$

$-15 \div -3 = 5$

Example 9: Evaluate

$$\frac{-36}{-6}$$

The 36 is negative and the 6 is also negative. Since the two signs are the same, the quotient will be positive.

$$+(36 \div 6) = +6$$

$$\frac{-36}{-6} = 6$$

Example 10: Evaluate

$$-6/3$$

The 6 is negative and the 3 is positive. Since the two signs are different, the quotient will be negative.

$$-(6 \div 3) = -2$$

$$-6/3 = -2$$

Multiplying Rational Numbers Rules:

1. Turn all mixed numbers and integers into improper fractions.
2. Multiply numerators together and denominators together. Follow the rules for multiplying and dividing integers to find the sign.
3. Simplify the fraction (make sure the numerator and denominator are not divisible by the same values).
4. If your answer is an improper fraction turn it into a mixed number.

Example 11: Evaluate

$$\left(-\frac{1}{2}\right)\left(1\frac{1}{3}\right)$$

1. $\left(-\frac{1}{2}\right)\left(\frac{4}{3}\right)$ **Remember to turn $1\frac{1}{3}$ into an improper fraction, we use $1 \cdot 3 + 1 = 4$ to find the numerator.
2. $-\frac{1\cdot 4}{2\cdot 3} = -\frac{4}{6}$ **One negative and one positive will make our answer negative.
3. 4 and 6 are both divisible by 2.
   $-\frac{4 \div 2}{6 \div 2} = -\frac{2}{3}$
4. $-\frac{2}{3}$ is not an improper fraction, so we can skip this step.

$$\left(-\frac{1}{2}\right)\left(1\frac{1}{3}\right) = -\frac{2}{3}$$

Example 12: Evaluate

$$\frac{1}{8} \cdot -\frac{7}{5}$$

1. There are no mixed numbers or integers, so we can skip this step.
2. $-\frac{1\cdot 7}{8\cdot 5} = -\frac{7}{40}$ **One positive and one negative will make our answer negative.
3. 7 and 40 do not have a common divisor, so we can skip this step.
4. $-\frac{7}{40}$ is not an improper fraction, so we can skip this step.

$$\frac{1}{8} \cdot -\frac{7}{5} = -\frac{7}{40}$$

Example 13: Evaluate

$$-\frac{4}{5} \cdot -2\frac{1}{2}$$

1. $-\frac{4}{5} \cdot -\frac{5}{2}$ **Remember to turn $2\frac{1}{2}$ into an improper fraction, we use $2 \cdot 2 + 1 = 5$ to find the numerator.
2. $+\frac{4\cdot 5}{5\cdot 2} = \frac{20}{10}$ **Two negatives will make our answer positive.
3. 20 and 10 are both divisible by 10.
   $\frac{20 \div 10}{10 \div 10} = \frac{2}{1}$

4. $\frac{2}{1}$ is an improper fraction. So, we divide. $2 \div 1 = 2$

$$-\frac{4}{5} \cdot -2\frac{1}{2} = 2$$

Example 14: Evaluate

$$\left(4\frac{5}{8}\right)\left(1\frac{1}{7}\right)$$

1. $\frac{37}{8} \cdot \frac{8}{7}$ ** $4 \cdot 8 + 5 = 37$ and $1 \cdot 7 + 1 = 8$
2. $+\frac{37 \cdot 8}{8 \cdot 7}$ **Two positives will make our answer positive.
3. I did not calculate the multiplication in step 2, because there is an 8 in the numerator and there is also an 8 in the denominator. When that situation occurs, you can cross both 8s out and be left with $\frac{37}{7}$.
4. $\frac{37}{7}$ is an improper fraction. So, we divide.

$$\begin{array}{r} 5 \\ 7\overline{)37} \\ \underline{-35} \\ 2 \end{array} \qquad 5\frac{2}{7}$$

$$\left(4\frac{5}{8}\right)\left(1\frac{1}{7}\right) = 5\frac{2}{7}$$

Dividing Rational Numbers Rules:

1. Turn all mixed numbers and integers into improper fractions.
2. Rewrite the problem as the first fraction multiplied by the reciprocal (flip) of the second fraction.
3. Multiply numerators together and denominators together. Follow the rules for multiplying and dividing integers to find the sign.
4. Simplify the fraction (make sure the numerator and denominator are not divisible by the same values).
5. If your answer is an improper fraction turn it into a mixed number.

Example 15: Evaluate

$$-\frac{3}{4} \div \frac{1}{2}$$

1. There are no mixed numbers or integers, so we can skip this step.
2. $-\frac{3}{4} \cdot \frac{2}{1}$
3. $-\frac{3 \cdot 2}{4 \cdot 1} = -\frac{6}{4}$ **One negative and one positive will make our answer negative.
4. 6 and 4 are both divisible by 2.
   $-\frac{6 \div 2}{4 \div 2} = -\frac{3}{2}$
5. $-\frac{3}{2}$ is an improper fraction. So, we divide. (Ignore the negative for the algorithm and add it back into your final answer.)

   $2\overline{)3}$ with quotient 1, -2, remainder 1 $\qquad 1\frac{1}{2}$

$$-\frac{3}{4} \div \frac{1}{2} = -1\frac{1}{2}$$

Example 16: Evaluate

$$\frac{-\frac{1}{3}}{-\frac{5}{6}}$$

1. There are no mixed numbers or integers, so we can skip this step.
2. $-\frac{1}{3} \cdot -\frac{6}{5}$
3. $+\frac{1 \cdot 6}{3 \cdot 5} = \frac{6}{15}$ **Two negatives will make our answer positive.
4. 6 and 15 are both divisible by 3.
   $\frac{6 \div 3}{15 \div 3} = \frac{2}{5}$
5. $\frac{2}{5}$ is not an improper fraction, so we can skip this step.

$$\frac{-\frac{1}{3}}{-\frac{5}{6}} = \frac{2}{5}$$

Example 17: Evaluate

$$1\frac{2}{5} \div -\frac{7}{9}$$

1. $\frac{7}{5} \div -\frac{7}{9}$ ** $1 \cdot 5 + 2 = 7$
2. $\frac{7}{5} \cdot -\frac{9}{7}$
3. $-\frac{7 \cdot 9}{5 \cdot 7}$ **One positive and one negative will make our answer negative.
4. I did not calculate the multiplication in step 3, because there is a 7 in the numerator and there is also a 7 in the denominator. When that situation occurs, you can cross both 7s out and be left with $-\frac{9}{5}$.
5. $-\frac{9}{5}$ is an improper fraction. So, we divide. (Ignore the negative for the algorithm and add it back into your final answer.)

$$\begin{array}{r} 1 \\ 5\overline{)\,9} \\ \underline{-5} \\ 4 \end{array} \qquad 1\frac{4}{5}$$

$$1\frac{2}{5} \div -\frac{7}{9} = 1\frac{4}{5}$$

Example 18: Evaluate

$$\frac{-\frac{1}{2}}{2}$$

1. $\frac{-\frac{1}{2}}{\frac{2}{1}}$ **Remember to turn 2 into an improper fraction, we put the integer over 1.
2. $-\frac{1}{2} \cdot \frac{1}{2}$
3. $-\frac{1 \cdot 1}{2 \cdot 2} = -\frac{1}{4}$ ** One negative and one positive will make our answer negative.
4. 1 and 4 do not have a common divisor, so we can skip this step.
5. $-\frac{1}{4}$ is not an improper fraction, so we can skip this step.

$$\frac{-\frac{1}{2}}{2} = -\frac{1}{4}$$