Ratio and Proportion

Ratio: A ratio is a comparison of two numbers by division.

We write ratios in one of three ways:

- x to y
- $x: y$
- x/y or $\frac{x}{y}$

Examples of ratios:

- 2 cups of flour to 1 egg
- 2:1
- 2/1 or $\frac{2}{1}$

Proportion: A proportion is an equation stating that two ratios are equal.

Example of a proportion: $\frac{4}{2} = \frac{8}{4}$

Example 1:

Determine whether the ratios form a proportion.

$\frac{4}{5}, \frac{24}{30}$

To determine whether the ratios form a proportion, we just need to determine if the ratios (fractions) are equal.

Since the first fraction is smaller, we will determine what we need to multiply the numerator and denominator by to obtain the second fraction's numerator and denominator.

Numerator:

$4 \cdot ? = 24$

$? = 6$

Denominator:

$5 \cdot ? = 30$

$? = 6$

Since the numerator and denominator are both multiplied by the same thing, we know that the ratios **do form a proportion.**

Example 2:

Determine whether the ratios form a proportion.

$\frac{6}{10}, \frac{2}{5}$

To determine whether the ratios form a proportion, we just need to determine if the ratios (fractions) are equal.

Since the first fraction is larger, we will determine what we need to divide the numerator and denominator by to obtain the second fraction's numerator and denominator.

Numerator:

$6 \div ? = 2$

$? = 3$

Denominator:

$10 \div ? = 5$

$? = 2$

Since the numerator and denominator are NOT divided by the same thing, we know that the ratios **do not form a proportion.**

Example 3:

Determine whether the ratios form a proportion.

$\frac{1}{6}, \frac{5}{30}$

To determine whether the ratios form a proportion, we just need to determine if the ratios (fractions) are equal.

Since the first fraction is smaller, we will determine what we need to multiply the numerator and denominator by to obtain the second fraction's numerator and denominator.

Numerator:

$1 \cdot ? = 5$

$? = 5$

Denominator:

$6 \cdot ? = 30$

$? = 5$

Since the numerator and denominator are both multiplied by the same thing, we know that the ratios **do form a proportion.**

Not all proportions are multiplied or divided by whole numbers. So, we have another method of determining whether two ratios are proportions. It is called a cross product.

Cross Product: If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example 4:

Use cross products to determine whether each pair of ratios forms a proportion.

$\frac{0.4}{0.8}, \frac{0.7}{1.4}$

If these two fractions are equal, then their cross products will also be equal.

$\frac{0.4}{0.8}, \frac{0.7}{1.4}$

$0.4 \cdot 1.4 \overset{?}{\Leftrightarrow} 0.8 \cdot 0.7$

$0.56 = 0.56$

Since the cross products are equal, we know that the pair of rations **do form a proportion.**

Example 5:

Use cross products to determine whether each pair of ratios forms a proportion.

$\frac{6}{8}, \frac{24}{28}$

If these two fractions are equal, then their cross products will also be equal.

$$\frac{6}{8}, \frac{24}{28}$$

$$6 \cdot 28 \overset{?}{\Leftrightarrow} 8 \cdot 24$$

$$168 \neq 192$$

Since the cross products are not equal, we know that the pair of rations **do not form a proportion.**

<u>Example 6:</u>

Solve the proportion.

$$\frac{n}{15} = \frac{24}{16}$$

In order to solve this proportion, we will use cross products.

$$\frac{n}{15} = \frac{24}{16}$$

$$n \cdot 16 = 15 \cdot 24$$

$$16n = 360$$

From here, the solving should be a familiar problem.

$$16n = 360$$

$$\frac{16n}{16} = \frac{360}{16}$$

$$n = 22\frac{1}{2}$$

We can plug our solution in and check that it works.

$$\frac{n}{15} = \frac{24}{16}$$

$$\frac{22\frac{1}{2}}{15} = \frac{24}{16}$$

If we divide both fractions:

$1.5 = 1.5$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{n = 22\frac{1}{2}}$.

<u>Example 7:</u>

Solve the proportion.

$$\frac{r}{8} = \frac{25}{40}$$

In order to solve this proportion, we will use cross products.

$$\frac{r}{8} = \frac{25}{40}$$

$r \cdot 40 = 8 \cdot 25$

$40r = 200$

From here, the solving should be a familiar problem.

$40r = 200$

$$\frac{40r}{40} = \frac{200}{40}$$

$r = 5$

We can plug our solution in and check that it works.

$$\frac{r}{8} = \frac{25}{40}$$

$$\frac{5}{8} = \frac{25}{40}$$

If we divide both fractions:

$0.625 = 0.625$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{r = 5}$.

Example 8:

Solve the proportion.

$$\frac{3.2}{4} = \frac{2.6}{n}$$

In order to solve this proportion, we will use cross products.

$$\frac{3.2}{4} = \frac{2.6}{n}$$

$3.2 \cdot n = 4 \cdot 2.6$

$3.2n = 10.4$

From here, the solving should be a familiar problem.

$3.2n = 10.4$

$$\frac{3.2n}{3.2} = \frac{10.4}{3.2}$$

$n = 3.25$

We can plug our solution in and check that it works.

$$\frac{3.2}{4} = \frac{2.6}{n}$$

$$\frac{3.2}{4} = \frac{2.6}{3.25}$$

If we divide both fractions:

$0.8 = 0.8$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{n = 3.25}$.

Example 9:

Trent goes on a 30-mile bike ride every Saturday. He rides this distance in 4 hours. At this rate, how far can he ride in 6 hours?

For this problem, we need to set up a proportion. Remember that a proportion is just two equal ratios. So, when I'm setting up the ratio for how much Trent rides every Saturday, it can be time to miles or miles to time (it doesn't matter which way).

$\frac{30\ miles}{4\ hours}$ or $\frac{4\ hours}{30\ miles}$

When I set up the proportion, I need to make sure that both ratios are set up the same way.

$\frac{30\ miles}{4\ hours} = \frac{x\ miles}{6\ hou}$ or $\frac{4\ hours}{30\ miles} = \frac{6\ hour}{x\ miles}$

Notice how miles are both in the numerator in the first equation and hours are both in the denominator. Notice a similar situation in the second equation, except the numerators and denominators are flipped from the first equation. Either equation will give you the same answer. So, I will solve only the first one.

$$\frac{30\ miles}{4\ hours} = \frac{x\ miles}{6\ hours}$$

Let's remove the units while we solve.

$$\frac{30}{4} = \frac{x}{6}$$

In order to solve this proportion, we will use cross products.

$$\frac{30}{4} = \frac{x}{6}$$

$$30 \cdot 6 = 4 \cdot x$$

$$180 = 4x$$

$$\frac{180}{4} = \frac{4x}{4}$$

$$45 = x$$

Trent can ride 45 miles in 6 hours.

<u>Example 10:</u>

Determine whether each pair of ratios forms a proportion. Write yes or no.

$$\frac{4}{11}, \frac{12}{33}$$

We can solve this one of two ways:

Option #1:

Numerator:

$4 \cdot ? = 12$

$? = 3$

Denominator:

$11 \cdot ? = 33$

$? = 3$

Since the numerator and denominator are both multiplied by the same thing, we know that the ratios **do form a proportion.**

Options #2:

If these two fractions are equal, then their cross products will also be equal.

$$\frac{4}{11}, \frac{12}{33}$$

$$4 \cdot 33 \overset{?}{\Leftrightarrow} 11 \cdot 12$$

$$132 = 132$$

Since the cross products are equal, we know that the pair of rations **do form a proportion.**

Yes.

<u>Example 11:</u>

Determine whether each pair of ratios forms a proportion. Write yes or no.

$$\frac{16}{8}, \frac{8}{9}$$

We can solve this one of two ways:

Option #1:

Numerator:

$16 \div ? = 8$

$? = 2$

Denominator:

$8 \div ? = 9$

$? = \frac{8}{9}$

Since the numerator and denominator are not both divided by the same thing, we

Option #2:

If these two fractions are equal, then their cross products will also be equal.

$$\frac{16}{8}, \frac{8}{9}$$

$$16 \cdot 9 \overset{?}{\Leftrightarrow} 8 \cdot 8$$

$$144 \neq 64$$

know that the ratios **do not form a proportion.**

No.

Since the cross products are not equal, we know that the pair of rations **do not form a proportion.**

Example 12:

Determine whether each pair of ratios forms a proportion. Write yes or no.

$\frac{2.1}{3.5}, \frac{0.5}{0.7}$

Unlike the previous two problems, this one isn't one that I can know what I would multiply or divide off of the top of my head, so I will use cross products to solve.

If these two fractions are equal, then their cross products will also be equal.

$\frac{2.1}{3.5}, \frac{0.5}{0.7}$

$2.1 \cdot 0.7 \overset{?}{\Leftrightarrow} 3.5 \cdot 0.5$

$1.47 \neq 1.75$

Since the cross products are not equal, we know that the pair of rations do not form a proportion.

No.

Example 13:

Solve the proportion. If necessary, round to the nearest hundredth.

$\frac{3}{4} = \frac{6}{x}$

In order to solve this proportion, we will use cross products.

$\frac{3}{4} = \frac{6}{x}$

$3 \cdot x = 4 \cdot 6$

$3x = 24$

$\frac{3x}{3} = \frac{24}{3}$

$x = 8$

We can plug our solution in and check that it works.

$\frac{3}{4} = \frac{6}{x}$

$\frac{3}{4} = \frac{6}{8}$

If we divide both fractions:

$0.75 = 0.75$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{x = 8}$.

<u>Example 14:</u>

Solve the proportion. If necessary, round to the nearest hundredth.

$\frac{a}{45} = \frac{5}{15}$

In order to solve this proportion, we will use cross products.

$\frac{a}{45} = \frac{5}{15}$

$a \cdot 15 = 45 \cdot 5$

$15a = 225$

$\frac{15a}{15} = \frac{225}{15}$

$a = 15$

We can plug our solution in and check that it works.

$\frac{a}{45} = \frac{5}{15}$

$$\frac{15}{45} = \frac{5}{15}$$

If we divide both fractions:

$0.\overline{3} = 0.\overline{3}$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{a = 15}$.

Example 15:

Solve the proportion. If necessary, round to the nearest hundredth.

$$\frac{0.6}{1.1} = \frac{n}{8.47}$$

In order to solve this proportion, we will use cross products.

$$\frac{0.6}{1.1} = \frac{n}{8.47}$$

$0.6 \cdot 8.47 = 1.1 \cdot n$

$5.082 = 1.1n$

$$\frac{5.082}{1.1} = \frac{1.1n}{1.1}$$

$4.62 = n$

We can plug our solution in and check that it works.

$$\frac{0.6}{1.1} = \frac{n}{8.47}$$

$$\frac{0.6}{1.1} = \frac{4.62}{8.47}$$

If we divide both fractions:

$0.\overline{54} = 0.\overline{54}$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{n = 4.62}$.

Example 16:

Solve the proportion. If necessary, round to the nearest hundredth.

$$\frac{3}{12} = \frac{2}{y+6}$$

In order to solve this proportion, we will use cross products.

$$\frac{3}{12} = \frac{2}{y+6}$$

$3 \cdot (y+6) = 12 \cdot 2$ **It is important to remember to put parentheses around the $y + 6$.

$3y + 18 = 24$

$\quad -18 \quad -18$

$3y = 6$

$$\frac{3y}{3} = \frac{6}{3}$$

$y = 2$

We can plug our solution in and check that it works.

$$\frac{3}{12} = \frac{2}{y+6}$$

$$\frac{3}{12} = \frac{2}{2+6}$$

$$\frac{3}{12} = \frac{2}{8}$$

If we divide both fractions:

$0.25 = 0.25$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{y = 2}$.

Example 17:

Solve the proportion. If necessary, round to the nearest hundredth.

$$\frac{m-1}{8} = \frac{2}{4}$$

In order to solve this proportion, we will use cross products.

$$\frac{m-1}{8} = \frac{2}{4}$$

$(m-1) \cdot 4 = 8 \cdot 2$ **It is important to remember to put parentheses around the $m-1$.

$4m - 4 = 16$ $**(m-1) \cdot 4 = 4 \cdot (m-1) = 4(m-1) = 4m - 4$

$\quad +4 \quad +4$

$4m = 20$

$$\frac{4m}{4} = \frac{20}{4}$$

$m = 5$

We can plug our solution in and check that it works.

$$\frac{m-1}{8} = \frac{2}{4}$$

$$\frac{5-1}{8} = \frac{2}{4}$$

$$\frac{4}{8} = \frac{2}{4}$$

If we divide both fractions:

$0.5 = 0.5$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{m = 5}$.

<u>Example 18:</u>

Solve the proportion. If necessary, round to the nearest hundredth.

$$\frac{5}{12} = \frac{x+1}{4}$$

In order to solve this proportion, we will use cross products.

$$\frac{5}{12} = \frac{x+1}{4}$$

$5 \cdot 4 = 12 \cdot (x + 1)$ **It is important to remember to put parentheses around the $x + 1$.

$20 = 12x + 12$

$-12 \qquad -12$

$8 = 12x$

$\frac{8}{12} = \frac{12x}{12}$

$0.\overline{6} = x$

Since we need to round to the nearest hundredth: $0.67 = x$.

We can plug our solution in and check that it works.

$\frac{5}{12} = \frac{x + 1}{4}$

$\frac{5}{12} = \frac{0.67 + 1}{4}$

$\frac{5}{12} = \frac{1.67}{4}$

If we divide both fractions AND round to the nearest hundredth:

$0.42 = 0.42$ ✓

Since our solution checks out, we know we have the correct solution: $\boldsymbol{x = 0.67}$.

Example 19:

The Lehmans' minivan requires 5 gallons of gasoline to travel 120 miles. How much gasoline will they need for a 350-mile trip?

For this problem, we need to set up a proportion. Remember that a proportion is just two equal ratios. So, when I'm setting up the ratio for how much gas I know the minivan uses, it can be gallons to miles or miles to gallons (it doesn't matter which way).

$\frac{5\ gallons}{120\ miles}$ or $\frac{120\ miles}{5\ gallons}$

When I set up the proportion, I need to make sure that both ratios are set up the same way.

$\frac{5\ gallons}{120\ miles} = \frac{x\ gallons}{350\ miles}$ or $\frac{120\ miles}{5\ gallons} = \frac{350\ miles}{x\ gallons}$

Notice how gallons are both in the numerator in the first equation and miles are both in the denominator. Notice a similar situation in the second equation, except the numerators and denominators are flipped from the first equation. Either equation will give you the same answer. So, I will solve only the first one.

$$\frac{5\ gallons}{120\ miles} = \frac{x\ gallons}{350\ miles}$$

Let's remove the units while we solve.

$$\frac{5}{120} = \frac{x}{350}$$

In order to solve this proportion, we will use cross products.

$$\frac{5}{120} = \frac{x}{350}$$

$$5 \cdot 350 = 120 \cdot x$$

$$1750 = 120x$$

$$\frac{1750}{120} = \frac{120x}{120}$$

$$14.58\overline{3} = x$$

The Lehmans will need 14.59 gallons of gasoline to travel 350 miles.

<u>Example 20:</u>

Ysidra paints a room that has 400 square feet of wall space in $2\frac{1}{2}$ hours. At this rate, how long will it take her to paint a room that has 720 square feet of wall space?

When I'm setting up the ratio for how much time it takes to paint space, it can be square feet to hours or hours to square feet (it doesn't matter which way).

$$\frac{400\ square\ feet}{2\frac{1}{2}\ hours} \text{ or } \frac{2\frac{1}{2}\ hours}{400\ square\ feet}$$

When I set up the proportion, I need to make sure that both ratios are set up the same way.

$$\frac{400\ square\ feet}{2\frac{1}{2}\ hours} = \frac{720\ square\ feet}{x\ hours} \text{ or } \frac{2\frac{1}{2}\ hours}{400\ square\ feet} = \frac{x\ hours}{720\ square\ feet}$$

$$\frac{400\ square\ feet}{2\frac{1}{2}\ hours} = \frac{720\ square\ feet}{x\ hours}$$

Let’s remove the units while we solve.

$$\frac{400}{2\frac{1}{2}} = \frac{720}{x}$$

In order to solve this proportion, we will use cross products.

$$\frac{400}{2\frac{1}{2}} = \frac{720}{x}$$

$$400 \cdot x = 2\frac{1}{2} \cdot 720$$

$$400x = 1800$$

$$\frac{400x}{400} = \frac{1800}{400}$$

$$x = 4.5$$

It will take Ysidra 4.5 hours to paint 720 square feet.