Solving Equations by Using Multiplication and Division

Multiplication Property of Equality:

If an equation is true and the same number is multiplied to each side, the resulting equation is true.

Example: $7 = 7$

$$7 \cdot 3 = 7 \cdot 3$$

$$21 = 21$$

For any numbers $a, b,$ and c, if $a = b$, then $a \cdot c = b \cdot c$

Division Property of Equality:

If an equation is true and the same number is divided from each side, the resulting equation is true.

Example: $27 = 27$

$$\frac{27}{-9} = \frac{27}{-9}$$

$$-3 = -3$$

For any numbers $a, b,$ and c, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$

**The basic idea of both the multiplication and division properties of equality is that, in order to maintain equality in an equation, if we multiply or divide something on one side of the equation we must do the same on the other side of the equation.

Example 1:

Solve: $\frac{t}{3} = 7$

When we solve an equation, we are looking for the value of the variable. The most efficient way to determine the value of the variable is to isolate the variable on one side of the equation.

To isolate the variable in an addition or subtraction situation, we are looking to create the additive identity (zero). To isolate the variable in a multiplication or division situation we are looking to create the multiplicative identity (one).

If we look at the equation, the 3 is on the same side of the equation as the variable. To turn the $/3$ into a one, we need to multiply by 3. We will need to multiply this to both sides of the equation.

$$\frac{t}{3} = 7$$

$$3 \cdot \frac{t}{3} = 7 \cdot 3$$

Since, $\frac{3}{3} = 1$, we could write $1t$ on the left side of the equation, but we don't need to write the 1 since it doesn't have an effect (one times any number or variable is just equal to the number of variable). So, we can simply write t on the left side.

$t = 21$ **On the right side of the equation, we need to calculate $7 \cdot 3$.

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$$\frac{t}{3} = 7$$

$$\frac{21}{3} = 7$$

$7 = 7$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{t = 21}$.

<u>Example 2:</u>

Solve: $13x = 195$

When we solve an equation, we are looking for the value of the variable. The most efficient way to determine the value of the variable is to isolate the variable on one side of the equation.

To isolate the variable in an addition or subtraction situation, we are looking to create the additive identity (zero). To isolate the variable in a multiplication or division situation we are looking to create the multiplicative identity (one).

If we look at the equation, the 13 is on the same side of the equation as the variable. Remember that there are several ways to indicate multiplication. One of those is to put nothing between the number and the variable. So, I know that the 13 is multiplied to the x. To turn the $13 \cdot$ into a one, we need to divide by 13. We will need to divide this from both sides of the equation.

$$13x = 195$$

$$\frac{13x}{13} = \frac{195}{13}$$

Since, $\frac{13}{13} = 1$, we could write $1x$ on the left side of the equation, but we don't need to write the 1 since it doesn't have an effect (one times any number or variable is just equal to the number of variable). So, we can simply write x on the left side.

$x = 15$ **On the right side of the equation, we need to calculate $195 \div 13$.

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$13x = 195$

$13(15) = 195$ **Remember that the 13 is multiplied into x. I can also indicate multiplication by placing one number in a set of parentheses as I have done here.

$195 = 195$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{x = 15}$.

<u>Example 3:</u>

Solve: $\left(2\frac{1}{4}\right)g = \frac{1}{2}$

To isolate the variable in this multiplication or division situation we are looking to create the multiplicative identity (one).

If we look at the equation, the $2\frac{1}{4}$ is on the same side of the equation as the variable. Remember that there are several ways to indicate multiplication. One of those is to put one of the two multiplied things into parentheses. So, I know that the $2\frac{1}{4}$ is multiplied to the g.

When working with fractions, it is often easiest to convert all mixed numbers into improper fractions prior to doing any calculations.

$\left(2\frac{1}{4}\right)g = \frac{1}{2}$ **Remember that to turn a mixed number into an improper fraction, you calculate $2 \cdot 4 + 1 = 9$ for the numerator and you leave the denominator a 4.

$$\left(\frac{9}{4}\right)g = \frac{1}{2}$$

To turn the $\frac{9}{4}\cdot$ into a one, we need to divide by $\frac{9}{4}$. We will need to divide this from both sides of the equation.

$$\left(\frac{9}{4}\right)g = \frac{1}{2}$$

$$\frac{\left(\frac{9}{4}\right)g}{\frac{9}{4}} = \frac{\frac{1}{2}}{\frac{9}{4}}$$

Since, $\frac{\left(\frac{9}{4}\right)}{\frac{9}{4}} = 1$, we can write g on the left side.

$g = \frac{2}{9}$ **On the right side of the equation, we need to calculate $\frac{1}{2} \div \frac{9}{4}$.

$$\frac{1}{2} \div \frac{9}{4} = \frac{1}{2} \cdot \frac{4}{9} = \frac{4}{18} = \frac{2}{9}$$

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$\left(\frac{9}{4}\right)g = \frac{1}{2}$ **I will use the version of the equation that has improper fractions rather than mixed numbers for ease of calculation.

$\left(\frac{9}{4}\right)\left(\frac{2}{9}\right) = \frac{1}{2}$ **Remember that the $\frac{9}{4}$ is multiplied into g. I can also indicate multiplication by placing both numbers in a set of parentheses as I have done here.

$$\left(\frac{9}{4}\right)\left(\frac{2}{9}\right) = \frac{18}{36} = \frac{1}{2}$$

$\frac{1}{2} = \frac{1}{2}$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{g = \frac{2}{9}}$.

<u>Example 4:</u>

Solve: $-3x = 12$

If we look at the equation, the -3 is on the same side of the equation as the variable. Remember that there are several ways to indicate multiplication. One of those is to put nothing between the number and the variable. So, I know that the -3 is multiplied to the x. To turn the $-3\cdot$ into a one, we need to divide by -3. We will need to divide this from both sides of the equation.

$-3x = 12$

$$\frac{-3x}{-3} = \frac{12}{-3}$$

Since, $\frac{-3}{-3} = 1$, we can write x on the left side.

$x = -4$ **On the right side of the equation, we need to calculate $12 \div -3$.

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$-3x = 12$

$-3(-4) = 12$

$12 = 12$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{x = -4}$.

Example 5:

Negative eighteen times a number equals -198. Find the number.

We need to turn the verbal equation into a numerical equation.

Negative eighteen times a number equals -198

$-18 \quad \cdot \quad n \quad = \quad -198$

$-18n = -198$ **I do not need to write the "·", writing the number and variable directly next to each other indicates multiplication.

If we look at the equation, the -18 is on the same side of the equation as the variable. The -18 is multiplied to the n. To turn the $-18 \cdot$ into a one, we need to divide by -18. We will need to divide this from both sides of the equation.

$-18n = -198$

$$\frac{-18n}{-18} = \frac{-198}{-18}$$

Since, $\frac{-18}{-18} = 1$, we can write n on the left side.

$n = 11$ **On the right side of the equation, we need to calculate $-198 \div -18$.

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$-18n = -198$

$-18(11) = -198$

$-198 = -198$ ✓

Since our solution checks out, we know we have the correct solution. **The number is 11**.

Example 6:

Solve: $\frac{t}{7} = -5$

If we look at the equation, the 7 is on the same side of the equation as the variable. To turn the $/7$ into a one, we need to multiply by 7. We will need to multiply this to both sides of the equation.

$$\frac{t}{7} = -5$$

$$7 \cdot \frac{t}{7} = -5 \cdot 7$$

Since, $\frac{7}{7} = 1$, we can just write t on the left side.

$t = -35$ **On the right side of the equation, we need to calculate $-5 \cdot 7$.

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$$\frac{t}{7} = -5$$

$$\frac{-35}{7} = -5$$

$-5 = -5$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{t = -35}$.

Example 7:

Solve: $\frac{a}{36} = \frac{4}{9}$

If we look at the equation, the 36 is on the same side of the equation as the variable. To turn the $/36$ into a one, we need to multiply by 36. We will need to multiply this to both sides of the equation.

$$\frac{a}{36} = \frac{4}{9}$$

$$36 \cdot \frac{a}{36} = \frac{4}{9} \cdot 36$$

Since, $\frac{36}{36} = 1$, we can just write a on the left side.

$a = 16$ **On the right side of the equation, we need to calculate $\frac{4}{9} \cdot 36$.

$$\frac{4}{9} \cdot 36 = \frac{4}{9} \cdot \frac{36}{1} = \frac{144}{9} = \frac{16}{1} = 16$$

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$$\frac{a}{36} = \frac{4}{9}$$

$$\frac{16}{36} = \frac{4}{9}$$

$$\frac{16}{36} = \frac{4}{9}$$

$\frac{4}{9} = \frac{4}{9}$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{a = 16}$.

Example 8:

Solve: $\frac{2}{3}n = 10$

If we look at the equation, the $\frac{2}{3}$ is on the same side of the equation as the variable. The $\frac{2}{3}$ is multiplied to the n. To turn the $\frac{2}{3}\cdot$ into a one, we need to divide by $\frac{2}{3}$. We will need to divide this from both sides of the equation.

$$\frac{2}{3}n = 10$$

$$\frac{\frac{2}{3}n}{\frac{2}{3}} = \frac{10}{\frac{2}{3}}$$

Since, $\frac{\frac{2}{3}}{\frac{2}{3}} = 1$, we can just write n on the left side.

$n = 15$ **On the right side of the equation, we need to calculate $10 \div \frac{2}{3}$.

$$10 \div \frac{2}{3} = \frac{10}{1} \div \frac{2}{3} = \frac{10}{1} \cdot \frac{3}{2} = \frac{30}{2} = \frac{15}{1} = 15$$

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$$\frac{2}{3}n = 10$$

$$\frac{2}{3}(15) = 10$$

$$\frac{2}{3}\left(\frac{15}{1}\right) = \frac{30}{3} = 10$$

$10 = 10$ ✓

Since our solution checks out, we know we have the correct solution $\boldsymbol{n = 15}$.

Example 9:

Solve: $\frac{8}{9} = \frac{4}{5}k$

In this equation, the variable is on the right-hand side of the equation. This does not change the way that we solve it. Keep in mind that the goal is to isolate the variable on one side of the equation. In this case, that will be on the right-hand side.

If we look at the equation, the $\frac{4}{5}$ is on the same side of the equation as the variable. The $\frac{4}{5}$ is multiplied to the k. To turn the $\frac{4}{5}\cdot$ into a one, we need to divide by $\frac{4}{5}$. We will need to divide this from both sides of the equation.

$$\frac{8}{9} = \frac{4}{5}k$$

$$\frac{\frac{8}{9}}{\frac{4}{5}} = \frac{\frac{4}{5}k}{\frac{4}{5}}$$

Since, $\frac{\frac{4}{5}}{\frac{4}{5}} = 1$, we can just write k on the right side.

$1\frac{1}{9} = k$ **On the left side of the equation, we need to calculate $\frac{8}{9} \div \frac{4}{5}$.

$$\frac{8}{19} \div \frac{4}{5} = \frac{8}{9} \cdot \frac{5}{4} = \frac{40}{36} = \frac{10}{9} = 1\frac{1}{9}$$

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$$\frac{8}{9} = \frac{4}{5}k$$

$$\frac{8}{9} = \frac{4}{5}\left(1\frac{1}{9}\right)$$

$$\frac{4}{5}\left(1\frac{1}{9}\right) = \frac{4}{5}\left(\frac{10}{9}\right) = \frac{40}{45} = \frac{8}{9}$$

$\frac{8}{9} = \frac{8}{9}$ ✓

Since our solution checks out, we know we have the correct solution $\boldsymbol{k = 1\frac{1}{9}}$.

Note that $k = 1\frac{1}{9}$ and $1\frac{1}{9} = k$ are the same thing. We can write that in either order and it is correct.

<u>Example 10:</u>

Solve: $12 = \frac{x}{-3}$

This equation has the variable on the right side of the equation again. Remember that we will just isolate the variable on that side.

If we look at the equation, the -3 is on the same side of the equation as the variable. To turn the $/-3$ into a one, we need to multiply by -3. We will need to multiply this to both sides of the equation.

$$12 = \frac{x}{-3}$$

$$-3 \cdot 12 = \frac{x}{-3} \cdot -3$$

Since, $\frac{-3}{-3} = 1$, we can just write x on the right side.

$-36 = x$ **On the left side of the equation, we need to calculate $-3 \cdot 12$.

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$$12 = \frac{x}{-3}$$

$$12 = \frac{-36}{-3}$$

$$12 = 12 \quad \checkmark$$

Since our solution checks out, we know we have the correct solution: $\boldsymbol{x = -36}$.

<u>Example 11:</u>

Solve: $-\frac{r}{4} = \frac{1}{7}$

It is important to note that $-\frac{r}{4} = \frac{-r}{4} = \frac{r}{-4}$. All three of these statements say the same thing. So, in the interests of efficiency, we will treat the negative like it goes with the denominator so that we can solve the equation in one step instead of two.

$$\frac{r}{-4} = \frac{1}{7}$$

If we look at the equation, the -4 is on the same side of the equation as the variable. To turn the $/-4$ into a one, we need to multiply by -4. We will need to multiply this to both sides of the equation.

$$\frac{r}{-4} = \frac{1}{7}$$

$$-4 \cdot \frac{r}{-4} = \frac{1}{7} \cdot -4$$

Since, $\frac{-4}{-4} = 1$, we can just write r on the left side.

$r = -\frac{4}{7}$ **On the right side of the equation, we need to calculate $\frac{1}{7} \cdot -4$.

$$\frac{1}{7} \cdot -4 = \frac{1}{7} \cdot \frac{-4}{1} = -\frac{4}{7}$$

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$$\frac{r}{-4} = \frac{1}{7}$$

$$\frac{-\frac{4}{7}}{-4} = \frac{1}{7}$$

$$\frac{-\frac{4}{7}}{-4} = -\frac{4}{7} \div -4 = -\frac{4}{7} \div -\frac{4}{1} = -\frac{4}{7} \cdot -\frac{1}{4} = \frac{4}{28} = \frac{1}{7}$$

$\frac{1}{7} = \frac{1}{7}$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{r = -\frac{4}{7}}$.

Example 12:

The discharge of a river is defined as the product of its width, its average depth, and its speed. At one location in St. Louis, the Mississippi River is 533 meters wide, its speed is 0.6 meter per second, and its discharge is 3198 cubic meters per second. How deep is the Mississippi River at this location?

Let's take the first sentence and convert it into an equation.

The discharge of a river is defined as the product of its width, its average depth, and its speed

The discharge of a river	is defined as	the product of	its width, its average depth, and its speed
R	=	Multiply everything that follows	W x D x S

So, our equation is $R = WDS$

They give us some values to plug in:

R = discharge of river

W = width of river

D = depth of river

S = speed of river

At one location in St. Louis, the Mississippi River is 533 meters wide, its speed is 0.6 meter per second, and its discharge is 3198 cubic meters per second

We have something to plug in for all variables, except the depth of the river, since that is what we are asked to find. So, let's rewrite our equation with those variables that we know replaced with values.

$R = WDS$

$3198 = (533)(D)(0.6)$

Since we can change the order of multiplication on one side of the equation, let's do that so that the numbers are together.

$3198 = (533)(0.6)(D)$

Now, let's multiply $(533)(0.6) = 319.8$.

$3198 = 319.8D$

Now, we can solve this as we would any other equation using multiplication or division.

If we look at the equation, the 319.8 is on the same side of the equation as the variable. The 319.8 is multiplied to the D. To turn the $319.8 \cdot$ into a one, we need to divide by 319.8. We will need to divide this from both sides of the equation.

$3198 = 319.8D$

$$\frac{3198}{319.8} = \frac{319.8D}{319.8}$$

Since, $\frac{319.8}{319.8} = 1$, we can write D on the right side.

$10 = D$ **On the left side of the equation, we need to calculate $3198 \div 319.8$.

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$3198 = (533)(10)(0.6)$

$3198 = (5330)(0.6)$

$3198 = 3198$ ✓

Since our solution checks out, we know we have the correct solution. We now just need to determine units. Since we have calculated depth of the river (which is a measurement of length) and we know that width of the river (which is also a measurement of length) was given in terms of meters, our answer will have units of meters.

The river has an average depth of 10 meters at this location in St. Louis.

Example 13:

Casey and Juanita are both solving the equation $8n = -72$. Who is correct? Why?

Casey

$8n = -72$

$8n(8) = -72(8)$

$n = -576$

Juanita

$8n = -72$

$$\frac{8n}{8} = \frac{-72}{8}$$

$n = -9$

Juanita solved the equation correctly. She correctly identified the operation between 8 and the variable as multiplication and used a division to create the multiplicative identity of one. Casey should have actually ended up with $64n$ on the left side of the equation.

Example 14:

Solve: $-2g = -84$

If we look at the equation, the -2 is on the same side of the equation as the variable. The -2 is multiplied to the g. To turn the $-2\,\cdot$ into a one, we need to divide by -2. We will need to divide this from both sides of the equation.

$-2g = -84$

$$\frac{-2g}{-2} = \frac{-84}{-2}$$

Since, $\frac{-2}{-2} = 1$, we can write g on the left side.

$g = 21$ **On the right side of the equation, we need to calculate $-84 \div -2$.

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$-2g = -84$

$-2(21) = -84$

$-84 = -84$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{g = 21}$.

Example 15:

Five times a number is 120. What is the number?

We need to turn the verbal equation into a numerical equation.

Five times a number is 120

$5 \quad \cdot \quad n \quad = 120$

$5n = 120$ **I do not need to write the "·", writing the number and variable directly next to each other indicates multiplication.

If we look at the equation, the 5 is on the same side of the equation as the variable. The 5 is multiplied to the n. To turn the $5\cdot$ into a one, we need to divide by 5. We will need to divide this from both sides of the equation.

$$5n = 120$$

$$\frac{5n}{5} = \frac{120}{5}$$

Since, $\frac{5}{5} = 1$, we can write n on the left side.

$n = 24$ **On the right side of the equation, we need to calculate $120 \div 5$.

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$$5n = 120$$

$$5(24) = 120$$

$$120 = 120 \quad \checkmark$$

Since our solution checks out, we know we have the correct solution. **The number is 24**.

Example 16:

One third equals negative seven times a number. What is the number?

We need to turn the verbal equation into a numerical equation.

One third equals negative seven times a number

$$\underbrace{\text{One third}}_{\frac{1}{3}}\ \underbrace{\text{equals}}_{=}\ \underbrace{\text{negative seven}}_{-7}\ \underbrace{\text{times}}_{\cdot}\ \underbrace{\text{a number}}_{n}$$

$\frac{1}{3} = -7n$ **I do not need to write the "·", writing the number and variable directly next to each other indicates multiplication.

If we look at the equation, the -7 is on the same side of the equation as the variable. The -7 is multiplied to the n. To turn the $-7\cdot$ into a one, we need to divide by -7. We will need to divide this from both sides of the equation.

$$\frac{1}{3} = -7n$$

$$\frac{\frac{1}{3}}{-7} = \frac{-7n}{-7}$$

Since, $\frac{-7}{-7} = 1$, we can write n on the right side.

$-\frac{1}{21} = n$ **On the left side of the equation, we need to calculate $\frac{1}{3} \div -7$.

$$\frac{1}{3} \div -7 = \frac{1}{3} \div -\frac{7}{1} = \frac{1}{3} \cdot -\frac{1}{7} = -\frac{1}{21}$$

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$$\frac{1}{3} = -7n$$

$$\frac{1}{3} = -7\left(-\frac{1}{21}\right)$$

$$-7\left(-\frac{1}{21}\right) = -\frac{7}{1}\left(-\frac{1}{21}\right) = \frac{7}{21} = \frac{1}{3}$$

$\frac{1}{3} = \frac{1}{3}$ ✓

Since our solution checks out, we know we have the correct solution. **The number is $-\frac{1}{21}$.**