Solving Equations by Using Addition and Subtraction

Addition Property of Equality:

If an equation is true and the same number is added to each side, the resulting equation is true.

Example: $7 = 7$

$$7 + 3 = 7 + 3$$

$$10 = 10$$

For any numbers $a, b,$ and c, if $a = b$, then $a + c = b + c$

Subtraction Property of Equality:

If an equation is true and the same number is subtracted from each side, the resulting equation is true.

Example: $7 = 7$

$$7 - 8 = 7 - 8$$

$$-1 = -1$$

For any numbers $a, b,$ and c, if $a = b$, then $a - c = b - c$

**The basic idea of both the addition and subtraction properties of equality is that, in order to maintain equality in an equation, if we add or subtract something on one side of the equation we must do the same on the other side of the equation.

Example 1:

Solve: $m - 48 = 29$

When we solve an equation, we are looking for the value of the variable. The most efficient way to determine the value of the variable is to isolate the variable on one side of the equation.

To isolate the variable in this problem, we need to add or subtract something to both sides of the equation so that we make a zero of anything on the same side as the variable.

If we look at the equation, the -48 is on the same side of the equation as the variable. To turn the -48 into a zero, we need to add 48. We will need to add this to both sides of the equation.

$m - 48 = 29$

$+48 \quad +48$

Since, $-48 + 48 = 0$, we could write $m + 0$ on the left side of the equation, but we don't need to write the $+0$ since it doesn't have an effect. So, we can simply write m on the left side.

$m = 77$ **On the right side of the equation, we need to calculate $29 + 48$.

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$m - 48 = 29$

$77 - 48 = 29$

$29 = 29$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{m = 77}$.

Example 2:

Solve: $21 + q = -18$

When we solve an equation, we are looking for the value of the variable. The most efficient way to determine the value of the variable is to isolate the variable on one side of the equation.

To isolate the variable in this problem, we need to add or subtract something to both sides of the equation so that we make a zero of anything on the same side as the variable.

If we look at the equation, the 21 is on the same side of the equation as the variable. To turn the 21 into a zero, we need to subtract 21. We will need to subtract this from both sides of the equation.

$21 + q = -18$

$-21 \quad -21$

Since, $21 - 21 = 0$, we can simply write q on the left side.

$q = -39$ **On the right side of the equation, we need to calculate $-18 - 21$.

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$21 + q = -18$

$21 + (-39) = -18$

$-18 = -18$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{q = -39}$.

Example 3:

Solve: $g + \frac{3}{4} = -\frac{1}{8}$

We need to isolate the variable on one side of the equation.

If we look at the equation, the $\frac{3}{4}$ is on the same side of the equation as the variable. To turn the $\frac{3}{4}$ into a zero, we need to subtract $\frac{3}{4}$ from both sides of the equation.

$$g + \frac{3}{4} = -\frac{1}{8}$$
$$-\frac{3}{4} \qquad -\frac{3}{4}$$

Since, $\frac{3}{4} - \frac{3}{4} = 0$, we can simply write g on the left side.

$g = -\frac{7}{8}$ **On the right side of the equation, we need to calculate $-\frac{1}{8} - \frac{3}{4}$.

Common Denominator: $-\frac{1}{8} - \frac{3 \cdot 2}{4 \cdot 2}$

$-\frac{1}{8} - \frac{6}{8}$

Combine Numerators: $-\frac{7}{8}$

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$$g + \frac{3}{4} = -\frac{1}{8}$$
$$-\frac{7}{8} + \frac{3}{4} = -\frac{1}{8}$$
$$-\frac{7}{8} + \frac{6}{8} = -\frac{1}{8}$$
$-\frac{1}{8} = -\frac{1}{8}$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{g = -\frac{7}{8}}$.

Example 4:

A number increased by 5 is equal to 42. Find the number.

We need to turn the verbal equation into a numerical equation.

A number increased by 5 is equal to 42

$n \quad + \quad 5 \quad = \quad 42$

$n + 5 = 42$

We need to isolate the variable on one side of the equation.

If we look at the equation, the 5 is on the same side of the equation as the variable. To turn the 5 into a zero, we need to subtract 5 from both sides of the equation.

$n + 5 = 42$

$-5 \quad -5$

$n = 37$ **On the right side of the equation, we need to calculate $42 - 5$.

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$n + 5 = 42$

$37 + 5 = 42$

$42 = 42$ ✓

Since our solution checks out, we know we have the correct solution. **The number is 37**.

Example 5:

Solve: $n - 20 = 5$

The -20 is on the same side of the equation as the variable. We will need to add 20 to both sides of the equation.

$n - 20 = 5$

$+20 \quad +20$

$n = 25$

Check:

$n - 20 = 5$

$25 - 20 = 5$

$5 = 5$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{n = 25}$.

Example 6:

Solve: $104 = y - 67$

The -67 is on the same side of the equation as the variable. We will need to add 67 to both sides of the equation.

$104 = y - 67$

$+67 \quad + 67$

$171 = y$ **Even though the variable is on the right side of the equation, it is still isolated, so we have solved the equation.

Check:

$104 = y - 67$

$104 = 171 - 67$

$104 = 104$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{y = 171}$.

**Note: $171 = y$ and $y = 171$ are the same thing.

Example 7:

Solve: $-4 + t = -7$

The -4 is on the same side of the equation as the variable. We will need to add 4 to both sides of the equation.

$-4 + t = -7$

$+4 \qquad +4$

$t = -3$

Check:

$-4 + (-3) = -7$

$-7 = -7$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{t = -3}$.

Example 8:

Solve: $33 = g + 5$

The 5 is on the same side of the equation as the variable. We will need to subtract 5 from both sides of the equation.

$33 = g + 5$

$-5 \quad -5$

$28 = g$ **Even though the variable is on the right side of the equation, it is still isolated, so we have solved the equation.

Check:

$33 = 28 + 5$

$33 = 33$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{g = 28}$.

**Note: $28 = g$ and $g = 28$ are the same thing.

Example 9:

Solve: $19 + p = 6$

The 19 is on the same side of the equation as the variable. We will need to subtract 19 from both sides of the equation.

$19 + p = 6$

$-19 \quad -19$

$p = -13$

Check:

$19 + (-13) = 6$

$6 = 6$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{p = -13}$.

Example 10:

Solve: $15 = b - (-65)$

**We see a $-(-$ in the equation, so we need to rewrite the equation and change that to an addition before we begin solving.

$15 = b + 65$

The 65 is on the same side of the equation as the variable. We will need to subtract 65 from both sides of the equation.

$15 = b + 65$

$-65 \quad -65$

$-50 = b$

Check:

$15 = -50 - (-65)$

$15 = -50 + 65$

$15 = 15$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{b = -50}$.

Example 11:

Solve: $h + \frac{2}{5} = \frac{7}{10}$

The $\frac{2}{5}$ is on the same side of the equation as the variable. We need to subtract $\frac{2}{5}$ from both sides of the equation.

$$h + \frac{2}{5} = \frac{7}{10}$$
$$-\frac{2}{5} \quad -\frac{2}{5}$$
$$h = \frac{3}{10}$$

**On the right side of the equation, we need to calculate $\frac{7}{10} - \frac{2}{5}$.

Common Denominator: $\frac{7}{10} - \frac{2 \cdot 2}{5 \cdot 2}$

$\frac{7}{10} - \frac{4}{10}$

Combine Numerators: $\frac{3}{10}$

Check:

$$\frac{3}{10} + \frac{2}{5} = \frac{7}{10}$$
$$\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$$
$$\frac{7}{10} = \frac{7}{10} \quad \checkmark$$

Since our solution checks out, we know we have the correct solution: $\boldsymbol{h = \frac{3}{10}}$.

Example 12:

Solve: $-6 = \frac{1}{4} + m$

The $\frac{1}{4}$ is on the same side of the equation as the variable. We need to subtract $\frac{1}{4}$ from both sides of the equation.

$-6 = \frac{1}{4} + m$

$-\frac{1}{4} \quad -\frac{1}{4}$

$-6\frac{1}{4} = m$

**On the right side of the equation, we need to calculate $-6 - \frac{1}{4}$.

Since both numbers are negative and we will have a bigger number, it is easy to just combine the two into a mixed number $-6\frac{1}{4}$.

Check:

$-6 = \frac{1}{4} + \left(-6\frac{1}{4}\right)$

$-6 = -6$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{m = -6\frac{1}{4}}$.

Example 13:

Solve: $-\frac{2}{3} + w = 1\frac{1}{2}$

The $-\frac{2}{3}$ is on the same side of the equation as the variable. We need to add $\frac{2}{3}$ to both sides of the equation.

$-\frac{2}{3} + w = 1\frac{1}{2}$

$+\frac{2}{3} \qquad +\frac{2}{3}$

$h = 2\frac{1}{6}$

**On the right side of the equation, we need to calculate $1\frac{1}{2} + \frac{2}{3}$.

Mixed number to improper fraction: $\frac{3}{2} + \frac{2}{3}$

Common Denominator: $\frac{3\cdot3}{2\cdot3} + \frac{2\cdot2}{3\cdot2}$

$\frac{9}{6} + \frac{4}{6}$

Combine Numerators: $\frac{13}{6}$

Check:

$\frac{3}{10}+\frac{2}{5}=\frac{7}{10}$

$\frac{3}{10}+\frac{4}{10}=\frac{7}{10}$

$\frac{7}{10}=\frac{7}{10}$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{h=\frac{3}{10}}$.

Example 14:

Twenty-one subtracted from a number is -8. Find the number.

We need to turn the verbal equation into a numerical equation.

Twenty-one subtracted from a number is -8

21 **Read side note n = -8

**The phrase "subtracted from" is one of those phrases that makes us re-order our equation. So, we will need a subtraction as the operation. However, saying that twenty-one is subtracted from means that the twenty-one should come after the subtraction and the "a number" that follows the phrase "subtracted from" should come before the subtraction operation.

$n-21=-8$

We need to isolate the variable on one side of the equation.

If we look at the equation, the -21 is on the same side of the equation as the variable. To turn the -21 into a zero, we need to add 21 to both sides of the equation.

$n-21=-8$

$+21 \quad +21$

$n=13$ **On the right side of the equation, we need to calculate $-8+21$.

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$n - 21 = -8$

$13 - 21 = -8$

$-8 = -8$ ✓

Since our solution checks out, we know we have the correct solution. **The number is 13**.

Example 15:

A number increased by 91 is 37. Find the number.

We need to turn the verbal equation into a numerical equation.

A number increased by 91 is 37

n + 91 = 37

$n + 91 = 37$

We need to isolate the variable on one side of the equation.

If we look at the equation, the 91 is on the same side of the equation as the variable. To turn the 91 into a zero, we need to subtract 91 from both sides of the equation.

$n + 91 = 37$

$-91 \quad -91$

$n = -54$ **On the right side of the equation, we need to calculate $37 - 91$.

Since our variable is isolated, we have found the value of our variable. However, it's always a good idea to check and make sure we didn't make a mistake somewhere in our solving.

$n + 91 = 37$

$-54 + 91 = 37$

$37 = 37$ ✓

Since our solution checks out, we know we have the correct solution. **The number is -54**.