Solving Single-Step Inequalities

KEY CONCEPT *Addition Property of Inequalities*

Words If any number is added to each side of a true inequality, the resulting inequality is also true.

Symbols For all numbers a, b, and c, the following are true.
1. If $a > b$, then $a + c > b + c$.
2. If $a < b$, then $a + c < b + c$.

This property is also true when $>$ and $<$ are replaced with $\geq$ and $\leq$.

KEY CONCEPT *Subtraction Property of Inequalities*

Words If any number is subtracted from each side of a true inequality, the resulting inequality is also true.

Symbols For all numbers a, b, and c, the following are true.
1. If $a > b$, then $a - c > b - c$.
2. If $a < b$, then $a - c < b - c$.

This property is also true when $>$ and $<$ are replaced with $\geq$ and $\leq$.

KEY CONCEPT *Multiplying by a Positive Number*

Words If each side of a true inequality is multiplied by the same positive number, the resulting inequality is also true.

Symbols If a and b are any numbers and c is a positive number, the following are true.
If $a > b$, then $ac > bc$, and if $a < b$, then $ac < bc$.

This property also holds for inequalities involving $\geq$ and $\leq$.

KEY CONCEPT *Multiplying by a Negative Number*

Words If each side of a true inequality is multiplied by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is also true.

Symbols If a and b are any numbers and c is a negative number, the following are true.
If $a > b$, then $ac < bc$, and if $a < b$, then $ac > bc$.

This property also holds for inequalities involving $\geq$ and $\leq$.

KEY CONCEPT ***Dividing by a Positive Number***

Words If each side of a true inequality is divided by the same positive number, the resulting inequality is also true.

Symbols If a and b are any numbers and c is a positive number, the following are true.

If $a > b$, then $\frac{a}{c} > \frac{b}{c}$, and if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.

KEY CONCEPT ***Dividing by a Negative Number***

Words If each side of a true inequality is divided by the same negative number, the direction of the inequality symbol must be *reversed* so that the resulting inequality is also true.

Symbols If a and b are any numbers and c is a negative number, the following are true.

If $a > b$, then $\frac{a}{c} < \frac{b}{c}$, and if $a < b$, then $\frac{a}{c} > \frac{b}{c}$.

These properties also hold for inequalities involving $\geq$ and $\leq$.

****Notice that there are two rules when we are solving inequalities using multiplication and division. The rules for multiplying and dividing by a positive number differ from the rules of multiplying by a negative number. This is because the negative number line looks exactly opposite of the positive number line.

<u>Example 1:</u>

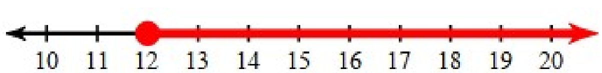
$t - 5 \geq 7$

Just like when we are solving equations, we are trying to isolate the variable. So, the -45 is on the same side of the inequality as the variable.

$t - 5 \geq 7$

$\quad +5 \quad +5$ **Add 5 to both sides so that it makes a 0 on the side with t.

$\quad t \geq 12$



**Remember from the first lesson that we fill in the circle to indicate that this can be equal to 12, and we shade to the right because the variable is bigger than 12.

Example 2:

$$\frac{n}{14} \leq -20$$

$$14 \cdot \frac{n}{14} \leq -20 \cdot 14$$ **We multiply to get rid of the division by 14.

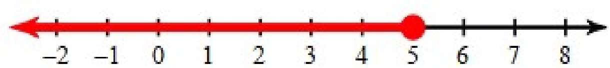
$$n \leq -280$$

Example 3:

$$-7 \geq -2 + x$$

$$+2 \quad\quad +2$$ **Add 2 to both sides so that it makes a 0 on the side with x.

$$-5 \geq x$$



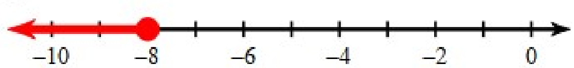
**Fill in the circle to indicate that this can be equal to 5, and shade to the left because the variable is smaller than 5.

Example 4:

$$-80 \geq 10x$$

$$-\frac{80}{10} \geq \frac{10}{10}$$ **We divide to get rid of the multiplication by 10.

$$-8 \geq x$$



Example 5:

$$-56 < -14a$$

$$\frac{-56}{-14} < \frac{-14a}{-14}$$

$$4 > a$$

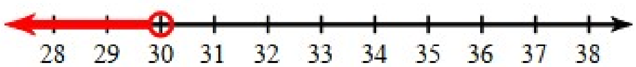
**When we multiply or divide by a negative number, we must flip the inequality. Remember that the inequality is the sign that tells us whether things are greater or less than ($<, >, \leq, \geq$).

Example 6:

$x + 23 < 53$

$-23 \quad -23$ **Subtract 23 from both sides so that it makes a 0 on the side with x.

$x < 30$



**Leave the circle open to indicate that this cannot be equal to 30, and shade to the left because the variable is smaller than 30.

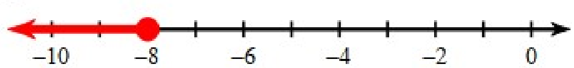
Example 7:

$\frac{c}{-2} \geq 4$

$-2 \cdot \frac{c}{-2} \geq 4 \cdot -2$

$c \leq -8$

**When we multiply or divide by a negative number, we must flip the inequality. Remember that the inequality is the sign that tells us whether things are greater or less than ($<, >, \leq, \geq$).



Example 8:

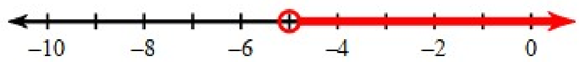
$2n > -10$

**We know we will divide to get rid of the multiplication between the 2 and the n

$\frac{2n}{2} > -\frac{10}{2}$

**It is at this point when we are doing our calculations where we ask ourselves if we have multiplied or divided by a negative number. We don't care if the answer ends up being negative, we only care about the number that we have actually multiplied or divided. In this case, we divided by a positive two, so we do not flip the inequality.

$n > -5$

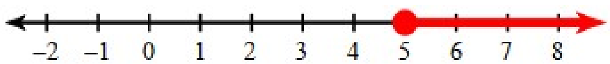


Example 9:

$9 \leq b + 4$

$-4 \qquad -4$ **Subtract 4 from both sides so that it makes a 0 on the side with b.

$5 \leq b$



**Fill in the circle to indicate that this can be equal to 5, and shade to the right because the variable is bigger than 5.

Example 10:

$-120 > -10v$

**We know we will divide to get rid of the multiplication between the -10 and the v.

$\frac{-120}{-10} > \frac{-10v}{-10}$

**It is at this point when we are doing our calculations where we ask ourselves if we have multiplied or divided by a negative number. We don't care if the answer ends up being negative, we only care about the number that we have actually multiplied or divided. In this case, we divided by a negative ten, so we *do* flip the inequality.

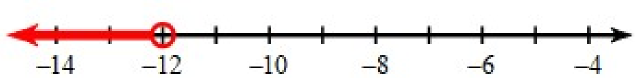
$12 < v$

Example 11:

$-26 > m - 14$

$+14 \qquad + 14$

$-12 > m$



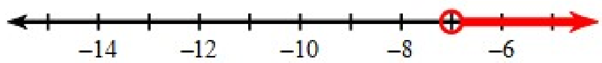
**Leave the circle open to indicate that this cannot be equal to -12, and shade to the left because the variable is smaller than -12.

Example 12:

$x + 8 > 1$

$\quad -8 \quad -8$

$x > -7$



**Leave the circle open to indicate that this cannot be equal to -7, and shade to the right because the variable is bigger than -7.

Example 13:

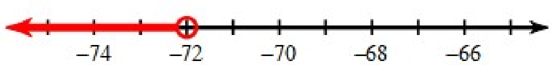
$\frac{k}{9} < -8$

**We know we will multiply to get rid of the division between the k and 9.

$9 \cdot \frac{k}{9} < -8 \cdot 9$

**It is at this point when we are doing our calculations where we ask ourselves if we have multiplied or divided by a negative number. We don't care if the answer ends up being negative, we only care about the number that we have actually multiplied or divided. In this case, we multiplied by a positive nine, so we *do not* flip the inequality.

$k < -72$



Example 14:

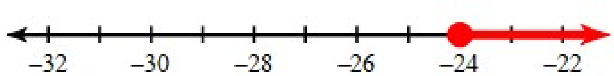
$-2 \leq \frac{n}{12}$

**We know we will multiply to get rid of the division between the n and 12.

$12 \cdot -2 \leq \frac{n}{12} \cdot 12$

**We multiplied by a positive twelve, so we *do not* flip the inequality.

$-24 \leq n$



Example 15:

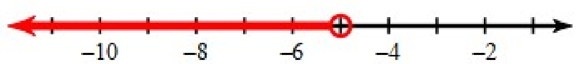
$-4k > 20$

**We know we will divide to get rid of the multiplication between -4 and k.

$$\frac{-4k}{-4} > \frac{20}{-4}$$

**We divided by a negative four, so we *do* flip the inequality.

$k < -5$



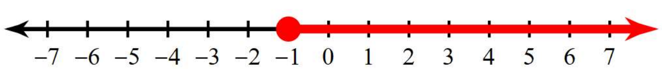
Example 16:

$x-(-2) \geq 1$ **Remember that a $-(-$ turns into a $+$

$x+2 \geq 1$

$-2 \quad -2$

$x \geq -1$



** Fill in the circle to indicate that this can be equal to -1, and shade to the right because the variable is bigger than -1.

Example 17:

$$-\frac{16}{11} \geq -\frac{12}{11}n$$

**We know we will divide to get rid of the multiplication between $-\frac{12}{11}$ and n.

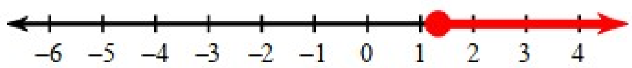
$$\frac{-\frac{16}{11}}{-\frac{12}{11}} \geq \frac{-\frac{12}{11}n}{-\frac{12}{11}}$$

**We divided by a negative twelve-elevenths, so we *do* flip the inequality.

***Let's do some math off to the side for calculating purposes:

$$\frac{-\frac{16}{11}}{-\frac{12}{11}} = -\frac{16}{11} \cdot -\frac{11}{12} = \frac{16}{12} = \frac{16 \div 4}{12 \div 4} = \frac{4}{3}$$

$\frac{4}{3} \leq n$



<u>Example 18:</u>

$\frac{13}{2}n < -\frac{91}{2}$

**We know we will divide to get rid of the multiplication between $\frac{13}{2}$ and n.

$$\frac{\frac{13}{2}n}{\frac{13}{2}} < \frac{-\frac{91}{2}}{\frac{13}{2}}$$

**We divided by a positive thirteen-halves, so we *do not* flip the inequality.

***Let's do some math off to the side for calculating purposes:

$$\frac{-\frac{91}{2}}{\frac{13}{2}} = -\frac{91}{2} \cdot \frac{2}{13} = \frac{-91}{13} = \frac{-91 \div 13}{13 \div 13} = \frac{-7}{1}$$

$n < -7$

Some important vocabulary:

At most means "≤"

At least means "≥"

<u>Example 19:</u>

A number decreased by seven is at most negative seven.

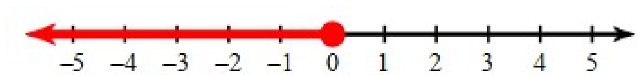
$n \quad - \quad 7 \quad \leq \quad -7$

$n - 7 \leq -7$

$n - 7 \leq -7$

$\quad +7 \qquad +7$

$\qquad n \leq 0$



<u>Example 20:</u>

Negative three times a number is at least negative six.

$-3 \quad \cdot \quad n \quad \geq \quad -6$

$-3n \geq -6$

$-3n \geq -6$

$$\frac{-3n}{-3} \geq \frac{-6}{-3}$$

$\quad n \leq 2$

**When we multiply or divide by a negative number, we must flip the inequality. Remember that the inequality is the sign that tells us whether things are greater or less than $(<, >, \leq, \geq)$.