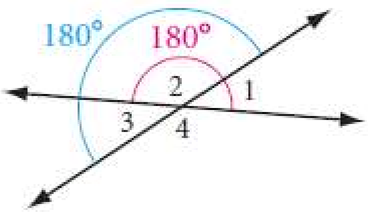
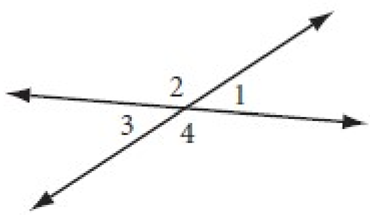
<u>Lesson 2.5 – Angle Relationships</u>

Linear Pair Conjecture - If two angles form a linear pair, then the measure of the angles add up to 180°.

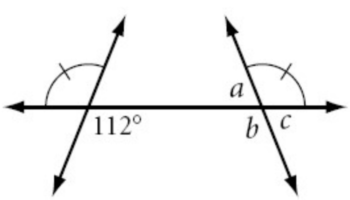


Vertical Angles Conjecture - If two angles are vertical angles, then they are congruent.

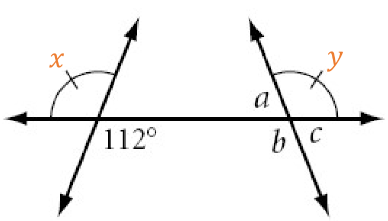


$\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

<u>Example 1: Find each lettered angle measure without a protractor.</u>



Let's label a couple of angles so that we can talk about them more clearly.



$x = 112°$ x and the $112°$ angle are vertical angles and are therefore congruent.

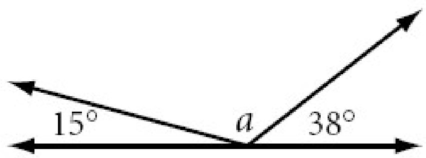
$y = 112°$ y is marked congruent to x, so it must equal the same degree measure.

$\boldsymbol{a = 68°}$ a and y are a linear pair of angles and must add to $180°$ ($a + 112 = 180$).

$\boldsymbol{b = 112°}$ y and b are vertical angles and are therefore congruent.

$\boldsymbol{c = 68°}$ c and a are vertical angles and are therefore congruent.

<u>Example 2: Find each lettered angle measure without a protractor.</u>

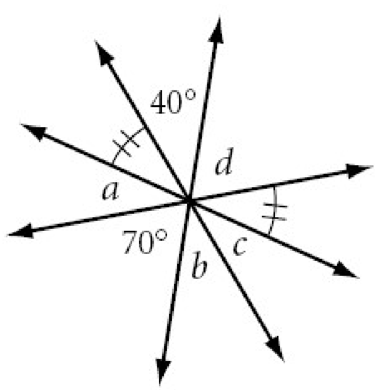


$\boldsymbol{a = 127°}$ a, the $15°$ angle and the $38°$ angles are supplementary because together they form a line. So, they must add to $180°$.

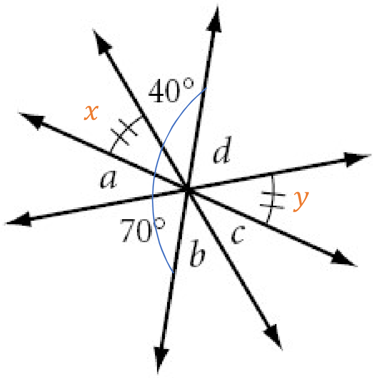
$$a + 15 + 38 = 180$$

$$a + 53 = 180$$

<u>Example 3: Find each lettered angle measure without a protractor.</u>



Let’s label a couple of angles so that we can talk about them more clearly.



a is a vertical angle to y, so those two must be congruent. y is marked congruent to x, therefore a, x and y are all congruent to each other and have the same measure.

$\boldsymbol{a = 35°}$ a, x, the 70° angle and the 40° angle are supplementary because together they form a line. So, they must add to 180°.

$$a + x + 70 + 40 = 180$$

$$a + x + 110 = 180$$

$$2a + 110 = 180$$

$$2a = 70$$

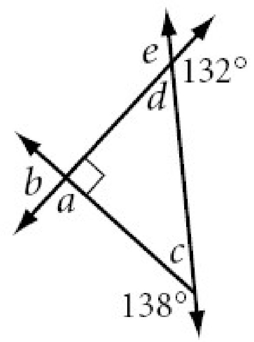
Since x and a have the same measure, we can say that $a + x$ is the same as having 2 a.

$\boldsymbol{b = 40°}$ The 40° angle and b are vertical angles and are therefore congruent.

$\boldsymbol{c = 35°}$ c and x are vertical angles and are therefore congruent. We established that x has the same measure as a.

$\boldsymbol{d = 70°}$ d and the 70° angle are vertical angles and are therefore congruent.

Example 4: Find each lettered angle measure without a protractor.



$\boldsymbol{a = 90°}$ a and the right angle (90°) are a linear pair of angles and must add to 180°.

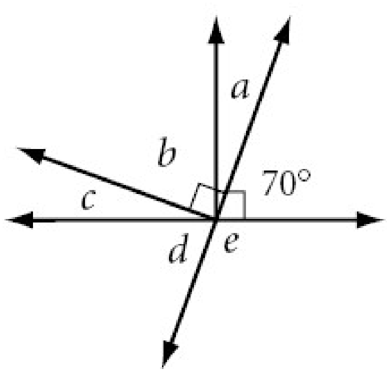
$\boldsymbol{b = 90°}$ b and the right angle are vertical angles and are therefore congruent.

$\boldsymbol{c = 42°}$ c and the 138° angle are a linear pair of angles and must add to 180°.

$\boldsymbol{d = 48°}$ d and the 132° angle are a linear pair of angles and must add to 180°.

$\boldsymbol{e = 132°}$ e and the 132° angle are vertical angles and are congruent.

Example 5: Find each lettered angle measure without a protractor.



$\boldsymbol{a = 20°}$ a and the 70° angle are a complementary pair of angles (notice the right angle marking that passes through a and the 70° angle) and must add to 90°.

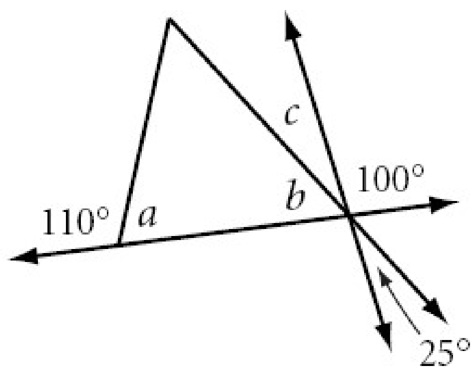
$\boldsymbol{b = 70°}$ b and a are a complementary pair of angles (notice the right angle marking that passes through a and b) and must add to 90°.

$\boldsymbol{c = 20°}$ c, b, a and the 70° angle are supplementary because they form a line when all combined together. So, they must add to 180°.

$\boldsymbol{d = 70°}$ d and the 70° angle are vertical angles and are congruent.

$\boldsymbol{e = 110°}$ e and d are a linear pair of angles and must add to 180°.

Example 6: Find each lettered angle measure without a protractor.



$a = 70°$ a and the $110°$ angle are a linear pair of angles and must add to $180°$.

$b = 55°$ $c, b,$ and the $100°$ angle are supplementary because they form a line when all combined together. So, they must add to $180°$. Find the measure of c first, and then calculate b.

$c = 25°$ c and the $25°$ angle are vertical angles and are congruent.

Example 7: Tell whether each statement is always (A), sometimes (S), or never (N) true.

________ The sum of the measures of two acute angles equals the measure of an obtuse angle.

The key in a problem like this is to try some examples. Acute angles are less than $90°$. So, if I think about $15° + 36° = 51°$, I have two acute angles that add to another acute angle. So, we know that this statement is not "always true". Now we must decide between sometimes or never true. So, I want to see if I can think of an example where two acute angles would equal the measure of an obtuse angle. $50° + 51° = 101°$. Since I can think of an example where the statement is true, it cannot be "never true". Therefore, our answer is (S), sometimes true.

Example 8: Tell whether each statement is always (A), sometimes (S), or never (N) true.

________ If two angles form a linear pair, then they are complementary.

A linear pair of angles form a line and add to $180°$. A complementary pair of angles form a right angle and add to $90°$. Since the two things are not the same, our answer is (N), never true.

Example 9: Fill in each blank to make a true statement.

If one angle of a linear pair is obtuse, then the other is _________________.

A linear pair of angles form a line and add to $180°$. An obtuse angle measures more than $90°$. If you think about subtracting a number larger than $90°$ from $180°$, you will be left with a value that is less than $90°$, which is the definition of an acute angle.

acute

Example 10: Fill in each blank to make a true statement.

If $\angle A \cong \angle B$ and the supplement of $\angle B$ has measure $22°$, then $m\angle A =$_________________.

$\angle A \cong \angle B$ means that those angles have the same measure. A supplement means that that angles are supplementary and add to $180°$. So, $m\angle B + 22° = 180°$. By solving, we find that $m\angle B = 158°$. Since we established that $\angle A \cong \angle B$, then $m\angle A = m\angle B$.

$\boldsymbol{m\angle A = 158°}$

Example 11: Fill in each blank to make a true statement.

If $\angle P$ is a right angle and $\angle P$ and $\angle Q$ form a linear pair, then $m\angle Q$ is _________________.

$\angle P$ is a right angle, so $m\angle P = 90°$. A linear pair of angles form a line and add to $180°$. So, $m\angle P + m\angle Q = 180°$. Since we know $m\angle P = 90°$, we can write $90° + m\angle Q = 180°$. By solving, we find that $m\angle Q = 90°$.

$\boldsymbol{90°}$

Example 12: Fill in each blank to make a true statement.

If $\angle S$ and $\angle T$ are complementary and $\angle T$ and $\angle U$ are supplementary, then $\angle U$ is a(n)_________________ angle.

A complementary pair of angles form a right angle and add to $90°$, so $m\angle S + m\angle T = 90°$. Two angles can only be complementary if they are each acute. A supplementary pair of angles add to $180°$, so $m\angle T + m\angle U = 180°$. Since we know $\angle P$ is acute, that means $\angle U$ must be obtuse.

obtuse